ISSN: 0974-5823 Vol. 7 (Special Issue 5, April 2022)

International Journal of Mechanical Engineering

SINGULARLY PERTURBED ONE D USING NON-POLYNOMIAL SPLINE

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Abstract

Herein, talk over the two- position schemes in spatial discretization with anon-polynomial spline for cracking two-parameter singularly perturbed 1D parabolic equations. It has been constitute that the developed algorithm gives accurate results and a advanced numerical rate of confluence. Stability analysis has also shown that the scheme is unreservedly stable. Maximum absolute crimes for (α, β, γ) can be doped . Fluid mechanics and the convection- diffusion process should be resolved. α, σ, and p values are calculated with N per every 20 intervals.

Keywords: parabolic equations, absolute errors, polynomial spline.

Introduction

These problems occur naturally in various fields of science and engineering, such as, nuclear engineering, control theory, elasticity, quantum mechanics, optimal control, chemical reactor theory, fluid mechanics and convection-diffusion process. Many authors like [1-3].Al- Said [4-6],Fyfe, D.J. [7-8] andislam , [9-13] and Loscalzo, F.R. and Talbot [17] et al, have developed approximate methods for the solution of singularly perturbed problems. A uniform convergent numerical method with respect to the diffusion parameter is given [18-25] and [14-16]. To solve the one-dimensional time-dependent convectiondiffusion problems we discussed.

Mathematical Details of the Discretization& Observation.

Consider a uniform mesh with grid points k_i on (a, b) such that

$$a = k_0 < k_1 < k_2 < .... < k_{n-1} < k_n = b \qquad ....3.1$$

Where

$$ki = a + ih, \quad i = 0,1 \dots, n \text{ and } h = (b-a)/n$$

Let consider the

$$Pi(k) = a_i \cos T\left(k - k_{i-\frac{1}{2}}\right) + b_i \sin T\left(k - k_{i-\frac{1}{2}}\right) + C_i \quad ....3.3$$

on solving

$$P_i(k_i) = l_i \; P_i(k_{i-1/2}) = T_{i-1/2} \qquad ....3.4$$

$$P_i(k_i) = M_i \; i = 0,1,...,n-1 \qquad ....3.5$$

$$a_i = -\frac{1}{T^2} Mi \sec\left(\frac{\theta}{2}\right) - \frac{1}{T} T_{i-1/2} \tan\left(\frac{\theta}{2}\right) \quad ...3.6$$

$$b_i = \frac{1}{T} T_{i-1/2} \qquad ....3.7$$

$$c_i = y_{i+1} - \frac{1}{T^2} M_i \quad \text{……..3.8}$$

on goes on solving

$$\alpha = \frac{\sec(\frac{\theta}{2}) - 1}{\theta^2} \quad \text{……..3.9}$$

$$\beta = \frac{4\sec(\frac{\theta}{2}) Sin^2(\frac{\theta}{2}) + 2(1 - \sec(\frac{\theta}{2}))}{\theta^2} \quad \text{……..3.10}$$

$$\gamma = \alpha$$

$$l_{i-1} - 2l_i + l_{i+1} = \frac{1}{2} h^2 (M_{i-1} + 6M_{i+1}),\ i = 1, 2, ..., n-1 \quad (3.2.4)$$

$$l_i - \in_d l_{xx} + \in_c p(k) l_x + q(k) l = f(k,t), (k,t) \in P_T \quad (3.3.1)$$

$$l = 0, (k,t) \in \partial\Omega \times I \quad \text{……3.11}$$

On substitution we get

$$\in_d \frac{d^2 l}{dk^2} = \in_c p(k) \frac{dl}{dk} + q(k) l - f(k) .$$
$$= F(k, l, l) \quad \text{…3.12}$$

$$Al_{i-1} + Bl_i + Cl_{i+1} = -h^2(\alpha f_{i-1} + \beta f_i + \gamma f_{i+1}), i = 1, 2, ..., n-1 \quad \text{……..3.13}$$

$$A_1 l_{i-1}^{j+1} + A_2 l_i^{j+1} = A_4 l_{i-1}^j + A_5 l_i^j + A_6 l_i^j - h^2(\alpha f_{i-1}^j + \beta f_i^i + \gamma f_{i+1}^j),$$
$$i = 1, 2, ..., n-1 \quad \text{…3.14}$$

$$A_1 = \frac{-h^2 \alpha}{k} + \frac{1}{2}(\in d + \frac{3}{2} h\alpha \in_c p_{i-1} - \alpha h^2 q_{i-1} + \frac{h}{2}\beta \in_c p_i - \frac{h}{2}\gamma \in_c p_{i+1}) \quad \text{…..3.15}$$

$$A_2 = \frac{-h^2 \beta}{k} + \frac{1}{2}(-2 \in_d - 2h\alpha \in_c p_{i-1} - \beta h^2 q_i + 2h\gamma \in_c p_{i+1}) \quad \text{…..3.16}$$

$$A_3 = \frac{-h^2 \gamma}{k} + \frac{1}{2}(\in d + \frac{1}{2} h\alpha \in_c p_{i-1} - \gamma h^2 q_{i-1} - \frac{h}{2}\beta \in_c p_i - \frac{3h}{2}\gamma \in_c p_{i+1}) \quad \text{….3.17}$$

On simplifying

$$t_i = \left[h^2\left[1 - (\alpha + \beta + \gamma)\right] M_x^2 - \frac{1}{2} k \left[\alpha h^2 q_{i-1} + \beta h^2 q_i + \gamma h^2 q_{i+1}\right] M_t + h^3 \left[\alpha - \lambda\right] M \frac{3}{x}\right.$$
$$+ h^4 \left[\frac{1}{12} - \frac{\alpha + \gamma}{2}\right] M_x^4 - \frac{1}{4} k^2 [\alpha h^2 q_{i-1} + \beta h^2 q_i + \gamma h^2 q_{i+1}] M_t^2 + h^5 [\frac{\alpha - \gamma}{3!}] M_x^5$$
$$\left. + h^6 [\frac{1}{360} - \frac{\alpha + \gamma}{24}] M_x^6 + ...\right] l_i^j, i = 1, 2, ..., n-1. \quad \text{……...3.18}$$

We get

$$A_1 L_{i-1}^{j+1} + A_2 L_{i+1}^{j+1} = A_4 L_{i-1}^j + A_5 L_i^j + A_6 L_{i+1}^j - h^2(\alpha f_{i-1}^j + \beta f_i^j + \gamma f_{i+1}^j), i = 1, 2..., n-1. \quad \text{…….3.19}$$

$$A_1 e_{i-1}^{j+1} + A_2 e_i^{j+1} + A_3 e_{i-1}^{j+1} = A_4 e_{i-1}^j + A_5 e_i^j + A_6 e_{i+1}^j, i = 1, 2, ..., n-1. \quad \text{………..3.20}$$

$$\varepsilon = \frac{A_4 e^{-i\delta} + A_5 + A_6 e^{i\delta}}{A_1 e^{-i\delta} + A_2 + A_3 e^{i\delta}} \quad .....3.21$$

Up on substitution

$$\varepsilon = \frac{-\frac{h^2}{k_{\in d}}(\alpha+\delta) - \frac{h^2}{c_d}(\alpha q_{i-1} - \beta q_i + \gamma q_{i+1}) + 2B_1 \sin^2(\frac{\delta}{2}) + iB_2 \sin(\frac{\delta}{2})}{-\frac{h^2}{k_{\in d}}(\alpha+\delta) + \frac{h^2}{\in_d}(\alpha q_{i-1} - \beta q_i + \gamma q_{i+1}) - 2B_1 \sin^2(\frac{\delta}{2}) + iB_2 \sin(\frac{\delta}{2})} \quad ...3.21(a)$$

$$B_1 = 1 + \frac{h^2}{k_{\in d}}(\alpha+\gamma) + h\frac{\in_c}{\in_d}(\alpha p_{i-1} - 2\gamma p_{i+1}) + h^2 \frac{1}{\in_d}(\alpha q_{i-1} + \gamma q_{i+1}) \quad .....3.22$$

$$B_2 = \frac{h^2}{k_{\in d}}(\alpha-\delta) + \frac{1}{2_{\in d}}[h_{\in c}(\alpha p_{i-1} + \beta p_i - \gamma q_{i+1}) + h^2(\gamma q_{i+1} - \alpha q_{i-1}) \quad ......3.23$$

Results

3.3.1 Table.1 Maximum absolute errors α, β, γ) = (1/8, 6/8, 1/8)

Method	N \€d	2^{0}	2^{-2}	2^{-4}	2^{-6}
Our method	16	7.2294e – 04	1.2280e–03	1.0487e–02	1.0921e–01
Clavero et al.		1.3076e–03	1.7398e–02	4.0133e–02	5.9664e–02
	ρ˜n	1.7534	1.9008	2.5426	1.3506
Our method	32	2.3156e–04	3.2887e–04	1.8000e–03	4.2823e–02
Claveroetal.[20]		7.9078e–04	9.6845e–03	2.5552e–02	3.7372e–02
	ρ˜n	1.8952	1.8851	2.4931	2.0663
Our method	64	6.2255e–05	8.9033e–05	3.1971e–04	1.0225e–02
Clavero et al.[20]		3.6986e–04	5.1056e–03	1.5865e–02	2.1792e–02
	ρ˜n	1.9602	1.9124	2.4935	2.6020
Our method	128	1.5998e–05	2.3652e–05	5.6774e–05	1.6841e–03
Claveroetal.[20]		1.8894e–04	2.6223e–03	9.5603e–03	1.2381e–02
	ρ˜n	1.9638	1.9347	2.4473	2.5951

3.3.2 Table.2 Maximum absolute errors of example for (α, β, γ) = (1/8, 6/8, 1/8)

Ƹd	**2^{0}**			**2^{-2}**			**2^{-4}**		
Ƹ c / N	**10^{-3}**	**10^{-4}**	**10^{-5}**	**10^{-3}**	**10^{-4}**	**10^{-5}**	**10^{-3}**	**10^{-4}**	**10^{-5}**
16	5.7028e– 06	5.6372e– 06	5.6328e– 06	7.6099e– 03	7.5937e– 03	7.5920e– 03	3.0375e– 01	3.0260e– 01	3.0248e– 0
Ƿn	1.4451	1.4479	1.4482	1.8084	1.8084	1.8084	1.8258	1.8258	1.8258
32	2.0944e–	06 2.0671e–	06 2.0643e– 06	2.1726e– 03	2.1681e– 03	2.1676e– 03	8.5684e– 02	8.5356e– 02	8.5324e– 02
Ƿn	1.8444	1.8459	1.8461	1.8568	1.8568	1.8568	1.9140	1.9140	1.9140
64	5.8324e– 07	5.7502e– 07	5.7420e– 07	5.9986e– 04	5.9857e– 04	5.9845e– 04	2.2736e– 02	2.2650e– 02	2.2640e– 02
Ƿn	1.9487	1.9496	1.9497	1.9306	1.9306	1.9306	1.9572	1.9572	1.9572

3.3.3 Table.3Maximum absolute errors of example for (α, β, γ) = (1/12, 10/12, 1/12)

Method	N \€d	2^{-1}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}
Our method	2^4	7.2292e – 04	8.0024e – 04	8.2241e – 04	8.3576e – 04	8.4320e – 04	8.4712e – 0
Mohanty et al.[51]		0.2924e – 03	0.4454e – 03	0.4777e – 03	0.5054e – 03	0.5344e – 03	0.5615e – 03
Our method	2^5	1.2613e – 05	1.3899e – 05	1.4267e – 04	1.4488e – 04	1.4610e – 04	1.4675e – 04
Mohanty et al.[21]		0.7286e – 04	0.1129e – 03	0.1239e – 03	0.1410e – 03	0.1869e – 03	0.3134e – 03
Our method	2^6	2.2181e – 05	2.4391e – 05	2.5022e – 05	2.5721e – 05	2.5610e – 05	2.5721e – 05
Mohanty et al.[21]		0.1814e – 04	0.2835e – 04	0.3166e – 04	0.3984e – 04	0.9429e – 04	0.1684e – 03
Our method	2^7	3.9130e – 06	4.2982e – 06	4.4079e – 06	4.5295e – 06	4.5102e – 06	4.5295e – 06
Mohanty et al.[21]		0.4524e – 05	0.7091e – 05	0.7987e – 05	0.1088e – 04	0.4743e – 04	0.9120e – 04

Conclusion

We've developed the two-echelon schemes in spatial discretization with an anon-polynomial spline for unraveling two-parameter singularly perturbed 1D parabolic equations. It has been constituted that the developed algorithm gives accurate results and an advanced numerical rate of confluence. Stability analysis has also shown that the scheme is unreservedly stable. N€ d value of our system correlated with Mohanty's proposed data.

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