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Effect of imposed time periodic boundary temperature on the onset of Rayleigh-Bénard convection in a dielectric couple stress fluid

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ABSTRACT

The effect of imposed time-periodic boundary temperature of small amplitude on electroconvection under AC electric field in dielectric couple stress liquids is investigated by making a linear stability analysis. A regular perturbation method is used to arrive at an expression for the correction Rayleigh number that throws light on the possibility of sub-critical motions. The Venezian approach is adopted for obtaining eigen value of the problem. Three cases of oscillating temperature field are examined: (a) symmetric, so that the wall temperatures are modulated in-phase, (b) asymmetric, corresponding to out-of-phase modulation and (c) only the lower wall is modulated. It is shown that the system is most stable when the boundary temperatures are modulated out-of-phase.

Keywords: Imposed time periodic boundary temperature; electroconvection; small amplitude; dielectric; couple stress fluid.

1. Introduction

The investigation of convective heat transfer with electrical forces in non-Newtonian fluids is of practical importance. A systematic study through a proper theory is essential to understand the physics of the complex flow behavior of these fluids and also to obtain invaluable scaled up information for industrial applications. The study of non-Newtonian fluids has attracted much attention, because of their practical applications in engineering and industry particularly in extraction of crude oil from petroleum products.

The application of a strong electric field in a poorly conducting fluid can induce bulk motions. This phenomenon known as electro-convection or electro-hydrodynamics is gaining importance due to the technological stimulus of designing more efficient heat exchangers as required for jet engines. Electro-hydrodynamics convection is very attractive in applications to new field (see [1]). Since magnetic field and switching circuits are not required the dielectric fluid motor enhances size reduction and hence is an attractive source of mechanical energy in a micro machine. The convective heat transfers through polarized dielectric liquids were studied by [2-10]. In all the above studies uniform temperature gradient has been considered.

However, we find that in many practically important situations the temperature gradient is a function of both space and time. There are many works available in the literature, concerning how a time-periodic boundary temperature affects the onset of Rayleigh-Bénard convection. [11] was the first to study the effect of temperature modulation on the onset of thermal instability in a Newtonian fluid layer for small amplitude. He derived the onset criteria using a perturbation expansion in powers of the amplitude of oscillations. He has established that the onset of convection can be delayed or advanced by out-of-phase or in phase modulation of the boundary

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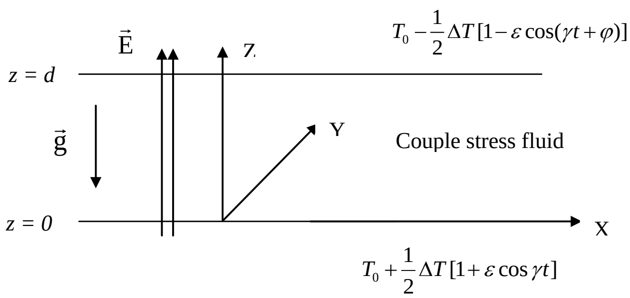

Figure 1: Physical Configuration

temperatures respectively, as compared to the unmodulated system. Later [12-21] have studied the effect of temperature modulation on the onset of thermal convection in a horizontal fluid layer**.** [22] have discussed the thermal instability in a layer of dielectric fluid when the boundaries of the layer are subjected to synchronous / asynchronous time – periodic temperatures.

An important class of fluid differs from that of Newtonian fluids, in that the relationship between the shear stress and flow field is more complicated. Such fluids are non-Newtonian. In the category of non-Newtonian fluids, couple stress fluid has distinct features, such as polar effects in addition to possessing large viscosity. The consideration of couple stress in addition to classical Cauchy stress, has led to the recent development of several theories of fluid micro-continua. One such couple stress theory of fluids was developed by [23] and represents the simplest generalization of the classical theory which allows for polar effects such as the presence of couple stresses and body couples. Couple stress is the consequence of assuming that mechanical action of one part of a body on another across a surface is equivalent to a force and moment distribution. In the classical non-polar theory, moment distribution is not considered and mechanical action is assumed to be equivalent to the force distribution only. The first work in couple stress fluid for Rayliegh – Bénard situation was reported by Siddheshwar and Pranesh [24].

Therefore, main object of the present investigation is to study the effect of imposed temperature modulation on the stability of convective flow in a couple stress dielectric liquid by considering free-free boundaries.

2. Mathematical Formulation

Consider a layer of Boussinesquian, dielectric couple stress fluid confined between two infinite horizontal surfaces separated by a distance d apart. The uniform AC electric field is directed along the z-axis. A Cartesian system is taken with origin in the lower boundary and z-axis vertically upward (figure 1).

The governing equations are:

Continuity Equation:

$\nabla \cdot \vec{q} = 0,$ (1)

Conservation of Linear Momentum:

$$\rho_0\left(\frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} - \mu'\nabla^4\vec{q} + \rho\vec{g} + (\vec{P}\cdot\nabla)\vec{E}, \tag{2.1}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)\, T = \chi\nabla^2 T, \tag{2.2}$$

Equation of State:

$$\rho = \rho_o \left(1 - \alpha \left(T - T_0\right)\right)_, \tag{2.3}$$

Electrical Equation:

$$\nabla \times \vec{E} = 0, \tag{2.4}$$

$$\nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = 0\,, \quad \vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}, \tag{2.5}$$

Equation of state for dielectric constant:

$$\varepsilon_r = (1 + \chi_e) - e\left(T - T_0\right), \tag{2.6}$$

The wall temperatures are time dependent, externally imposed and are taken as

$$T(0,t) = T_0 + \frac{1}{2}\Delta T\left[1 + \varepsilon\cos\gamma t\right], \tag{2.7}$$

$$T(d,t) = T_0 - \frac{1}{2}\Delta T\left[1 - \varepsilon\cos(\gamma t + \varphi)\right]. \tag{2.8}$$

We consider three types of thermal modulation namely:
Case (a): Symmetric(in-phase, $\varphi = 0$) ,
Case (b): Asymmetric (out-of-phase,$\varphi = \pi$) and
Case (c):Only lower wall temperature is modulated while the upper wall is held at constant temperature ($\varphi = -i\infty$).

2.1 *Basic State:*

The basic state of the fluid is quiescent and is described by

$$\vec{q_b} = \vec{0}\,, \rho = \rho_b\left(z,t\right)\,,\, T = T_b\left(z,t\right)\,,\; p = p_b\left(z,t\right)\,,\, \vec{E} = \vec{E}_b(z), \vec{P} = \vec{P}_b(z). \tag{2.9}$$

Substituting equation (10) into basic governing equations(1)-(7), we obtain the quiescent state solutions as:

$$\frac{\partial p_b}{\partial z} = \rho_b g + P_b\frac{\partial E_b}{\partial z}, \tag{2.10}$$

$$\frac{\partial T_b}{\partial t} = \chi\frac{\partial^2 T_b}{\partial z^2} \quad , \tag{2.11}$$

$$\left.\begin{array}{l} \rho_b = \rho_0\left(1 - \alpha\left(T_b - T_0\right)\right) \\ \varepsilon_r = (1+\chi_e) - e\left(T_b - T_0\right) \\ \vec{E}_b = \left[\frac{(1+\chi_e)E_0}{(1+\chi_e)+\frac{e\Delta T}{d}z}\right]\hat{k} \\ \vec{P}_b = \varepsilon_0 E_0(1+\chi_e)\left[1 - \frac{1}{(1+\chi_e)+\frac{e\Delta T}{d}z}\right]\hat{k} \end{array}\right\}. \tag{2.12}$$

The solution of equation (12) that satisfies the thermal boundary conditions (8) and (9) is

$$T_b = T_0 + \frac{\Delta T}{2}\left(1 - \frac{2z}{d}\right) + \varepsilon\, Re \quad \left\{\left[a(\lambda)\, e^{\frac{\lambda z}{d}} + a(-\lambda)\, e^{\frac{-\lambda z}{d}}\right] e^{-i\gamma t}\right\}, \tag{2.13}$$

where $\lambda = (1-i)\left(\frac{\gamma d^2}{2\chi}\right)^{\frac{1}{2}}$, (15)

$$a(\lambda) = \frac{\Delta T}{2}\left[\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right]$$

and *Re* stands for the real part.

We now superpose infinitesimal perturbations on this basic state and study the stability of the system.

2.2 *Linear Stability Analysis:*

Let the basic state be perturbed by an infinitesimal thermal perturbation so that

$$\vec{q} = \vec{q}_b + \vec{q}\,', \;\; \rho = \rho_b + \rho', p = p_b + p', T = T_b + T', \vec{P} = \vec{P}_b + \vec{P}', \;\; \vec{E} = \vec{E}_b + \vec{E}', \tag{2.14}$$

where the prime indicates that the quantities are infinitesimal perturbations. Let the components of perturbed polarization and electric field be $(P_1', P_2', P_b(z) + P_3')$and $(E_1', E_2', E_b(z) + E_3')$.

The second equation of (6), on linearization yields

$$\begin{array}{l} P_i' = \varepsilon_0 \chi_e E_i' \quad for\ i = 1, 2 \\ P_3' = \varepsilon_0 \chi_e E_3' - e\varepsilon_0 E_0 T' \end{array} \tag{2.15}$$

where it has been assumed that $e\Delta T << (1 + \chi_e)$.

Equation (5) implies one can write $\vec{E}' = \nabla\phi'$.

Substituting equation (16) into equations (1)-(7) and using the basic state equations, we get linearized equations governing the infinitesimal perturbations in the form:

$$\nabla \cdot \vec{q}\,' = 0, \tag{2.16}$$

$$\rho_0\left[\frac{\partial \vec{q}\,'}{\partial t}\right] = -\nabla p' + \mu\, \nabla^2 \vec{q}\,' - \mu'\, \nabla^4 \vec{q}\,' - \rho' g\hat{k} + \vec{P}_b \cdot \nabla \vec{E}' + \vec{P}' \cdot \nabla \vec{E}_b, \tag{2.17}$$

$$\frac{\partial T'}{\partial t} + w'\frac{\partial T_b}{\partial z} = \chi \nabla^2 T', \tag{2.18}$$

$$\rho' = -\alpha \rho_0 T', \tag{2.19}$$

$$\varepsilon' = -\varepsilon_o e T', \tag{2.20}$$

$$\nabla \cdot \left(\varepsilon_0 \vec{E}' + \vec{P}'\right) = 0. \tag{2.21}$$

Introducing the electric potential ϕ', eliminating the pressure p in equation (19) and incorporating the quiescent state solution, we obtain the perturbed state vorticity transport equation in the form:

$$\rho_0\left[\frac{\partial}{\partial t}\left(\nabla^2 w'\right)\right]+\alpha\rho_0 gT'-\frac{\varepsilon_0 e^2E_0^2}{1+\chi_e}\left(\frac{\partial T_b}{\partial z}\right)\nabla_1^2 T'$$
$$+\varepsilon_0 eE_0\left(\frac{\partial T_b}{\partial z}\right)\nabla_1^2\left(D\phi'\right)=\mu\nabla^4 w'-\mu'\,\nabla^6 w'. \tag{2.22}$$

Using equation (17) on equation (23), we obtain :

$$\left(1+\chi_e\right)\nabla^2\phi'-eE_0DT'=0. \tag{2.23}$$

The perturbation equations (20),(24) and (25) are non-dimensionalized using the following definitions:

$$(x^*,y^*,z^*)=\left(\frac{x'}{d},\frac{y'}{d},\frac{z'}{d}\right),\quad t^*=\frac{t'}{d^2/\chi},\quad w^*=\frac{w'}{\chi/d},\quad T^*=\frac{T'}{\Delta T},\ \phi^*=\frac{\phi'}{\frac{eE_0d\Delta T}{1+\chi_e}}\quad . \tag{2.24}$$

to obtain (after dropping the asterisk)

$$\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\left(\nabla^2 w\right)-L\nabla_1^2 T+L\frac{\partial(\nabla_1^2\phi)}{\partial z}=R\nabla_1^2 T+\nabla^4 w-C\nabla^6 w, \tag{2.25}$$

$$\frac{\partial T}{\partial t}+w\frac{\partial T_0}{\partial z}=\nabla^2 T, \tag{2.26}$$

$$\nabla^2\phi-\frac{\partial T}{\partial z}=0\quad . \tag{2.27}$$

The non-dimensional parameters Pr,L, $R$and C are given as
$\mathrm{Pr}=\dfrac{\mu}{\rho_0\chi}$ (Prandtl number),
$L=\dfrac{\varepsilon_0\left(eE_0\Delta Td\right)^2}{(1+\chi_e)\mu\chi}$ (Electric Rayleigh number),
$R=\dfrac{\rho_0\alpha gd^3\Delta T}{\mu\chi}$ (Rayleigh number),
$C=\dfrac{\mu'}{d^2\mu}$ (Couple stress parameter).
In equation (28), $\left(\dfrac{\partial T_0}{\partial z}\right)$is the non-dimensional form of$\left(\dfrac{\partial T_b}{\partial z}\right)$, where

$$\frac{\partial T_0}{\partial z}=-1+\varepsilon f(z), \tag{2.28}$$

$$f(z)=Re\left\{\left[A(\lambda)\,e^{\lambda z}+A(-\lambda)\,e^{-\lambda z}\right]e^{-i\gamma t}\right\}, \tag{2.29}$$

and $A(\lambda)=\dfrac{\lambda}{2}\left[\dfrac{e^{-i\varphi}-e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}\right]$. (32)
Equations (27) to (29) are solved subject to the conditions:

$w = \frac{\partial^2 w}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0 \mathrm{at} z = 0\,,\ \ 1$ (33)
Eliminating T and ϕ from equations (27)-(29), we get a differential equation of order 10 for w in the form:

$$\left\{ \left(\frac{\partial}{\partial t} - \nabla^2 \right) \left[\frac{1}{\mathrm{Pr}} \frac{\partial}{\partial t} - \nabla^2 + C\nabla^4 \right] \nabla^4 + L \frac{\partial T_0}{\partial z} \nabla_1^4 + R\nabla^2 \frac{\partial T_0}{\partial z} \nabla_1^2 \right\} w = 0. \tag{2.30}$$

In dimensionless form, the velocity boundary conditions for solving equation (34)are obtained from equations (27) to (29) and (33) in the form:
$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^4 w}{\partial z^4} = \frac{\partial^6 w}{\partial z^6} = \frac{\partial^8 w}{\partial z^8} = 0.\ \mathrm{at} z = 0,\ \ 1$ (35)

3. Method of Solution

We now seek the eigen-function w and eigen-values R of the equation (34) for the basic temperature distribution (30) that departs from the linear profile $\frac{\partial T_0}{\partial z} = -1$ by quantities of order ε. Thus, the eigen-values of the present problem differ from those of the ordinary Bénard convection by quantities of orderε. We seek the solution of equation (34) in the form:

$$(R, w) = (R_0, w_0) + \varepsilon\,(R_1, w_1) + \varepsilon^2\,(R_2, w_2) + ... \quad . \tag{3.1}$$

The expansion (36) is substituted into equation (34) and the coefficients of various powers of ε are equated on either side of the equation. The resulting system of equation is

$$L_1 w_0 = 0, \tag{3.2}$$

$$L_1 w_1 = \left[-Lf\nabla_1^2 + (R_1 - R_0 f)\,\nabla^2 \right] \nabla_1^2 w_0, \tag{3.3}$$

$$L_1 w_2 = -L\,f\,\nabla_1^4 w_1 - f\,R_0 \nabla^2\,\nabla_1^2\,w_1 +\ R_1 \nabla^2\,\nabla_1^2\,w_1 - f\,R_1\,\nabla^2\,\nabla_1^2 w_0 + R_2\,\nabla^2\,\nabla_1^2\,w_0, \tag{3.4}$$

where

$$L_1 = \left(\frac{\partial}{\partial t} - \nabla^2 \right) \left[\frac{1}{\mathrm{Pr}} \frac{\partial}{\partial t} - \nabla^2 + c\,\nabla^4 \right] \nabla^4 - L\,\nabla_1^4 - R_0 \nabla^2\,\nabla_1^2. \tag{3.5}$$

3.1 *Solution To The Zeroth Order Problem*

The zeroth order problem is equivalent to the Rayleigh-Bénard problem of couple stress fluid with electric field in the absence of temperature modulation. The linear analysis of Rayleigh-Bénard convection in couple stress fluid without electric field has been thoroughly investigated by [26]. The stability of the system in the absence of thermal modulation is investigated by introducing vertical velocity perturbation $w_0$corresponding to lowest mode of convection as:

$$w_0 = Sin\,(\pi z) \exp\left[i(lx + my) \right] \quad . \tag{3.6}$$

Substituting equation (41) into equation (37) we obtain the expression for Rayleigh number in the form

$$R_0 = \frac{(K_1^2)^3 (1 + C\,K_1^2)}{a^2} - \frac{L\,a^2}{K_1^2}. \tag{3.7}$$

3.2 *Solution To The First Order Problem*

Equation (38) for $w_1$now takes the form

$$L_1 w_1 = \left[R_1 a^2 K_1^2 - f R_0 a^2\, K_1^2 - L f a^4\right] w_0. \tag{3.8}$$

If the above equation is to have a solution, the right hand side must be orthogonal to the null-space of the operator L_1. This implies that the time independent part of the RHS of the equation (43) must be orthogonal to$\sin(\pi z)$. Since f varies sinusoidal with time, the only steady term on the RHS of equation (43) is$R_1 a^2 K_1^2 \sin(\pi z)$, so that$R_1 = 0$. It follows that all the odd coefficients i.e. $R_1 = R_3 = ... = 0$ in equation (36).

To solve equation (43), we expand the right-hand side using Fourier series expansion and obtain w_1 by inverting the operator L_1 term by term as:

$$w_1 = \left\{-R_0 a^2 K_1^2 - L a^4\right\} Re \left\{\sum \frac{B_n\left(\lambda\right)}{L_1\left(\gamma, n\right)} e^{-i\,\gamma\, t} \sin\left(n\pi z\right)\right\}, \tag{3.9}$$

where $B_n\left(\lambda\right) = A\left(\lambda\right) g_{n1}\left(\lambda\right) + A\left(-\lambda\right) g_{n1}\left(-\lambda\right)$

$$= \frac{2n\pi^2\lambda^2[e^{-\lambda} - e^{\lambda} + (-1)^n(e^{-\lambda - i\,\varphi} - e^{\lambda - i\,\varphi})]}{[e^{\lambda} - e^{-\lambda}][\lambda^2 + (n+1)^2\pi^2][\lambda^2 + (n-1)^2\pi^2]}, \tag{3.10}$$

$$L_1\left(\gamma, n\right) = \left(-\frac{\gamma^2}{\Pr} + K_n^2 X_1\right) X_2 - L a^4 - R_0 a^2 K_n^2 \, - i\,\gamma \left[X_1 + \frac{K_n^2}{\Pr}\right] X_2,$$

$$X_1 = (K_n^2) + c(K_n^2)^2,$$

$$X_2 = (K_n^2)^2,$$

and$K_n^2 = n^2\pi^2 + a^2$ (see [18,19]).

The equation forw_2,then becomes

$$L_1 w_2 = R_2 a^2 K_1^2 w_0 - a^2 f \left\{L a^2 + R_0 K_n^2\right\} w_1, \tag{3.11}$$

We shall not solve equation (46), but will use this to determineR_2. The solvability condition requires that the time-independent part of the right hand side of equation (46) must be orthogonal to $\sin\left(n\pi z\right)$, and this results in the following equation,

$$R_2 = \left(\frac{-R_0 K_1^2 - L a^2}{2K_1^2}\right) \sum \left[\left(L a^4 + R_0 a^2 K_n^2\right) \frac{|B_n\left(\lambda\right)|^2}{|L_1\left(\gamma, n\right)|^2} \left[\frac{L_1\left(\gamma, n\right) + L_1^*\left(\gamma, n\right)}{2}\right]\right] \quad , \tag{3.12}$$

where$L_1^*(\gamma, n)$ is the conjugate of$L_1\left(\gamma, n\right)$ respectively.

4. MINIMUM RAYLEIGH NUMBER FOR CONVECTION:

The value of Rayleigh number R obtained by this procedure is the eigenvalue corresponding to the eigen function w,which, though oscillating, remains bounded in time. Since R is a function of the horizontal wave number a and the amplitude of modulation ε, we have

$$R(a, \varepsilon) = R_0(a) + \varepsilon^2 R_2(a) + ... \tag{4.1}$$

It was shown by [11]that the critical value of thermal Rayleigh number is computed up to $O(\varepsilon^2)$, by evaluating R_0 and R_2 at $a = a_0$.It is only when one wishes to evaluate R_4 that $a_2$must be taken into account where $a = a_2$

minimizes R_2. To evaluate the critical value of R_2 (denoted byR_{2c}) one has to substitute $a = a_0$ in R_2, where a_0 is the value at which R_0 given by equation (42) is minimum.

We now evaluate R_{2c} for three cases:

Case (a): When the oscillating field is symmetric so that the wall temperatures are modulated in-phase with$\varphi = 0$. In this case, $B_n(\lambda) = b_n$ or 0, accordingly as n is even or odd.

Case (b): When the wall temperature field is antisymmetric corresponding to out-of-phase modulation with $\varphi = \pi$. In this case, $B_n(\lambda) = 0$ or b_n,accordingly as n is even or odd.

Case (c): When only the temperature of the bottom wall is modulated, the upper plate being held at constant temperature, with$\varphi = -i\infty$. In this case,$B_n(\lambda) = \dfrac{b_n}{2}$, for integer values of n.

where

$$b_n = \frac{-4n\pi^2\lambda^2}{[\lambda^2 + (1+n)^2\pi^2][\lambda^2 + (1-n)^2\pi^2]} \quad .$$

The variable λ defined in equation (15), in terms of the dimensionless frequency, reduces to

$$\lambda = (1-i)\left(\frac{\gamma}{2}\right)^{\frac{1}{2}}$$

and thus

$$|b_n|^2 = \frac{16n^2\pi^4\gamma^2}{[\gamma^2 + (1+n)^4\pi^4][\gamma^2 + (1-n)^4\pi^4]}.$$

Hence from equation (47) and using the above expression of$B_n(\lambda)$, we can obtain the following expression for R_{2c} in the form:

$$R_{2c} = \left(\frac{-R_0K_1^2 - La^2}{2K_1^2}\right)\sum\left[\left(La^4 + R_0a^2K_n^2\right)\frac{|b_n|^2}{|L_1(\gamma,n)|^2}\left[\frac{L_1(\gamma,n) + L_1^*(\gamma,n)}{2}\right]\right]. \quad (4.2)$$

In equation (49) the summation extends over even values of n for case (a), odd values of n for case (b) and for all values of n for case (c). The infinite series (49) converges rapidly in all cases. The variation of $R_{2c}$with γ for different values of C and L are depicted in figures 2-7.

5. Subcritical Instability

The critical value of Rayleigh number R_c is determine to be of order ε^2, by evaluating R_{oc} and R_{2c}, and is of the form

$$R_c = R_{oc} + \varepsilon^2 R_{2c} \quad (5.1)$$

where R_{oc} and R_{2c} can be obtained from equations (42) and (49) respectively.

If R_{2c} is positive, super critical instability exists and R_c has a minimum at $\varepsilon = 0$. When R_{2c} is negative, sub critical instabilities are possible. In this case from equation (48) we have

$$\varepsilon^2 < \frac{R_{oc}}{R_{2c}}. \quad (5.2)$$

Now, we can calculate the maximum range of ε, by assigning values to the physical parameters involved in the above condition. Thus, the range of the amplitude of modulation, which causes sub critical instabilities in different physical situations, can be explained.

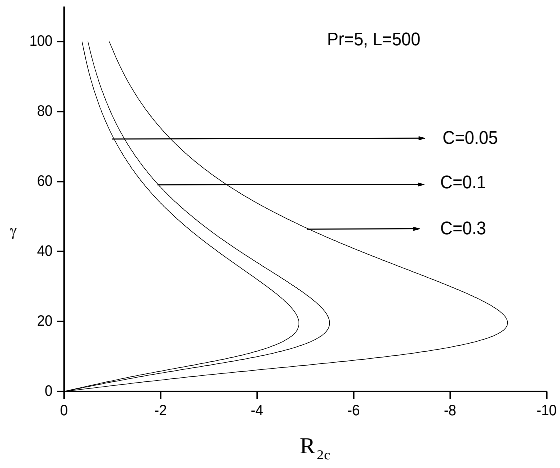

Figure 2: The plot of R_{2c} versus frequency of modulation γ for in-phase temperature modulation for different values of couple stress parameter C.

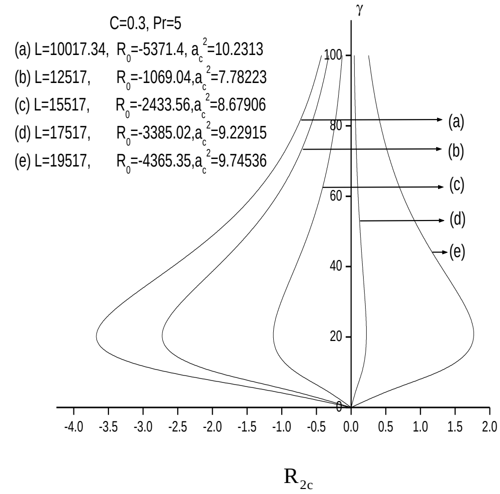

Figure 3: The plot of R_{2c} versus frequency of modulation γ for in-phase temperature modulation for different values of Electric Rayleigh number L.

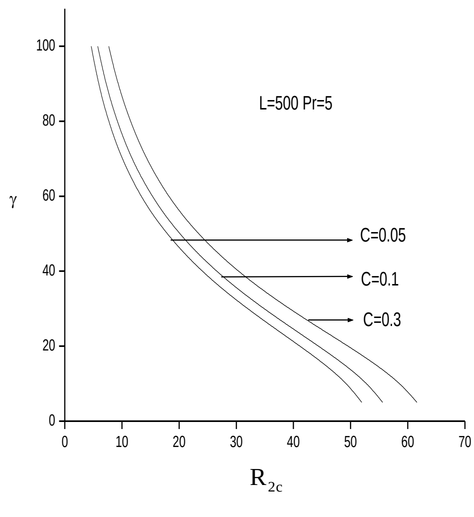

Figure 4: The plot of R_{2c} versus frequency of modulation γ for out-of-phase modulation for different values of couple stress parameter C.

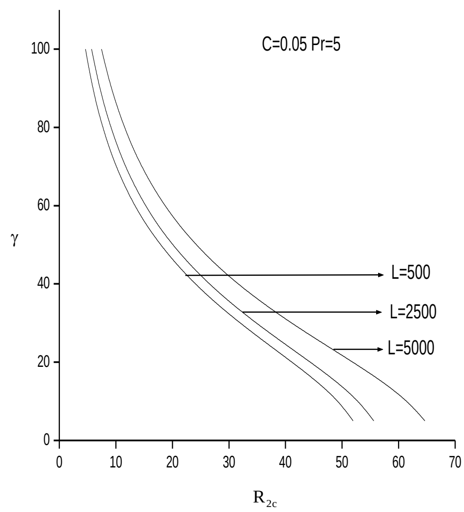

Figure 5: The plot of R_{2c} versus frequency of modulation γ for out-of-phase temperature modulation for different values of Electric Rayleigh number L.

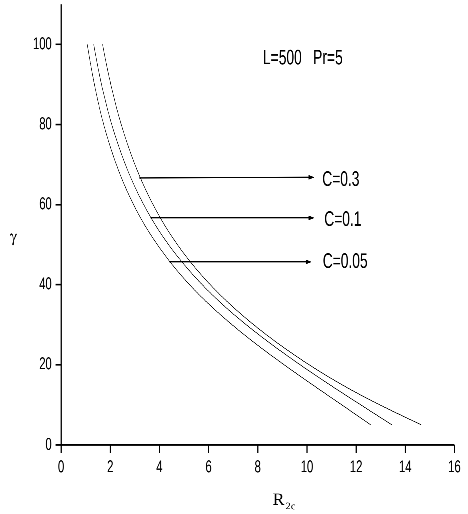

Figure 6: The plot of R_{2c} versus frequency of modulation γ for bottom wall modulation for different values of couple stress parameter C.

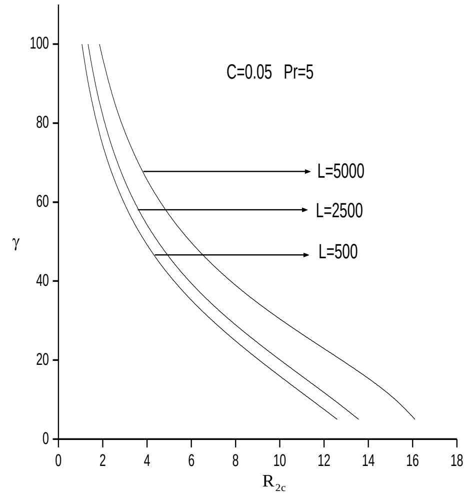

Figure 7: The plot of R_{2c} versus frequency of modulation γ for bottom wall temperature modulation for different values of Electric Rayleigh number L.

6. Results and Discussion

The effect of thermal modulation on the onset of convection in a horizontal dielectric couple stress fluid is examined using linear stability analysis. The expression for the critical correction Rayleigh number $R_{2c}$is computed as function of the frequency of the modulation for different parameters. The value ofR_{2c}has been calculated in the following three cases; (a) when the walls' temperature is modulated in- phase i.e., $\varphi = 0$, (b) when the modulation is out-of-phase, i.e., $\varphi = \pi$and (c) when only the lower wall temperature is modulated, the upper wall is held at constant temperature, i.e.,$\varphi = -i\infty$.

The analysis presented is based on the assumption that the amplitude of the modulating temperature is small compared with the imposed steady temperature difference. The validity of the results obtained here depends on the value of the modulating frequency γ. When $\gamma << 1$, the period of modulation is large and hence the disturbance grows to such an extent as to make finite amplitude effects important. When$\gamma \to \infty$, $R_{2c} \to 0$, thus the effect of modulation becomes small. In view of this, we choose only moderate values of γ in our present study.

The results have been presented in figures 2-7. From the figures we observe that the value of R_{2c} may be positive or negative. The sign of the correction Rayleigh number characterizes the stabilizing or destabilizing effect of modulation on R_{2c}. A positiveR_{2c}means the modulation effect is stabilizing while a negativeR_{2c}means the modulation effect is destabilizing compared to the system in which the modulation is absent.

The effect of in-phase modulation of wall temperature on the onset of convection in a horizontal layer of couple stress fluid is shown in figures 2 – 3. We find that for low frequency γ, R_{2c} becomes more and more negative indicating that the in-phase modulation for low frequency is destabilizing and for moderate values of γ, R_{2c} becomes less and less negative indicating that the in phase modulation for moderate frequency is stabilizing. Let γ_c be the frequency at which the R_{2c}changes from destabilizing to stabilizing, then the modulated system may be classified as destabilized or stabilized according as$\gamma < \gamma_c$ or $\gamma > \gamma_c$ when compared with unmodulated system. For some particular value of γ, R_{2c} becomes zero. This is due to the fact that when the frequency of modulation is low, the effect of modulation on the temperature field is felt throughout the fluid layer. If the plates are modulated in-phase, the temperature profile consists of the steady straight line section plus a parabolic profile which oscillates in time. As the amplitude of modulation increases, the parabolic part of the profile becomes more and more significant. It is known that a parabolic profile is subject to finite amplitude instabilities so that convection occurs at lower Rayleigh number than those predicted by the linear theory.

Figure (2) is the plot of R_{2c} versus γ for different values of couple stress parameter C, in the case of in-phase modulation. In the figure we observe that as C increases R_{2c}becomes more and more negative. C is indicative of the concentration of the suspended particles. The physical reason for the nature of effect of C on R_{2c} can be given by invoking the Einstein law on viscosity of suspensions which states that the viscosity of the carrier liquid is enhanced by a factor of 2.5ϕ($\phi$is the concentration of particles in the liquid) by adding suspended particles.

Figure (3) is the plot of R_{2c}versus γ for different values of electric Rayleigh number L with respect to in-phase modulation. The electric Rayleigh number$L$is the ratio of electric to gravitational forces. We see from the figure that when L is greater than 17215super critical motion occurs and R_{2c} increases with an increase in L at a given frequencyγ.Hence L has a stabilizing effect on the flow. When L is less than 17215 subcritical motion occurs. It is also interesting to see from the figure that for a given $L(L <$17215),R_{2c}first decreases with increase in γ,reaches a minimum and then increases with increase in γ and for a given L ($L >$17215) R_{2c}increases with increase in γ reaches the maximum and then decreases with increase in γ. This shows that for a weakly dielectric fluid, the flow is destabilized for small values of γ and stabilized for large values of γ. This is due to the fact that when the frequency of modulation is low, the effect of modulation is felt throughout the fluid.

The effect of out-of-phase modulation on the wall temperature on the onset of convection is shown in figures (4) – (5). We find that in general the effect is to stabilize the system. Thus C and L have opposing influences in in-phase and out-of-phase modulations. The above results are due to the fact that in the case of out-of-phase modulation the temperature field has essentially a linear gradient varying in time, so that the instantaneous Rayleigh number is super-critical for half a cycle and sub-critical during the other half cycle. Therefore, in general, sub-critical motions are ruled out in the case of out-of-phase modulation. The above results on the effect of various parameters on R_{2c} for out-of-phase modulation do not qualitatively change in the case of temperature modulation of just the lower boundary. This is illustrated with the help of figures (6) and (7).

7. CONCLUSION

From the study we conclude that:

Nomenclature			
d	depth of the fluid	T	time
$\vec{g}$	acceleration due to gravity	$\vec{q}$	velocity
l,m	wave numbers in xy plane with $a^2 = l^2 + m^2$	ρ_0	density of the fluid at temperature T=T_0
p	Pressure	μ	coefficient of viscosity
μ'	couple stress viscosity	ρ	Density
$\vec{E}$	AC electric field	$\vec{P}$	dielectric polarization field
T	Temperature	χ	Thermal diffusivity
$\left(\vec{P} \cdot \nabla\right) \vec{E}$	represents a polarization force called the dielectrophoretic force	ε_0	electric permittivity of free space
ε_r	dielectric constant	χ_e	electric susceptibility
$e = -\left(\dfrac{\partial \varepsilon_r}{\partial T}\right)_{T=T_0}$		E	Amplitude
α	Coefficient of thermal expansion	***Other symbols***	

ΔT	difference in temperature of the fluid between lower and upper plates	$\nabla^2 = \nabla_1^2 + D^2$, $\nabla_1^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$, $D = \frac{\partial}{\partial z}$, $K_1^2 = \pi^2 + a^2$	
γ	Frequency	***Subscripts***	
φ	phase angle	B	basic state
ϕ'	perturbed electric scalar potential	C	Critical
E_0	root mean square value of the electric field at the lower surface		

1. For in phase modulation, it is found that, when γ is small, the modulation effect is destabilizing and when γ is moderate modulating effect is stabilizing.
2. When boundary temperature is modulated out-of-phase or only lower wall is modulated, the system is more stable than that for in-phase modulation.
3. In-phase temperature modulation leads to sub-critical motions.
4. The effect of temperature modulation is found to stabilize or destabilize the system depending on the values of the parameters.
5. The effect of modulation disappears for large values of γ.
6. The problem throws light on an external means of controlling convection in dielectric couple stress fluid which is quite important from the application point of view.

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