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Effect of Two-Dimensional Multimode Quasimonochromatic Electric Fields on the Shape of Hydrogenic Spectral Lines

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ABSTRACT: The paper presents the effect of a two-dimensional multimode quasimonochromatic electric field on the hydrogenic Ly-alpha line. Analytical results are obtained for the Stark profile of the main line and of the satellites in the form of the one-fold integral. It is demonstrated analytically and by numerical calculations that the Stark profile of the main line and of its satellites has a cusp in its center. This atypical shape of the Stark profile is a counterintuitive result.

Key words: Stark broadening; two-dimensional multimode electric field; hydrogenic spectral lines

1. INTRODUCTION

Studies of the effects of various oscillatory electric fields on hydrogenic spectral lines, as well as applications of these studies to spectroscopic diagnostics of laboratory and astrophysical plasmas have a long history. As early as in 1933, Blochinzew [1] considered the splitting of a model hydrogen line, consisting of just one Stark component, under a *single-mode*, linearly-polarized electric field $\mathbf{E}_0\cos(\omega t)$. He showed that the Stark component splits in satellites separated by $p\omega$ ($p = \pm1, \pm2, \pm3, \ldots$) from the unperturbed frequency ω_0 of the spectral line (here p is the satellite number). Blochinzew's results result [1] was generalized to profiles of real, multicomponent hydrogenic spectral lines in paper [2] (presented later also in book [3], Sect. 3.1).

Theoretical and experimental results in this research area obtained before 1995 were summarized in book [3]. More recent theoretical and experimental results were presented in reviews [4-6].

The overwhelming majority of the theoretical studies was devoted *single-mode* QEFs. There are just two exceptions, to the best of our knowledge. In 1968 Lifshitz [57] analyzed the effect of a one-dimensional multimode QEF on a hydrogenic spectral line. In terms of the quasienergies (introduced by Zeldovich [8] and Ritus [9]), the outcome was zero quasienergies.

In 1973 Kim and Wilhelm [10] studied, in particular, the Stark profile of the hydrogen Ly-alpha line under the three-dimensional multimode QEF. In terms of the quasienergies, the primary result by Kim and Wilhelm [10] was that the quasienergies were zero, just like in the one-dimensional case studies by Lifshitz [7].

In Sect. 4.1.5 of book [3] there was analyzed the effect of a two-dimensional multimode QEF on the hydrogen Ly-alpha line. It was shown that already for the two-dimensional case, the quasienergies are non-zeros. Therefore it was noted that the oversimplified approach by Kim and Wilhelm [10] led to the incorrect result for quasienergies and was therefore not valid (more details are given below).

In the present paper we further develop the results from Sect. 4.1.5 of book [3]. We obtain an expression for the Stark profiles of the satellites and of the main line in the form of a one-fold integral. We show both analytically and by numerical calculations that these Stark profiles have a cusp at the center.

2. ANALYTICAL RESULTS

We study the profile of the hydrogenic Ly-alpha line under the following two-dimensional multimode QEF:

$$\mathbf{E}(t) = E_x(t)\mathbf{e}_x + E_z(t)\mathbf{e}_z,$$

$$E_x(t) = \sum_k E_k \cos(\omega t + \varphi_k) = \rho_x \cos(\omega t + \alpha_x), \quad (1)$$

$$E_x(t) = \Sigma\, E_p \cos(\omega t + \varphi_p) = \rho_z \cos(\omega t + \alpha_z).$$

Here α_x and α_z have uniform distribution within the $(0, 2\pi)$-interval. As for the amplitudes ρ_x and ρ_z, they have the Rayleigh distributions

$$W(\rho_x) = (2\rho_x/\rho_{x0}^2)\exp(-\rho_x^2/\rho_{xo}^2),\ W(\rho_z) = (2\rho_z/\rho_{z0}^2)\exp(-\rho_z^2/\rho_{zo}^2), \quad (2)$$

where ρ_{x0} is mean square of ρ_x and ρ_{zo} is the mean square of ρ_z.

In Sect. 4.1.5 of book [3] it was shown that at a fixed phase difference $\alpha_z - \alpha_x$, the Ly-alpha spectrum has the following form (here and below the atomic units $\hbar = e = m_e$ are used)

$$I_x(\Delta\omega) = \delta(\Delta\omega - K) + \delta(\Delta\omega + K), \qquad I_y(\Delta\omega) = 2\delta(\Delta\omega),$$

$$I_z(\Delta\omega) = \sum_{p=-\infty}^{\infty} \{2J_{2p+1}^2(v_z)\,\delta[\Delta\omega - (2p+1)\omega] + J_{2p}^2(v_z)[\delta(\Delta\omega - 2p\omega - K) + \delta(\Delta\omega - 2p\omega + K)]\}, \quad (3)$$

$$v_z = 3\rho_z/(Z\omega),$$

where

$$K = -(3\rho_x/Z)J_1(v_z)\sin(\alpha_z - \alpha_x). \quad (4)$$

In Eqs. (3) and (4), $J_q(u)$ are the Bessel functions, Z is the nuclear charge.

Thus the multidimensionality of the multimode QEF causes the nonzero quasienergies $\pm K$ in the spectrum. Consequently the results by Kim and Wilhelm [10] are incorrect.*/

The next step is the averaging over the phase difference $\alpha_z - \alpha_x$. Following the example from Sect. 4.1.5 of book [3] (where just one of the five terms in Eq. (3) was averaged over $\alpha_z - \alpha_x$), we obtain the following (there is no need to average $I_y(\Delta\omega) = 2\delta(\Delta\omega)$):

$$< I_x(\Delta\omega) >_{phase} = (2/\pi)\{[3\rho_x J_1(v_z)/Z\,]^2 - (\Delta\omega)^2\}^{-1/2}\theta[3\rho_x|J_1(v_z)|/Z - |\Delta\omega|],$$

$$<I_z(\Delta\omega)> =$$

$$(1/\pi) \sum_{p=-\infty}^{\infty} \{2J_{2p+1}^2(v_z)\ \{[(3\rho_x J_1(v_z)/Z]^2 - [\Delta\omega - (2p+1)\omega]^2\}^{-1/2}\theta[3\rho_x|J_1(v_z)|/Z - |\Delta\omega-(2p+1)\omega|] \quad (5)$$

$$+ 2J_{2p}^2(v_z)[(3\rho_x J_1(v_z)/Z\,]^2 - (\Delta\omega - 2p\omega)^2\}^{-1/2}\theta[(3\rho_x|J_1(v_z)|/Z - |\Delta\omega - 2p\omega|],$$

where $\theta(u)$ is the step-function.

It is seen that the phase-average spectrum consists of the following type of terms (with different statistical weights)

$$S_0(D) = (1/\pi)\{[3\rho_x J_1(v_z)/(Z\omega)\,]^2 - D^2\}^{-1/2}\theta[3\rho_x|J_1(v_z)|/(Z\omega) - |D|], \quad (6)$$

where

$$D = (\Delta\omega)/\omega,\ \text{or}\ D = (\Delta\omega)/\omega - (2p+1),\ \text{or}\ D = (\Delta\omega)/\omega - 2p. \quad (7)$$

While proceeding from Eq. (5) to Eqs. (6), (7), we chose the QEF frequency ω as the unit of the frequency scale.

Now let us integrate S(D) from Eq. (6) over the Rayleigh distribution $W(\rho_x)$ from Eq. (2). This integration can be performed analytically. As a result we obtain

$$<S_0(D)>_{\rho x} = \exp\{-D^2/[3\rho_{x0}J_1(v_z)/(Z\omega)]^2\}/[\pi^{1/2}3\rho_{x0}|J_1(v_z)|/(Z\omega)\ ]. \quad (8)$$

The last step is the averaging of the $<S_0(D)>_{\rho x}$ from Eq. (8) over the Rayleigh distribution $W(\rho_z)$ from Eq. (2):

$$S(D) = <<S_0(D)>_{\rho x}>_{\rho z} = (2/\pi^{1/2}) \int_0^\infty dw\ w\ \exp\{-w^2 - D^2/[aJ_1(bw)]^2\}/[\pi^{1/2}a|J_1(bw)|], \quad (9)$$

where

$$w = \rho_z/\rho_{z0}, \qquad a = 3\rho_{x0}/(Z\omega), \qquad b = 3\rho_{z0}/(Z\omega). \quad (10)$$

The integration in Eq. (9) cannot be performed analytically in the general form. However, under certain conditions it can be performed approximately by the method of the steepest descent.

Namely, let us consider the case where

$$b << 1, \quad ab/2 << |D| << a/(2b). \quad (11)$$

Assume that the argument of the exponential has a maximum at $w = w_0$, such that $bw_0 << 1$. Then the Bessel function in Eq. (9) can be approximated as $J_1(bw) = bw/2$. Then it easy to find that

$$w_0 = [2|D|/(ab)]^{1/2} \quad (12)$$

and the condition $bw_0 << 1$ is indeed satisfied due to the inequality $|D| << a/(2b)$ from Eq. (11).

In the vicinity of $w = w_0$, the argument of the exponential in Eq. (10) can be approximated as

$$w = -4|D|/(ab) - 4(w - w_0)^2. \quad (13)$$

Then, after denoting $u = w - w_0$, the Stark profile S(D) from Eq. (9) can be approximated as follows

$$S(D) = \{4\exp[-4|D|/(ab)]/(\pi^{1/2}ab)\} \int_{-\infty}^{\infty} du\ \exp(-4u^2) = 2\exp[-4|D|/(ab)]/(ab). \quad (14)$$

Thus, from the analytical result of Eq. (14) it is expected the Stark profile S(D) of each satellite (|p| > 0) and of the main line (p = 0) in the double-averaged Eq. (5) has a cusp in its central part. This analytical result is confirmed by numerical calculations for examples presented below.

Figure 1 shows the Stark profile S(D) for a = b = 1. It is seen that the profile indeed has the cusp at the scaled frequency $D = (\Delta\omega)/\omega = 0$.

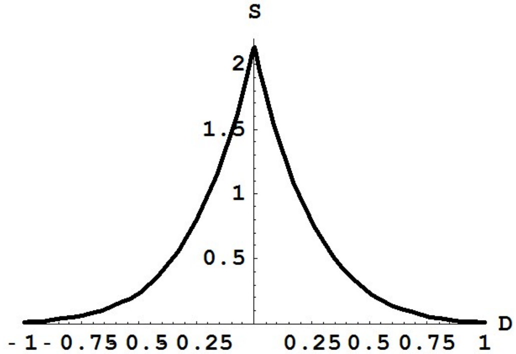

Fig. 1. Stark profile S(D) for a = b = 1, where $D = (\Delta\omega)/\omega$, $a = 3\rho_{x0}/(Z\omega)$, $b = 3\rho_{z0}/(Z\omega)$.

Figure 2 presents the Stark profiles S(D) for the pair a = 1, b = 0.1 (solid line) and for the pair a = 0.1, b = 1 (dashed line). It is seen that both profiles have the cusp at D = 0 and that these two profiles did not differ much from each other and have practically the same width.

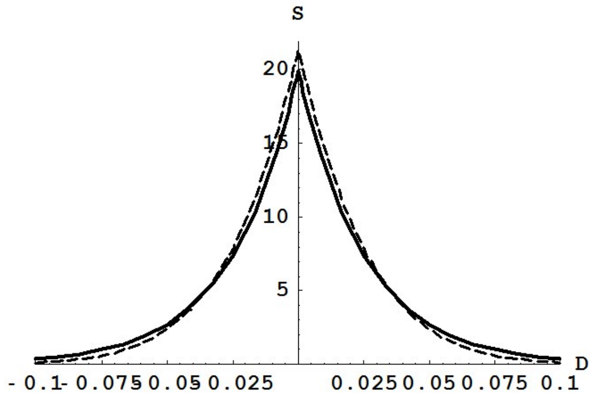

Fig. 2. The same as in Fig. 1, but for a = 1, b = 0.1 (solid line) and for a = 0.1, b = 1 (dashed line).

The cusp-shaped type of the Stark profile S(D) of the main line and of the satellites, obtained analytically for b = $3\rho_{z0}/(Z\omega)$ << 1, seems to be valid practically for any value of the parameter b. This atypical shape of the Stark profile is a *counterintuitive* result.

3. CONCLUSIONS

We studied the profile of the hydrogenic Ly-alpha line under the two-dimensional multimode QEF. We further developed the results from Sect. 4.1.5 of book [3] by averaging over the stochastic amplitudes of the QEF in each of the two dimensions. We obtained the Stark profile S(D) of the main line and of the satellites in the form of the one-fold integral.

In the case of $3\rho_{z0}/(Z\omega)$ << 1, we performed the remaining integration using the method of the steepest descent. The characteristic feature of the resulting analytical shape of the profile was a cusp at the center of the profile (at D = 0).

Alternatively we calculated that integral numerically. By the way of example, we found that the cusp at the center (at D = 0) remains as the characteristic feature of the Stark profiles practically for any value of the dimensionless parameter $3\rho_{z0}/(Z\omega)$. This kind of shape of the Stark profile is a *counterintuitive* result.

Notes

*/ As noted in Sect. 4.1.5 of book [3], the problem involving a three-dimensional multimode QEF can be always reduced to the corresponding two-dimensional problem – see the transition from Eq. (4.1.34) to Eq. (4.1.35) in book [3].

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