AUM BLOCK SUM LABELLING FOR SOME SPECIAL GRAPHS

A. Uma Maheswari[1], S. Azhagarasi[2]

[1]Associate Professor & Head,

PG & Research Department of Mathematics

[2]Research Scholar,

PG & Research Department of Mathematics

Quaid-E-Millath Government College for Women(Autonomous), Chennai-02

Abstract

AUM block sum labelling is a newly introduced labelling technique in recent times. It has been established that path, triangular snake graph, alternate triangular snake graph, double triangular snake graph, double alternate triangular snake graph, quadrilateral snake graph, alternate quadrilateral snake graph, double quadrilateral snake graph, double alternate quadrilateral snake graph, Friendship graph $F_n (n \geq 5)$, cycle cactus graph $C_4^k (k \geq 2)$ and $C_5^k (k \geq 2)$, perfect binary tree $T_{2,l}$ admit AUM block labelling. In this paper, we have obtained the AUM block sum labelling for star graph, bistar graph, comb graph and sunlet graph, Diamond snake graph, Pentagonal snake graph and Alternate pentagonal snake graph. Suitable illustrations are also given.

Keywords: AUM block sum labelling**,** bistar, comb, diamond snake graph, pentagonal snake, star, sunlet

1. INTRODUCTION

The concept of graph labelling initiated by Rosa.A[4] in 1967 has developed vastly in recent years mainly due to its interesting connection and applications to other fields like health science, medical fields, communication network, chemical structures, material science etc[1][3][22][23][24][30] and the detailed survey given in J.Gallein[28]. Various labelling technique can be seen in[5][6][7][8][9][10][11][12][13][14][16][20]. The study on AUM block sum labelling can be referred to [15][17][19], where in it was established that path, triangular snake graph, alternate triangular snake graph, double triangular snake graph, double alternate triangular snake graph, quadrilateral snake graph, alternate quadrilateral snake graph, double quadrilateral snake graph, double alternate quadrilateral snake graph, Friendship graph $F_n (n \geq 5)$, cycle cactus graph $C_4^k (k \geq 2)$, $C_5^k (k \geq 2)$ and perfect binary tree $T_{2,l}$ admit AUM block labelling. In [18] A.Uma Maheswari and et.al have presented a new algorithm for Encoding and Decoding using AUM block labelling.

In this paper, we have proved that bistar graph, comb graph, diamond snake graph, pentagonal snake, star graph and sunlet graph admit AUM block sum labelling. We also give explicit examples for AUM block labelled graphs.

Labelling of graphs finds its applications in heterogeneous fields including the field of material science particularly, in crystallography[22][21], difference labelling of signed graphs in cryptoanalysis[24], secured encryption of text on social media[30], secure transmission of messages[23], computer network[2][26], dental architecture[27].

2. PRELIMINARIES

Definition 1.1[15],[17]: AUM Block sum labelling

Let G be a graph with p vertices, q edges and b blocks, $p, q, b \geq 1$. Let $V(G) = \{v_1, v_2, \ldots, v_p\}$, $E(G) = \{e_1, e_2, \ldots, e_q\}$, $B(G) = \{B_1, B_2, \ldots, B_b\}$ denote the vertex set, edge set and the block set of G respectively.

We say the graph G has a block sum labeling if there exists a bijection

$f: V(G) \rightarrow \{1,2,\ldots,p\}$ and $f^*: E(G) \rightarrow \mathbb{Z}^+$ induced from f by $f^*(uv) = f(u) * f(v)$ and

$f^{**}: B(G) \rightarrow \mathbb{Z}^+$, defined as follows:

Let B_j be incident with the vertices $v_{j_1}, v_{j_2}, \ldots, v_{j_k}, 1 \leq j_k \leq p$ and edges $e_{j_1}, e_{j_2}, \ldots, e_{j_m}$, $1 \leq j_m \leq q$. Then $f^{**}(B_j) = \sum_{i=1}^{k} f(v_{j_i}) + \sum_{i=1}^{m} f^*(e_{j_i})$ and $f^{**}(B_j) \neq f^{**}(B_i)$ for $1 \leq i, j \leq b$ and $i \neq j$.

Definition 1.2[25]: star graph

A star S_n is the complete bipartite graph $K_{1,n}$. It is also defined as a tree with one internal node and n leaves

Definition 1.3[25]: Bi-star graph

The Bi-star $B(m,n)$ is a two-star graph, one with $m+1$ and other with $n+1$ vertices along with an edge joining the apex of the two-star graphs. The Bi-star $B(m,n)$ has $m+n+2$ vertices and $m+n+1$ edges.

Definition 1.4[28] sunlet graph

The *n*-sunlet graph is a graph on *2n* vertices is obtained by attaching *n*-pendent edges to the cycle C_n and it is denoted by S_n

Definition 1.5[25]: Comb graph $P_n \odot K_1$

Let P_n be a path on *n* vertices. The comb graph is defined as $P_n \odot K_1$. It has *2n* vertices and $2n-1$ edges.

Definition 1.6[5] Diamond snake graph

The graph G consists of collection of *n* cycles C_4, these cycles are connected in such a way that any two adjacent cycles sharing a common vertex, the resulting graph is called the diamond snake graph and it is denoted by D_n. A snake is a Eulerian path that has no chords.

Definition 1.7[29] pentagonal snake graph

The pentagonal snake $P(S_k)$is obtained from a path $u_1, u_2, u_3, \ldots, u_k$ by joining u_i and u_{i+1} for $1 \leq i \leq k-1$, to three new vertices v_i, w_i, x_i and then joining v_i, w_i, and x_i.That is the path P_n by replacing each edge of the path by a cycle C_5.

Definition 1.8[29] Alternate pentagonal snake graph

An alternate pentagonal snake $A(PS_k)$ is obtained from a path h $u_1, u_2, u_3, \ldots, u_k$by joining u_i and u_{i+1} to three new vertices v_i, w_i, x_i and then joining v_i, w_i, and x_i respectively. That is, every alternate edge of a path is replaced by a cycle C_5.

3. MAIN RESULTS

In this section, we associate AUM block labelling for some standard graphs

Theorem 3.1: *Every star graph $S_n, n \geq 4$ admits AUM block sum labeling.*

Proof: Let G be the star graph, $S_n, n \geq 4$. Let $V(G) = \{v_0, v_1, v_2, \ldots., v_{n-1}\}$. $|V(G)| = n$ and $|E(G)| = n-1$.

Let us associate the AUM block sum labelling to blocks of G as follows.

Define the function $f: V(G) \to \{1,2,\ldots,n\}$ by

$$f(v_0) = 1$$

$$f(v_i) = i+1, 1 \leq i \leq n-1$$

Obtain the induced function $f^*: E(G) \to \mathbb{Z}^+$ from f as

$$f^*(v_0 v_i) = i+2, 1 \leq i \leq n-1$$

Now we label the blocks from the sum labelling of vertices and edges.

Define $f^{**}: B(G) \to \mathbb{Z}^+$ by

$$f^{**}(B_i) = 2(i+2), 1 \leq i \leq n-1$$

For $i \neq j$, $f^{**}(B_i) \neq f^{**}(B_j)$

Implying the block labels, $f^{**}(B_i)s$ are distinct. Hence G admits AUM block labelling as shown in Fig.1

Note: For star graph S_1 labelling is trivial. The star graph S_2 *and* S_3 are same as path P_2 *and* P_3 which admit AUM block sum labelling[15].

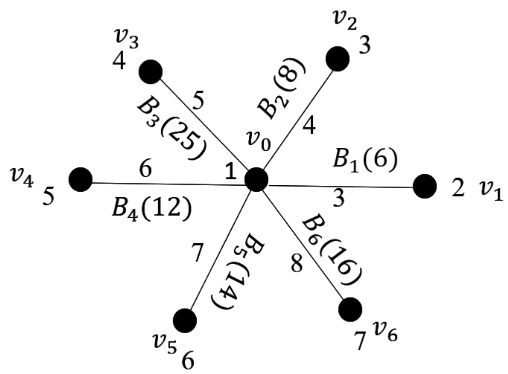

Fig.1 S_7 AUM block sum labelling

Theorem 3.2: *Every bistar graph $B_{m,n}$ admits AUM block sum labelling.*

Proof: Consider the two copies of $K_{1,n}$. Let $v_1, v_2, \ldots, v_m$ and $u_1, u_2, \ldots, u_n$ are vertices of each copy of $K_{1,n}$ with apex vertex *v* and *u*.

$|V(G)| = m + n + 2$ and $|E(G)| = m + n + 1$. We consider in three cases:

Case 1: If $m = n$ then $B_{n,n}$ admits AUM block sum labelling

Define the function $f: V(G) \to \{1,2,\ldots,|V(G)|\}$ by

$$f(v) = 1, f(u) = n + 2$$
$$f(v_i) = i + 1, 1 \leq i \leq n$$
$$f(u_i) = n + i + 2, 1 \leq i \leq n$$

Obtain the induced function $f^*: E(G) \to \mathbb{Z}^+$ from f as

$$f^*(vu) = 2n + 6$$
$$f^*(vv_i) = i + 2, 1 \leq i \leq n$$
$$f^*(uu_i) = 2\text{n} + i + 4, 1 \leq i \leq n$$

Now we label the blocks from the sum labelling of vertices and edges.

Define $f^{**}: B(G) \to \mathbb{Z}^+$ by

$$f^{**}(B_1) = 2\text{n} + 6$$
$$f^{**}(B_{i+1}) = 2\text{i} + 4, 1 \leq i \leq n$$
$$f^{**}(\text{B}'_{i+1}) = 4\text{n} + 2\text{i} + 8, 1 \leq i \leq n$$

Hence G admits AUM block labelling as shown in Fig.2

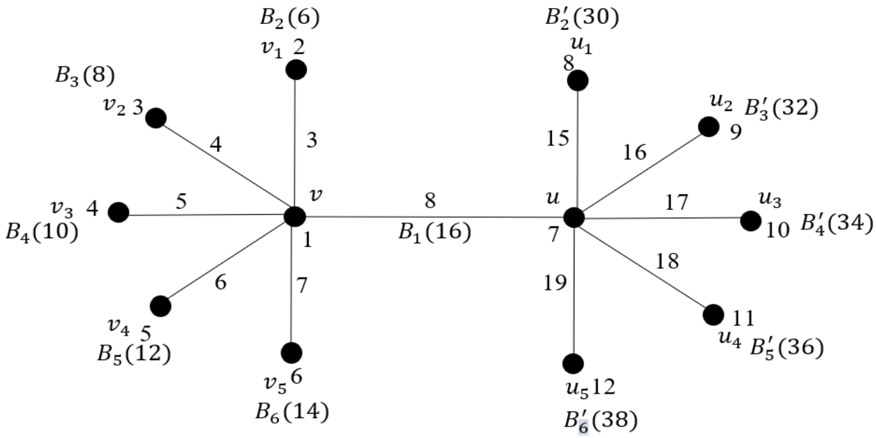

Fig.2 $B_{5,5}$ AUM block sum labelling

Case 2: If $m > n$ then $B_{m,n}$ admits AUM block sum labelling

Define the function $f: V(G) \to \{1,2,\ldots,|V(G)|\}$ by

$$f(v) = 1, f(u) = m + 1$$
$$f(v_i) = i + 1, 1 \leq i \leq m$$
$$f(u_i) = m + i + 1, 1 \leq i \leq n$$

Obtain the induced function $f^*: E(G) \to \mathbb{Z}^+$ from f as

$$f^*(vu) = m + 2$$
$$f^*(vv_i) = i + 2, 1 \leq i \leq m$$
$$f^*(uu_i) = 2m + i + 2, 1 \leq i \leq n$$

Now we label the blocks from the sum labelling of vertices and edges.

Define $f^{**}: B(G) \to \mathbb{Z}^+$ by

$f^{**}(B_1) = 2(m + 2)$

$f^{**}(B_{i+1}) = 2\text{i} + 4, 1 \leq i \leq m$

$f^{**}(B'_{i+1}) = 4m + 2i + 4, 1 \leq i \leq n$

Hence G admits AUM block labelling as shown in Fig.3

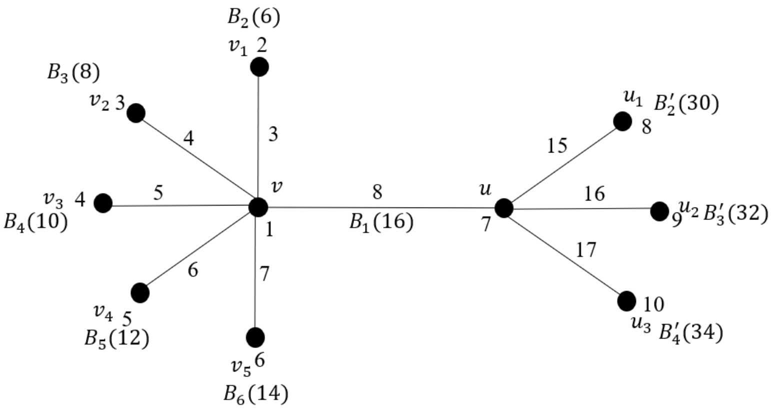

Fig. 3 $B_{5,3}$ AUM block sum labelling

Case 3: If $m < n$ then $B_{m,n}$admits AUM block sum labelling

Define the function $f: V(G) \rightarrow \{1,2, \dots, |V(G)|\}$ by

$$f(v) = 1, f(u) = m + 1$$

$$f(v_i) = i + 1, 1 \leq i \leq m$$

$$f(u_i) = m + i + 1, 1 \leq i \leq n$$

Obtain the induced function $f^*: E(G) \rightarrow \mathbb{Z}^+$ from f as

$$f^*(vu) = m + 2$$

$$f^*(vv_i) = i + 2, 1 \leq i \leq m$$

$$f^*(uv_i) = 2m + i + 2, 1 \leq i \leq n$$

Now we label the blocks from the sum labelling of vertices and edges.

Define $f^{**}: B(G) \rightarrow \mathbb{Z}^+$ by

$$f^{**}(B_1) = 2(m + 2)$$

$$f^{**}(B_{i+1}) = 2i + 4, 1 \leq i \leq m$$

$$f^{**}(B'_{i+1}) = 4m + 2i + 4, 1 \leq i \leq n$$

Hence G admits AUM block labelling as shown in Fig.4

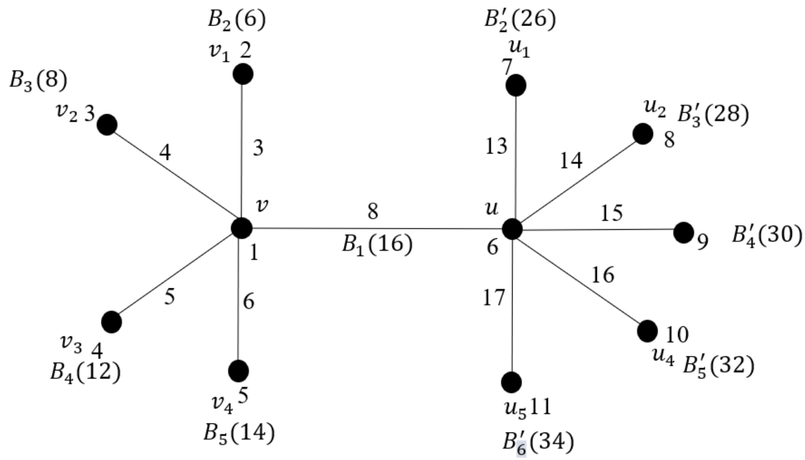

Fig.4 $B_{4,5}$ Aum block sum labelling

Theorem 3.3: *Every Sunlet graph ($S_n, n = odd$) admits AUM block sum labelling.*

Proof: Let *G* be a sunlet graph with *2n* vertices and *2n* edges. Let $v_1, v_2, v_3, \dots v_n$ are the vertices of the cycle sunlet graph, $v'_1, v'_2, v'_3, \dots, v'_n$ are the pendent vertices of sunlet graph. $|V(G)| = 2n$, $|E(G)| = 2n$

Now we shall define AUM block sum labelling for the blocks of *G* as follows.

Define the function $f: V(G) \rightarrow \{1,2,\ldots,2\mathrm{n}\}$ by

$$f(v_i) = 2i - 1, 1 \leq i \leq n$$

$$f(v_i') = 2i, 1 \leq i \leq n$$

Define the induced function $f^*: E(G) \rightarrow \mathbb{Z}^+$ from f as

$$f^*(v_1 v_n) = 2n$$

$$f^*(v_i v_{i+1}) = 4i,\ 1 \leq i \leq n - 1$$

$$f^*(v_i' v_i) = 4i - 1, 1 \leq i \leq n$$

Now we label the blocks obtained from the labelling of vertices and edges.

Define $f^{**}: B(G) \rightarrow \mathbb{Z}^+$ by

$$f^{**}(B_1) = 3n^2$$

$$f^{**}(B_{i+1}) = 8i - 2, 1 \leq i \leq n$$

For $i \neq j$, $f^{**}(B_i) \neq f^{**}(B_j)$.

Hence G admits AUM block labelling as shown in Fig.5

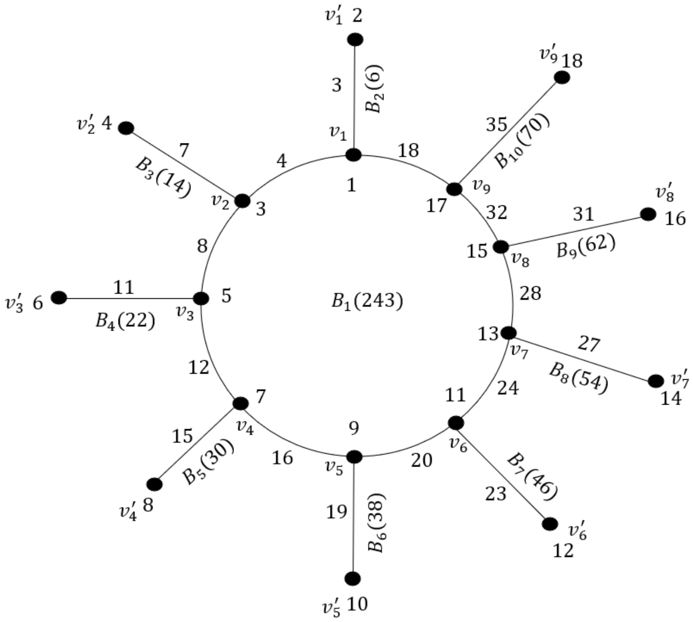

Fig.5 S_9 AUM block sum labelling

Theorem 3.4: *Every Comb graph $P_n \odot K_1$, admits AUM block sum labelling*

Proof: Let $G = P_n \odot K_1$ be the comb graph. Consider $\{v_1, v_2, v_3, \ldots v_n\}$ are the vertices of the path P_n and $\{u_1, u_2, u_3, \ldots, u_n\}$ are the pendent vertices (adjacent vertices) to each vertex of the path P_n. Hence, $|V(G)| = 2n$ and $|E(G)| = 2n - 1$.

Now we define AUM block sum labelling for the blocks of G as follows.

Define the function $f: V(G) \rightarrow \{1,2,\ldots,2\mathrm{n}\}$ by

$$f(v_i) = 2i - 1, 1 \leq i \leq n$$

$$f(u_i) = 2i, 1 \leq i \leq n$$

Define the induced function $f^*: E(G) \rightarrow \mathbb{Z}^+$ from f as

$$f^*(v_i v_{i+1}) = 4i, 1 \leq i \leq n - 1$$

$$f^*(v_i u_i) = 4i - 1, 1 \leq i \leq n$$

Now we label the blocks obtained from the labelling of vertices and edges.

Define $f^{**}: B(G) \rightarrow \mathbb{Z}^+$ by

$$f^{**}(B_{2i-1}) = 2(4\mathrm{i} - 1), 1 \leq i \leq n$$

$$f^{**}(B_{2i}) = 8i, 1 \leq i \leq n - 1$$

For $i \neq j$, $f^{**}(B_i) \neq f^{**}(B_j)$.

Hence G admits AUM block sum labelling as shown in Fig.6

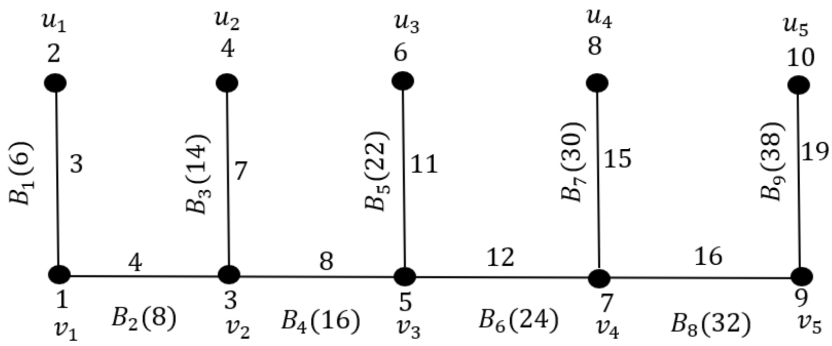

Fig.6 $P_5 \odot K_1$ AUM block sum labelling

Theorem 3.5: *For any Diamond snake graph D_n admits AUM block sum labelling.*

Proof: Let D_n be a graph G. The cycles C_4 are connected by sharing a common vertex adjacently. Therefore, the diamond snake graph D_n is obtained. Let $v_1, v_2, v_3, \ldots v_{3n+1}$be the vertices of the graph. Hence, $|V(G)| = 3n + 1$ and $|E(G)| = 4n$.

Now we define AUM block sum labelling for the blocks of *G* as follows.

Define the function $f: V(G) \to \{1,2, \ldots, 3\mathrm{n} + 1\}$ by

$$f(v_i) = i, 1 \leq i \leq 3n + 1$$

Define the induced function $f^*: E(G) \to \mathbb{Z}^+$ from f as

$$f^*(v_{3i-2}v_{3i-1}) = 6i - 3, 1 \leq i \leq n$$

$$f^*(v_{3i-1}v_{3i+1}) = 6i, 1 \leq i \leq n$$

$$f^*(v_{3i-2}v_{3i}) = 6i - 2, 1 \leq i \leq n$$

$$f^*(v_{3i}v_{3i+1}) = 6i + 1, 1 \leq i \leq n$$

we label the blocks obtained from the labelling of vertices and edges.

Define $f^{**}: B(G) \to \mathbb{Z}^+$ by

$$f^{**}(B_i) = 6(6\mathrm{i} - 1), 1 \leq i \leq n$$

For $i \neq j, f^{**}(B_i) \neq f^{**}(B_j)$.

Hence G admits AUM block sum labelling as shown in Fig.7

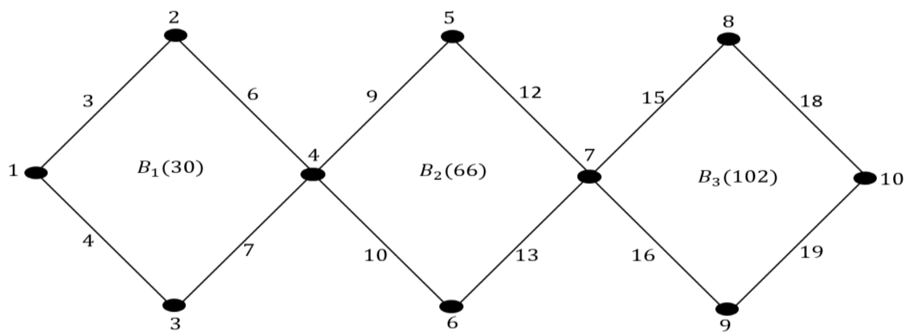

Fig.7 D_3 AUM block sum labelling

Theorem 3.6: *Pentagonal snake graph PS_n admits AUM block sum labelling.*

Proof: Consider the path P_n, $v_1, v_2, v_3, \ldots, v_n$. Every edge is replaced by cycle C_5. Therefore, the graph G, PS_n is obtained. The graph has $v_1, v_2, v_3, \ldots, v_{4n-3}$, vertices. Hence, $|V(G)| = 4n - 3$, $|E(G)| = 5n - 5$.

Now we define AUM block sum labelling for the blocks of *G* as follows.

Define the function $f: V(G) \to \{1,2, \ldots, 4\mathrm{n} - 3\}$ by

$$f(v_i) = i, 1 \leq i \leq 4\mathrm{n} - 3$$

Define the induced function $f^*: E(G) \to \mathbb{Z}^+$ from f as

$$f^*(v_i v_{i+1}) = 2i + 1, 1 \leq i \leq 4(n - 1)$$

$$f^*(v_{4i-3}v_{4i+1}) = 2(4i-1), 1 \le i \le n-1$$

we label the blocks obtained from the labelling of vertices and edges.

Define $f^{**}: B(G) \to \mathbb{Z}^+$ by

$$f^{**}(B_i) = 5(12\text{i} - 3), 1 \le i \le n-1$$

For $i \ne j$, $f^{**}(B_i) \ne f^{**}(B_j)$.

Hence G admits AUM block sum labelling as shown in fig. 8

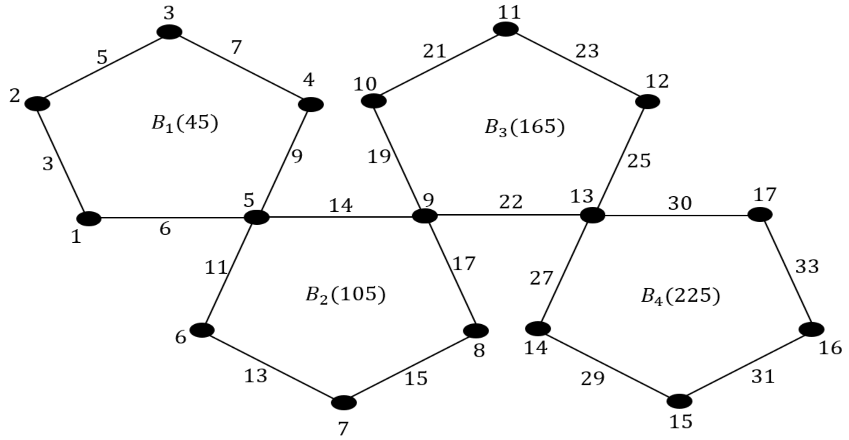

Fig.8 PS_5 AUM block sum labelling

Theorem 3.7: *Alternate Pentagonal snake graph $A(PS_n)$,admits AUM block sum labelling*

Proof: Consider the path P_n, $v_1, v_2, v_3, \dots, v_n$. Alternately edges are replaced by cycle C_5. Therefore, the graph G, $A(PS_n)$ is obtained. The graph has $v_1, v_2, v_3, \dots, v_{4n-3}$, vertices. Hence, if n is even $|V(G)| = 5\left(\frac{n}{2}\right)$ $|E(G)| = 3n - 1$ and if n is odd $|V(G)| = 5\left(\frac{n-1}{2}\right) + 1$ $|E(G)| = 3(n-1)$.

Now we define AUM block sum labelling for the blocks of G as follows.

Define the function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by

$f(v_i) = i, 1 \le i \le 5\left(\frac{n}{2}\right)$; if n is even, $1 \le i \le 5\left(\frac{n-1}{2}\right) + 1$; if n is odd

Define the induced function $f^*: E(G) \to \mathbb{Z}^+$ from f as

$$f^*(v_i v_{i+1}) = 2i + 1, 1 \le i \le 4(n-1)$$

$$f^*(v_{4i-3}v_{4i+1}) = 2(4i-1), 1 \le i \le n-1$$

we label the blocks obtained from the labelling of vertices and edges.

Define $f^{**}: B(G) \to \mathbb{Z}^+$ by

$$f^{**}(B_{2i-1}) = 15(5\text{i} - 2), 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f^{**}(B_{2i}) = 2(10\text{i} + 1), 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor$$

For $i \ne j$, $f^{**}(B_i) \ne f^{**}(B_j)$.

Hence G admits AUM block sum labelling as shown in Fig.9

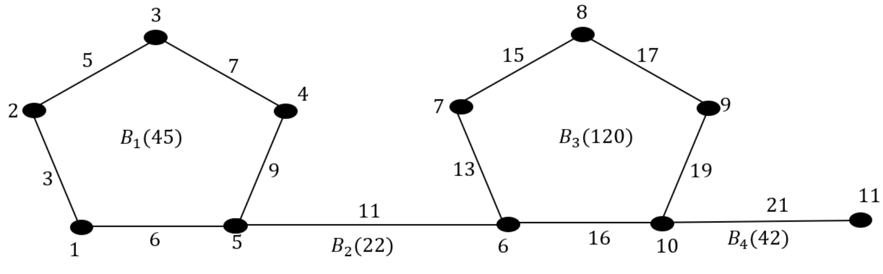

Fig.9 $A(PS_5)$ AUM block sum labelling

Conclusion:

In this paper, we have proved that star, Bistar, Comb graph admits AUM block sum labelling. Also, we have proved that Diamond snake, Pentagonal snake, Alternate Pentagonal Snake, Sunlet graph admits AUM block sum labelling. This work has further scope for extending to other graphs with applications in many other fields including medical science, health science, chemical science, physical science, computer science.

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