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A single unit system with preventive maintenance and repair subject to maximum operation and repair times

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ABSTRACT

The purpose of the present study is to determine reliability measures of a single-unit system considering arbitrary distributions for the random variables associated with failure time, preventive maintenance and repair times, maximum operation time, maximum repair time and replacement time. The system fails completely either directly from normal mode or via partial failure. There is a single server who visits the system immediately to do repair activities. The partially failed operating unit is shutdown after pre-specific time (called maximum operation time) for preventive maintenance. However, repair of the unit is done after its complete failure. The completely failed unit is replaced by new one if server is unable to do its repair within a maximum repair time. The unit works as new after preventive maintenance and repair. The expressions for some important measures of system effectiveness are derived using semi-Markov process and regenerative point technique. The numerical results for a particular case are obtained to depict the behavior of MTSF, availability and profit function with respect to maximum operation time for fixed values of other parameters and costs.

Keywords: Single-unit System; Preventive Maintenance; Repair; Maximum Operation Time; Maximum Repair Time; Replacement Time and Reliability Measures

1. Introduction

In the past few years several research papers including Chander and Bansal [1] have been appeared on the reliability modeling of single-unit systems because of their practical importance and frequent use in day to day activities. And, in most of these papers, it is assumed that system has a constant failure rate and it can work for a long time without any maintenance. But hazard rates of many systems like rotating shaft and valves are of linearly increased nature due to wear out under mechanical stress and so their failure time may follow arbitrary distributions.

On the other hand, continuous operation and ageing of systems gradually reduce their performance and thus reliability. It is proved that preventive maintenance can show the deterioration process of a repairable system and restore the system in a younger age or state. Therefore, method of preventive maintenance can be used at any stage of operation of the system for improving its reliability. Malik et al. [3] obtained reliability measures of a single-unit system conducting preventive maintenance after a maximum operation time. Further, down time of a system can be reduced by making replacement of a failed unit if its repair is not possible by the server in a pre-specific time (called maximum repair time). Kumar and Malik [2] analyzed a computer system with the concept of maximum operation and repair times.

While considering above facts and practical situations in mind, here a single-unit system is investigated by taking arbitrary distributions for the random variables associated with failure time, preventive maintenance and repair times, maximum operation time, maximum repair time and replacement time. The system fails completely either

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directly from normal mode or via partial failure. There is a single server who visits the system immediately to do repair activities. The partially failed operating unit is shutdown after pre-specific time (called maximum operation time) for preventive maintenance. However, repair of the unit is done after its complete failure. The completely failed unit is replaced by new one if server is unable to do its repair within a maximum repair time. The unit works as new after preventive maintenance and repair. All random variables are statistically independent. The expressions for some important measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), reliability, availability, busy period of the server due to preventive maintenance, repair and replacement, expected number of repairs, replacements and preventive maintenances, expected number of visits and profit function are derived using semi-Markov process and regenerative point technique. The numerical results for a particular case are obtained to depict the behavior of MTSF, availability and profit function with respect to maximum operation time for fixed values of other parameters and costs. The applications of the present work can be visualized in the system of electric transformers.

2. Notation

E	Set of regenerative states
O	The unit is operative and in normal mode
PFO	The unit is partially failed and operative
$PFPm$	The unit is partially failed and under preventive
FUr	The unit is failed and under repair
$FUrp$	The unit is failed and under replacement
$f(t)$, $F(t)$	Probability density function (p.d.f.), cumulative distribution function (c.d.f.) of the failure time from normal mode to partial failure
$f_1(t)$, $F_1(t)$	p.d.f., c.d.f. of the failure time from normal mode to complete failure
$f_3(t)$, $F_3(t)$	p.d.f., c.d.f. of the failure time from partial failure to complete failure
$f_2(t)$, $F_2(t)$	p.d.f., c.d.f. of the preventive maintenance time of the unit
$g(t)$, $G(t)$	p.d.f., c.d.f. of the repair time of a failed unit
$z(t)$, $Z(t)$	p.d.f., c.d.f. of maximum operation time after partial failure
$m(t)$, $M(t)$	p.d.f., c.d.f. of maximum repair time after complete failure
$h(t)$, $H(t)$	p.d.f., c.d.f. of the replacement time of the unit
$*$	Laplace transforms
©	Convolution

$E_0(t) = \overline{F(t)F_1(t)}$ $\quad E_1(t) = \overline{Z(t)F_3(t)}$ $\quad E_2(t) = \overline{F_3(t)Z(t)}$

$E_3(t) = Z(t)\overline{F_3(t)}$ $\quad E_4(t) = Z(t)\overline{G(t)}$ $\quad E_5(t) = G(t)\overline{M(t)}$

The system may be in one of the following states:
Up states $S_0(O)$, $S_1(PFO)$, $S_3(PFPm)$ Down states $S_2(FUr)$, $S_4(FUrp)$.
Possible transitions between states along with cumulative distribution functions time are shown in Table 1.

Table 1

From	S_0	S_1	S_2	S_3	S_4
S_0	-	$F(t)$	$F_1(t)$	-	-
S_1	-	-	$F_3(t)$	$Z(t)$	-
S_2	$G(t)$	-	-	-	$M(t)$
S_3	$F_2(t)$	-	-	-	-
S_4	$H(t)$	-	-	-	-

3. Reliability Analysis

Let $R_i(t)$ be the probability that the system survives during $(0,t) \mid E_0(t) = S_i$. To determine reliability we regard the failed states as absorbing state. The equations determining the reliability of the system are as follows:

$$\begin{aligned} &R_0(t) = E_0(t) + F_1(t)©R_1(t) \\ &R_1(t) = E_1(t) \end{aligned} \tag{3.1}$$

By using Laplace transform technique, we can solve for $R_0^*(s)$ and is given by:

$$R_0^*(s) = E_0^*(s) + f_1^*(s)E_1^*(s) \tag{3.2}$$

The steady-state reliability of the system given by

$$R_0 = \lim_{s\to 0} sR_0^*(s) = \lim_{t\to\infty} R_0(t) \tag{3.3}$$

4. Availability Analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are given by:

$$\begin{aligned} &A_0(t) = E_0(t) + F_1(t)©A_1(t) \\ &A_1(t) = E_1(t) + E_2(t)©A_2(t) + E_3(t)©A_3(t) \\ &A_2(t) = E_5(t)©A_0(t) + E_4(t)©A_4(t) \\ &A_3(t) = F_2(t)©A_0(t) \\ &A_4(t) = h(t)©A_0(t) \end{aligned} \tag{4.1}$$

By taking Laplace transforms of the above equations and solving for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{4.2}$$

where

$$N_1(s) = E_0^*(s) + f_1^*(s)E_1^*(s), \;\; D_1(s) = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state availability of the system given by:

$$A_0^*(s) = \lim_{s\to 0} sA_0(s) = \lim_{t\to\infty} A_0(t) \tag{4.3}$$

5. Busy Period of the Server due to Repair

Let $B_i^R(t)$ is defined as the probability that the system is busy due to repair at epoch t starting from state $S_i \in E$. We have the following recursive relation:

$$\begin{aligned} &B_0^R(t) = F_1(t)©B_1^R(t) \\ &B_1^R(t) = E_2(t)©B_2^R(t) + E_3(t)©B_3^R(t) \\ &B_2^R(t) = \overline{G(t)} + E_5(t)©B_0^R(t) + E_4(t)©B_4^R(t) \\ &B_3^R(t) = F_2(t)©B_0^R(t) \end{aligned}$$

$$B_4^R(t) = h(t)ⓒB_0^R(t) \tag{5.1}$$

By taking Laplace transforms of the above equations and solving for $B_0^{R*}(s)$, we get

$$B_0^{R*}(s) = \frac{N_2^R(s)}{D_2^R(s)} \tag{5.2}$$

where

$$N_2^R(t) = f_1^* \overline{G^*}(s) E_2^*(s),\ D_2^R = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state busy period of the server due to repair is given by:

$$B_0^{R*}(s) = \lim_{s \to 0} sB_0^{R*}(s) = \lim_{t \to \infty} B_0^R(t) \tag{5.3}$$

6. Busy Period of the Server due to Preventive Maintenance

Let $B_i^P(t)$ is defined as the probability that the system is busy due to preventive maintenance at epoch t starting from state $S_i \in E$. We have the following recursive relation:

$$B_0^P(t) = F_1(t)ⓒB_1^P(t)$$
$$B_1^P(t) = E_2(t)ⓒB_2^P(t) + E_3(t)ⓒB_3^P(t)$$
$$B_2^P(t) = E_5(t)ⓒB_0^P(t) + E_4(t)ⓒB_4^P(t)$$
$$B_3^P(t) = \overline{F_2(t)} + F_2(t)ⓒB_0^P(t)$$
$$B_4^P(t) = h(t)ⓒB_0^P(t) \tag{6.1}$$

By taking Laplace transforms of the above equations and solving for B_0^{P*}, we get

$$B_0^{P*}(s) = \frac{N_2^P(s)}{D_2^P(s)} \tag{6.2}$$

where

$$N_2^P(s) = f_1^* \overline{F_2^*}(s) E_3^*(s),\ D_2^P(s) = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state busy period of the server due to preventive maintenance is given by:

$$B_0^{P*} = \lim_{s \to 0} sB_0^{P*} = \lim_{t \to \infty} B_0^P(t) \tag{6.3}$$

7. Busy Period of the Server Due to Replacement

Let $B_i^{RP}(t)$ is defined as the probability that the system is busy due to replacement at epoch t starting from state $S_i \in E$. We have the following recursive relation:

$$B_0^{RP}(t) = f_1(t)ⓒB_1^{RP}(t)$$
$$B_1^{RP}(t) = E_2(t)ⓒB_2^{RP}(t) + E_3(t)ⓒB_3^{RP}(t)$$
$$B_2^{RP}(t) = E_5(t)ⓒB_0^{RP}(t) + E_4(t)ⓒB_4^{RP}(t)$$
$$B_3^{RP}(t) = F_2(t)ⓒB_0^{RP}(t)$$

$$B_4^{RP}(t) = \overline{H(t)} + h(t)©B_0^{RP}(t) \tag{7.1}$$

By taking Laplace transforms of the above equations and solving for $B_0^{RP*}(s)$, we get

$$B_0^{RP*}(s) = \frac{N_2^{RP}(s)}{D_2^{RP}(s)} \tag{7.2}$$

where

$$N_2^{RP}(s) = f_1^*(s)\overline{H^*}(s)E_2^*(s)E_4^*(s),$$
$$D_2^{RP}(s) = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state busy period of the server due to replacement is given by:

$$B_0^{RP*} = \lim_{s\to 0} sB_0^{RP*} = \lim_{t\to\infty} B_0^{RP}(t) \tag{7.3}$$

8. Expected Number of Repair by the Server

Let $E_i^R(t)$ be the expected number of repairs by the server in $(0, t]$ given that the system entered in the regenerative state i at $t = 0$. The recursive relations for $E_i^R(t)$ are given as

$$E_0^R(t) = f_1(t)©E_1^R(t)$$
$$E_1^R(t) = E_2(t)©E_2^R(t) + E_3(t)©E_3^R(t)$$
$$E_2^R(t) = E_5(t)©[1 + E_0^R(t)] + E_4(t)©E_4^R(t)$$
$$E_3^R(t) = f_2(t)©E_0^R(t)$$
$$E_4^R(t) = h(t)©E_0^R(t) \tag{8.1}$$

By taking Laplace transforms of the above equations and solving for $E_0^{R*}(t)$, we get

$$E_0^{R*}(t) = \frac{N_3^R(s)}{D_3^R(s)} \tag{8.2}$$

where

$$N_3^R(s) = f_1^*(t)E_2^*(t)E_4^*(t),$$
$$D_3^R(s) = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state expected number of repairs by the server is given by:

$$E_0^{R*} = \lim_{s\to 0} sE_0^{R*}(s) = \lim_{t\to\infty} E_0^R(s) \tag{8.3}$$

9. Expected Number of Preventive Maintenances by the Server

Let $E_i^P(t)$ be the expected number of preventive maintenances by the server in $(0, t]$ given that the system entered in the regenerative state i at $t = 0$. The recursive relations for $E_i^P(t)$ are given as

$$E_0^P(t) = f_1(t)©E_1^P(t)$$
$$E_1^P(t) = E_2(t)©E_2^P(t) + E_3(t)©E_3^P(t)$$

$$E_2^P(t) = E_5(t)©E_0^P(t) + E_4(t)©E_4^P(t)$$
$$E_3^P(t) = f_2(t)©[1 + E_0^P(t)]$$
$$E_4^P(t) = h(t)©E_0^P(t) \tag{9.1}$$

By taking Laplace transforms of the above equations and solving for $E_0^{P*}(t)$, we get

$$E_0^{P*}(s) = \frac{N_3^P(s)}{D_3^P(s)} \tag{9.2}$$

where

$$N_3^P(s) = f_1^*(s)E_3^*(s)E_4^*(s), D_3^P(s) \quad = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state expected number of preventive maintenances by the server is given by:

$$E_0^{P*} = \lim_{s\to 0} sE_0^{P*}(s) = \lim_{t\to\infty} E_0^P(s) \tag{9.3}$$

10. Expected Number of Replacements by the Server

Let $E_i^{RP}(t)$ be the expected number of replacements by the server in $(0, t]$ given that the system entered in the regenerative state i at $t = 0$. The recursive relations for $E_i^{RP}(t)$ are given as

$$E_0^{RP}(t) = f_1(t)©E_1^{RP}(t)$$
$$E_1^{RP}(t) = E_2(t)©E_2^{RP}(t) + E_3(t)©E_3^{RP}(t)$$
$$E_2^{RP}(t) = E_5(t)©E_0^{RP}(t) + E_4(t)©E_4^{RP}(t)$$
$$E_3^{RP}(t) = f_2(t)©E_0^{RP}(t)$$
$$E_4^{RP}(t) = h(t)©[1 + E_0^{RP}(t)] \tag{10.1}$$

By taking Laplace transforms of the above equations and solving for $E_0^{RP*}(s)$, we get

$$E_0^{RP*}(s) = \frac{N_3^{RP}(s)}{D_3^{RP}(s)} \tag{10.2}$$

where

$$N_3^{RP} = f_1^*(s)E_3^*(s)f_2^*(s),$$
$$D_3^{RP} = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state expected number of replacements by the server is given by:

$$E_0^{RP*}(s) = \lim_{s\to 0} sE_0^{RP*} = \lim_{t\to\infty} E_0^{RP} \tag{10.3}$$

11. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. We have the following recursive relations for $N_i(t)$:

$$N_0(t) = f_1(t)©N_1(t)$$

$$N_1(t) = E_2(t)©[1 + N_2(t)] + E_3(t)©[1 + N_3(t)]$$
$$N_2(t) = E_5(t)©N_0(t) + E_4(t)©N_4(t)$$
$$N_3(t) = F_2(t)©N_0(t)$$
$$N_4(t) = h(t)©N_0(t) \tag{11.1}$$

By taking Laplace transforms of the above equations and solving for $N_0^*(s)$, we get:

$$N_0^*(s) = \frac{N_4(s)}{D_4(s)} \tag{11.2}$$

where

$$N_4(s) = f_1^*(s)[E_3^*(s) + E_2^*(s)],$$
$$D_4(s) = 1 - f_1^*(s)[f_2^*(s)E_3^*(s) + E_2^*(s)E_5^*(s) + E_2^*(s)h^*(s)E_4^*(s)]$$

The steady-state expected number of visits by the server is given by:

$$N_0^* = \lim_{s\to 0} sN_0^*(s) = \lim_{t\to\infty} N_0(s) \tag{11.3}$$

12. Profit Analysis

Any manufacturing industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major factors contributing to the total cost are availability, busy period of server and expected number of visits by the server. The cost of these individual items varies with reliability or mean time to system failure. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability. The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function in time t is given by:

$$P(t) = \text{Expected revenue in } (0, t] - \text{Expected total cost in } (0, t]$$

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as

$$\lim_{t\to\infty}\left(\frac{P(t)}{t}\right)$$

i.e.

profit per unit time = total revenue per unit time − total cost per unit time.

Considering the various costs, the profit equation is given as:

$$P = K1A_0 - K2B_0^R - K3B_0^P - K4B_0^{RP} - K5E_0^R - K6E_0^P - K7E_0^{RP} - K8N_0$$

where

$K1$ = Revenue per unit up-time of the system,

$K2$ = Cost per unit time for which server is busy in repair,

$K3 =$ Cost per unit time for which server is busy in preventive maintenance
$K4 =$ Cost per unit time for which server is busy in replacement,
$K5 =$ Cost per unit time repair of the unit,
$K6 =$ Cost per unit time preventive maintenance of the unit,
$K7 =$ Cost per unit time replacement of the unit,
$K8 =$ Cost per unit visit by the server.

Numerical Results

In this section, some of the results obtained for the above system are illustrated with a numerical example, we assume that

$$f(t) = \lambda e^{-\lambda t} \quad f_1(t) = \lambda_1 e^{-\lambda_1 t} \quad f_2(t) = \gamma e^{-\gamma t} \quad f_3(t) = \beta e^{-\beta t}$$
$$z(t) = \alpha e^{-\alpha t} \quad g(t) = \theta e^{-\theta t} \quad h(t) = \beta_1 e^{-\beta_1 t} \quad m(t) = \theta_1 e^{-\theta t}$$

From equation (3.2), the time-dependent reliability is given by:

$$R_0^*(t) = \sum_{i=1}^{3} \frac{[S_i(S_i + \alpha + \beta) + \lambda_1(2S_i + \alpha + \beta + \lambda_1 + \lambda)]e^{S_i t}}{\prod\limits_{j=1, i\neq j}^{3} (S_i - S_j)}$$

where S_i $(i = 1$ to $3)$ are the roots of the given equation.

$$S^3 + S^2(\alpha+\beta+\lambda+2\lambda_1) + S(\alpha\lambda+\beta\lambda+\lambda\lambda_1+2\alpha\lambda_1+2\beta\lambda_1+\lambda_1^2) + \alpha\lambda\lambda_1+\lambda\lambda_1\beta+\alpha\lambda_1^2+\beta\lambda_1^2 = 0$$

Hence mean time to system failure is calculated using the relation

$$MTSF = R_0^*(0) = \frac{[\alpha + \beta + \lambda_1 + \lambda)]}{\alpha\lambda + \lambda\beta + \lambda_1\alpha + \lambda_1\beta}$$

Now from equation (4.2) the time-dependent availability of the system is given by:

$$A_0^*(t) = \sum_{i=1}^{6} \frac{\begin{pmatrix} [(S_i^3 + S_i^2(\theta + \beta + \gamma + \theta_1) + S_i(\gamma\theta + \beta\theta + \beta\theta_1 + \gamma\theta_1 + \beta\gamma) + \beta\gamma\theta + \beta\gamma\theta_1) \\ (S_i^2 + S_i(\beta + \alpha + 2\lambda_1) + \beta\lambda_1 + \alpha\lambda_1 + \lambda\lambda_1 + \lambda_1^2)]e^{S_i t} \end{pmatrix}}{\prod\limits_{j=1, i\neq j}^{6} (S_i - S_j)}$$

where s_i $(i = 1$ to $6)$ are the roots of the equation

$$(S + \lambda + \lambda_1)[(S + \lambda_1)(S + \gamma)(S + \alpha + \beta)(S + \theta + \theta_1)(S + \beta_1) - \lambda_1\alpha\gamma(S + \theta + \theta_1)(S + \beta_1) - \lambda_1\beta\theta$$
$$(S + \lambda_1)(S + \gamma) - (S + \gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state availability of the system given by

$$A_0 = \frac{[\alpha + \beta + \lambda_1 + \lambda](\theta + \theta_1)\beta\gamma\lambda_1}{\begin{pmatrix} \beta\theta\gamma\beta_1\lambda_1 - \beta\lambda_1\theta\gamma_1\lambda_1 + \beta\lambda\gamma\beta_1\lambda_1 + \lambda\theta\gamma\beta_1\lambda_1 + \alpha\theta\lambda\gamma\beta_1 + \alpha\theta\lambda\lambda_1\beta_1 + \lambda\theta\gamma\beta_1\beta \\ +\lambda\lambda_1\theta\beta_1\beta + \lambda\lambda_1\theta_1\beta_1\gamma + \lambda\theta_1\beta_1\alpha\gamma + \lambda_1\theta_1\beta_1\alpha\lambda + \lambda\beta\gamma\theta_1\beta_1 + \lambda\beta\gamma\theta_1\lambda_1 - \lambda\beta\theta_1\lambda_1\lambda_1 \\ +\gamma\beta\beta_1\lambda_1\lambda_1 + \gamma\theta\beta_1\lambda_1\lambda_1 + \alpha\theta\lambda_1\beta_1\gamma + \alpha\theta\lambda_1\beta_1\lambda_1 + \beta\gamma\theta\beta_1\lambda_1 + \theta\beta\beta_1\lambda_1\lambda_1 + \gamma\theta_1\beta_1\lambda_1\lambda_1 \\ +\gamma\theta_1\beta_1\lambda_1\alpha + \alpha\theta_1\beta_1\lambda_1\lambda_1 + \beta\gamma\theta_1\beta_1\lambda_1 + \gamma\theta_1\beta\lambda_1\lambda_1 - \beta\theta\lambda_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (5.2) the time-dependent busy period analysis due to repair is given by:

$$B_0^{R*}(t) = \sum_{i=1}^{6} \frac{\begin{pmatrix} [\beta\lambda_1(S_i^3 + S_i^2(\theta + \beta_1 + \gamma + \theta_1) + S_i(\gamma\theta + \beta_1\theta + \beta_1\theta_1 + \gamma\theta_1 + \beta_1\gamma) + \beta_1\gamma\theta + \beta_1\gamma\theta_1) \\ (S_i^2 + S_i(\beta_1 + \alpha + 2\lambda_1) + \beta_1\lambda_1 + \alpha\lambda_1 + \lambda\lambda_1 + \lambda_1^2)]e^{S_i t} \end{pmatrix}}{\prod_{j=1, i\neq j}^{6} (S_i - S_j)}$$

where s_i $(i = 1$ to $6)$ are the roots of the equation

$$(S+\theta)[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta \\ (S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state busy period of the server due to repair is given by

$$B_0^R = \frac{(\theta+\theta_1)\beta\gamma\beta_1\lambda_1}{\begin{pmatrix} \beta\theta\gamma\beta_1\lambda_1 - \beta\lambda_1\theta\gamma_1\lambda_1 + \beta\theta\gamma\beta_1\lambda_1 + \theta\theta\gamma\beta_1\lambda_1 + \alpha\theta\theta\gamma\beta_1 \\ \alpha\theta\theta\lambda_1\beta_1 + \theta\theta\gamma\beta_1\beta + \theta\lambda_1\theta\beta_1\beta + \theta\lambda_1\theta_1\beta_1\gamma + \theta\theta_1\beta_1\alpha\gamma \\ +\lambda_1\theta_1\beta_1\alpha\theta + \lambda\beta\gamma\theta_1\beta_1 + \theta\beta\gamma\theta_1\lambda_1 - \theta\beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (6.2) the time-dependent busy period analysis due to preventive maintenance is given by:

$$B_0^{P*}(t) = \sum_{i=1}^{5} \frac{[\beta\lambda_1(S_i^2 + S_i(\theta + \beta_1 + \theta_1) + \beta_1\theta + \beta_1\theta_1)]e^{S_i t}}{\prod_{j=1, i\neq j}^{5} (S_i - S_j)}$$

where s_i $(i = 1$ to $5)$ are the roots of the equation

$$[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta \\ (S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state busy period of the server due to preventive maintenance is given by

$$B_0^P = \frac{(\theta+\theta_1)\beta\beta_1\lambda_1}{\begin{pmatrix} \beta\gamma\beta_1\lambda_1 + \theta\gamma\beta_1\lambda_1 + \alpha\theta\gamma\beta_1 + \alpha\theta\lambda_1\beta_1 + \theta\gamma\beta_1\beta + \lambda_1\theta\beta_1\beta + \lambda_1\theta_1\beta_1\gamma \\ +\theta_1\beta_1\alpha\gamma + \lambda_1\theta_1\beta_1\alpha + \lambda\beta\theta_1\beta_1 + \beta\gamma\theta_1\lambda_1 - \beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (7.2) the time-dependent busy period analysis due to replacement is given by:

$$B_0^{RP*}(t) = \sum_{i=1}^{5} \frac{[\beta\lambda_1\beta_1\theta_1(S_i+\gamma)]e^{S_i t}}{\prod_{j=1, i\neq j}^{5} (S_i - S_j)}$$

where s_i $(i = 1$ to $5)$ are the roots of the equation

$$[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta \\ (S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state busy period of the server due to replacement is given by

$$B_0^{RP} = \frac{\gamma\theta\beta\beta_1\lambda_1}{\begin{pmatrix} \beta\gamma\beta_1\lambda_1 + \theta\gamma\beta_1\lambda_1 + \alpha\theta\gamma\beta_1 + \alpha\theta\lambda_1\beta_1 + \theta\gamma\beta_1\beta + \lambda_1\theta\beta_1\beta + \lambda_1\theta_1\beta_1\gamma \\ +\theta_1\beta_1\alpha\gamma + \lambda_1\theta_1\beta_1\alpha + \lambda\beta\theta_1\beta_1 + \beta\gamma\theta_1\lambda_1 - \beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (8.2) the time-dependent expected number of repairs is given by:

$$E_0^{R*}(t) = \sum_{i=1}^{5} \frac{[\beta\lambda_1\theta_1(S_i^2 + S(\beta_1+\gamma) + \beta_1\gamma)]e^{S_i t}}{\prod\limits_{j=1, i\neq j}^{5} (S_i - S_j)}$$

where s_i $(i = 1$ to $5)$ are the roots of the equation

$$[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta$$
$$(S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state expected number of repairs is given by

$$E_0^R = \frac{\gamma\theta\beta\beta_1\lambda_1}{\begin{pmatrix} \beta\gamma\beta_1\lambda_1 + \theta\gamma\beta_1\lambda_1 + \alpha\theta\gamma\beta_1 + \alpha\theta\lambda_1\beta_1 + \theta\gamma\beta_1\beta + \lambda_1\theta\beta_1\beta + \lambda_1\theta_1\beta_1\gamma \\ +\theta_1\beta_1\alpha\gamma + \lambda_1\theta_1\beta_1\alpha + \lambda\beta\theta_1\beta_1 + \beta\gamma\theta_1\lambda_1 - \beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (9.2) the time-dependent expected number of preventive maintenance is given by:

$$E_0^{P*}(t) = \sum_{i=1}^{5} \frac{[\alpha\gamma\lambda_1(S_i^2 + S_i(\theta+\beta_1+\theta_1) + \beta_1\theta + \beta_1\theta_1)]e^{S_i t}}{\prod\limits_{j=1, i\neq j}^{5} (S_i - S_j)}$$

where s_i $(i = 1$ to $5)$ are the roots of the equation

$$[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta$$
$$(S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state expected number of preventive maintenance is given by

$$E_0^P = \frac{(\theta+\theta_1)\alpha\gamma\beta_1\lambda_1}{\begin{pmatrix} \beta\gamma\beta_1\lambda_1 + \theta\gamma\beta_1\lambda_1 + \alpha\theta\gamma\beta_1 + \alpha\theta\lambda_1\beta_1 + \theta\gamma\beta_1\beta + \lambda_1\theta\beta_1\beta + \lambda_1\theta_1\beta_1\gamma \\ +\theta_1\beta_1\alpha\gamma + \lambda_1\theta_1\beta_1\alpha + \lambda\beta\theta_1\beta_1 + \beta\gamma\theta_1\lambda_1 - \beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (10.2) the time-dependent expected number of replacements is given by:

$$E_0^{RP*}(t) = \sum_{i=1}^{5} \frac{[\beta_1\theta_1\beta\lambda_1(S_i+\gamma)]e^{S_i t}}{\prod\limits_{j=1, i\neq j}^{5} (S_i - S_j)}$$

where s_i $(i = 1$ to $5)$ are the roots of the equation

$$[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta$$
$$(S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state expected number of replacements is given by

$$E_0^{RP} = \frac{\beta\gamma\theta_1\beta_1\lambda_1}{\begin{pmatrix} \beta\gamma\beta_1\lambda_1 + \theta\gamma\beta_1\lambda_1 + \alpha\theta\gamma\beta_1 + \alpha\theta\lambda_1\beta_1 + \theta\gamma\beta_1\beta + \lambda_1\theta\beta_1\beta + \lambda_1\theta_1\beta_1\gamma \\ +\theta_1\beta_1\alpha\gamma + \lambda_1\theta_1\beta_1\alpha + \lambda\beta\theta_1\beta_1 + \beta\gamma\theta_1\lambda_1 - \beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

From equation (11.2) the time-dependent expected number of visits by the server is given by:

$$N_0^*(t) = \sum_{i=1}^{5} \frac{\begin{pmatrix} [(\alpha+\beta)\lambda_1(S_i^3 + S_i^2(\theta+\beta_1+\gamma+\theta_1) \\ +S_i(\gamma\theta+\beta_1\theta+\beta_1\theta_1+\gamma\theta_1+\beta_1\gamma)+\beta_1\gamma\theta+\beta_1\gamma\theta_1)]e^{S_i t} \end{pmatrix}}{\prod\limits_{j=1, i\neq j}^{5}(S_i - S_j)}$$

where s_i $(i = 1$ to $5)$ are the roots of the equation

$$[(S+\lambda_1)(S+\gamma)(S+\alpha+\beta)(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\alpha\gamma(S+\theta+\theta_1)(S+\beta_1) - \lambda_1\beta\theta$$
$$(S+\lambda_1)(S+\gamma) - (S+\gamma)\lambda_1\theta_1\beta_1\beta] = 0$$

In case steady-state expected number of visits by the server is given by

$$N_0 = \frac{\gamma\beta_1\lambda_1(\alpha+\beta)(\theta+\theta_1)}{\begin{pmatrix} \beta\gamma\beta_1\lambda_1+\theta\gamma\beta_1\lambda_1+\alpha\theta\gamma\beta_1+\alpha\theta\lambda_1\beta_1+\theta\gamma\beta_1\beta+\lambda_1\theta\beta_1\beta+\lambda_1\theta_1\beta_1\gamma \\ +\theta_1\beta_1\alpha\gamma+\lambda_1\theta_1\beta_1\alpha+\lambda\beta\theta_1\beta_1+\beta\gamma\theta_1\lambda_1-\beta\theta_1\lambda_1\lambda_1 \end{pmatrix}}$$

13. Conclusion

The expressions for various reliability measures are derived taking arbitrary distributions for the random variables. Later on, a particular case is considered to obtain numerical results for these measures assuming negative exponential distributions for the random variables. The graphical behavior of mean time to system failure (MTSF) with respect to maximum rate of operation time (α) is shown in Fig. 1. It is observed that MTSF decreases with the increase of α. And, there is a further decline in its values when direct failure rate (1) increases. Figs. 2 and 3 respectively revealed that availability and profit of the system model decrease with the increase of maximum rate of operation (α), direct failure rate (λ_1) and replacement rate (β_1) for fixed values of other parameters. However, values of these measures increase when repair rate (θ) increases. Thus it is concluded that a single-system can be made more reliable and economical to use either by conducting preventive maintenance of the system after a maximum operation time at its partial failure stage or by making replacement of the unit if server is not in a position to complete its repair in a given maximum repair time.

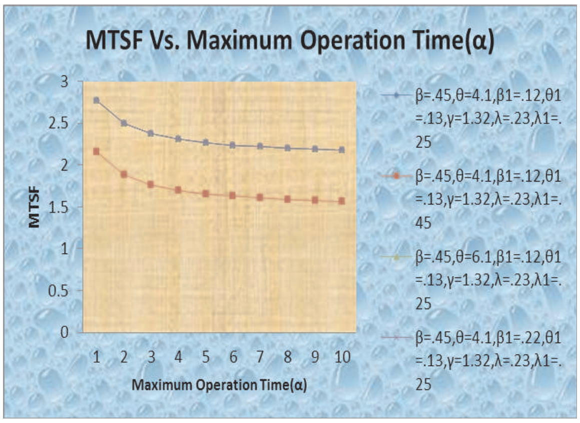

Figure 1: MTSF vs. Maximum Operation Time (α).

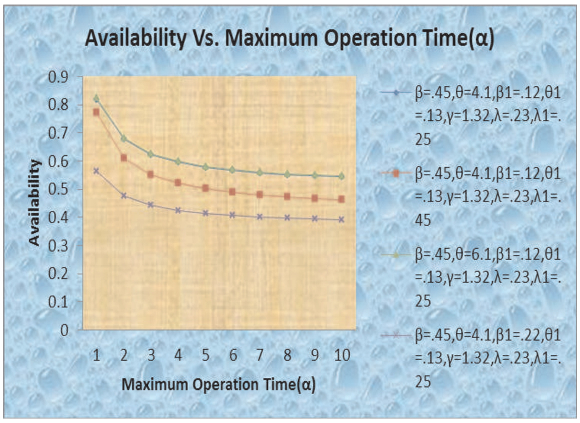

Figure 2: Availability vs. Maximum Operation Time (α).

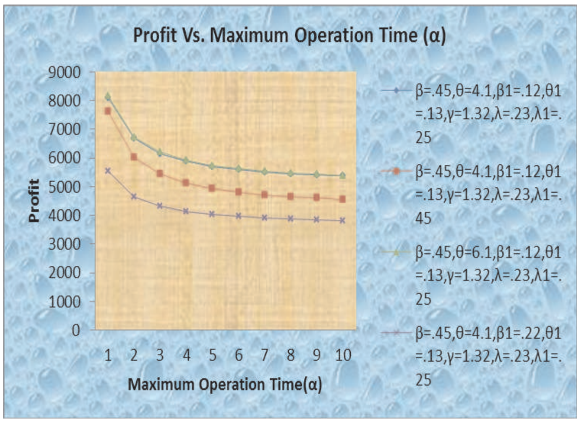

Figure 3: Profit vs. Maximum Operation Time (α).

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