ISSN: 0974-5823 Vol. 7 (Special Issue 2, Jan.-Feb. 2022)

International Journal of Mechanical Engineering

G- SUPPORT DOMINATION FUZZY GRAPH USING STRONG ARC - NEW APPROACH

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Abstract - *In this paper we extend the concept Global domination in fuzzy graph using strong arc to Global support domination in fuzzy graph using strong arc. Support of a vertex in a fuzzy graph is defined. Global support domination number using strong arc in fuzzy graph is defined and finding the global support domination using strong arc for some standard fuzzy graphs.*

Keywords - *Fuzzy graph, Strong arc, Non strong arc, Domination number using strong arc, Global Domination number using strong arc, Support of a vertex, Support Domination number, Global Support Domination number.*

AMS Classification : 05C72

1. INTRODUCTION

Fuzzy graph is the generalization of the ordinary graph. Therefore it is natural that though fuzzy graph inherits many properties similar to those of ordinary graph, it deviates at many places. The formal mathematical definition of domination was given by Ore.O in 1962. In 1975 A.Rosenfeld introduced the notion of fuzzy graph and several analogs of theoretic concepts such path, cycle and connected. A.Somasundaram and S.Somasundaram discussed the domination in fuzzy graph using effective arc. C.Y.Ponnappan and V.Senthil Kumar discussed the domination in fuzzy graph using strong arc. Before introducing new results in fuzzy graphs using strong arcs, we are placed few preliminary definitions and results for new one.

2. PRELIMINARIES

Definition : 2.1

Fuzzy graph G(σ ,μ) is pair of function $\sigma : V \rightarrow [0,1]$ and $\mu : E \rightarrow [0,1]$ where for all u ,v in V, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition: 2.2

The complement of a fuzzy graph G(σ,μ) is a subgraph $\underline{G} = (\underline{\sigma},\underline{\mu})$ where $\underline{\sigma} = \sigma$ and

$\underline{\mu} = \sigma(u)\wedge\sigma(v) - \mu(u,v)$ for all u,v in V. A fuzzy graph is self complementary if $G = \underline{G}$.

Definition: 2.3

Let u,v be two nodes in G(σ,μ). If they are connected by means of a path ρ then strength of that path is $\wedge_i^n \mu(ui-1, ui)$.

Definition 2.4

A strongest path joining any two nodes u,v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^\infty(u,v)$.

$\mu^\infty(u,v) = \sup \{ \mu^k(u,v) \mid k= 1,2,3\ldots\}$.

Definition 2.5

An arc (u,v) of the fuzzy graph G(σ,μ) is called a strong arc if $\mu(u,v) = \mu^\infty(u,v)$ else arc (u,v) is called non strong. Strong neighborhood of u∊V is $N_S(u) = \{ v \in V : \text{arc } (u,v) \text{ is strong}\}$.

$N_S[u]= N_S(u) \cup \{u\}$ is the closed neighborhood of u. The minimum cardinality of strong neighborhood $\delta_S(G) = \min\{|N_S(u)|: u\in V(G)\}$. Maximum cardinality of strong neighborhood $\Delta_S(G)= \max\{|N_S(u)|: u\in V(G)\}$.

Definition 2.6

Let G(σ,μ) be a fuzzy graph. Let u,v be two nodes of G (σ,μ). We say that u dominates v if edge (u,v) is a strong arc. A subset D of V is called a dominating set of G(σ,μ) if for every v ∊V- D , there exists u ∊ D such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality taken over all dominating sets of a graph G is called the strong arc dominating number and is denoted by $\gamma_s(G)$ and the corresponding set is called minimum strong arc dominating set. The number of elements in the minimum strong arc dominating set is denoted by $n[\gamma_s(G)]$.

The minimum fuzzy cardinality taken over all dominating sets of a graph $\underline{G}$,where $\underline{G}$ is the complement of the fuzzy graph G, is called the strong arc dominating number of $\underline{G}$ and is denoted by $\gamma_s(\underline{G})$ and the corresponding set is called the minimum strong arc dominating set of $\underline{G}$. The number of elements in the minimum strong arc dominating set is denoted by $n[\gamma_s(\underline{G})]$.

Fuzzy Global Dominating set using strong arc is the set which is the corresponding dominating set of min{ $\gamma_s(G)$, $\gamma_s(\underline{G})$ } and is denoted by $\gamma_{gs}(G)$. The number of elements of fuzzy global dominating set using strong is denoted by $n[\gamma_{gs}(G)]$.

Note: Here we consider the fuzzy graphs with non *effective edges.

*An arc (u,v) of the fuzzy graph $G(\sigma,\mu)$ is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$

Definition :2.7

The strong degree of a vertex u of a fuzzy graph is defined as the sum of the number of strong arcs incident with that vertex and is denoted by strong deg(u).

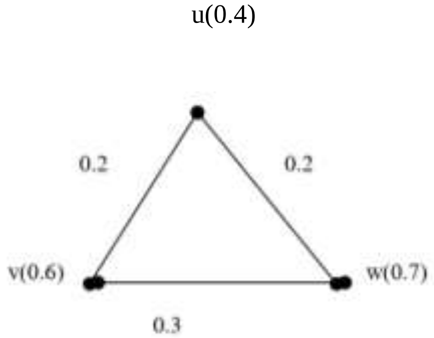


Fig- 2(a)

Here all arcs are strong arcs. Therefore strong deg(u) = strong deg(v) = strong deg(w) = 2.

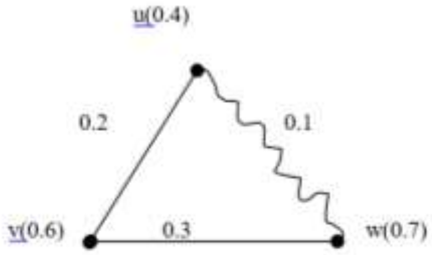


Fig- 2(b)

In the above graph if $\mu(u,w) = 0.1$, then the arc (u,w) is a non strong arc. In that case strong deg(u) = 1 ; strong deg(v) = 2 ; strong deg(w) = 1 ;

Definition : 2.8

Let G be a simple fuzzy graph with n number of vertices. A vertex $u \in G$ is said to be a full strong degree vertex if $n[N_S(u)] = n-1$.

In Fig 2(a) , $n[N_S(u)] = n[N_S(v)] = n[N_S(w)] = 2$.

In Fig 2(b) , $n[N_S(u)] = 1$; $n[N_S(v)] = 2$; $n[N_S(w)] = 1$.

Note : $[N_S(u)] = \{v \in V : \text{arc } (u,v) \text{ is strong}\}$

Definition : 2.9

Let G be a fuzzy graph. Then support of $u \in V(G)$ using strong arc is give by

Suups (u) = { $\sum$ deg (v_i) : $v_i \in N_S(u)$ }. And is labeled in fuzzy graph in [ ].

(ie) Supps (u) = $\sum_{v \in Ns(u)} \quad Strong\ deg(v)$

3. Global Support Domination in Fuzzy Graph Using Strong Arc

Definition : 3.1

Let G be a fuzzy graph. Let u,v be two strong neighborhood nodes of G. Then the vertex u is said to be support dominates the vertex v (using strong arc) if Supps(u) $\leq$ Supps(v).

A subset D of V is called a support dominating set of G using strong arc if for every v $\in$ V- D, there exists u $\in$ D such that u support dominates v using strong arc.

A support dominating set D using strong arc is called a minimal support dominating set using strong arc if no proper subset of D is a support dominating set using strong arc.

The minimum support fuzzy cardinality using strong arc taken over all minimal support dominating set using strong arc of G is called the support domination number using strong arc and is denoted by $\gamma_{supps}(G)$ and the corresponding dominating set is called minimum support dominating set using strong arc. The number of elements in the minimum support dominating set using strong arc is denoted by n[$\gamma_{supps}(G)$].

$\gamma_{supps}(G) = \min \{ \sum \text{Supps}(u) + \sigma(u)$, where $u \in D(G) \}$

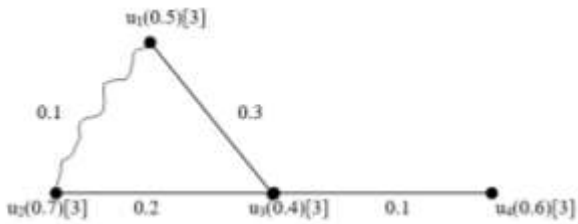
Similarly the minimum support fuzzy cardinality using strong arc taken over all minimal support dominating set using strong arc of $\underline{G}$ is called the support domination number using strong arc and is denoted by $\gamma_{supps}(\underline{G})$ and the corresponding dominating set is called minimum support dominating set using strong arc. The number of elements in the minimum support dominating set using strong arc is denoted by n[$\gamma_{supps}(\underline{G})$].

$\gamma_{supps}(\underline{G}) = \min \{ \sum \text{Supps}(v) + \sigma(v)$, where $v \in D(\underline{G}) \}$

Then Global Support Domination number of G using strong arc is defined as the minimum of $\gamma_{supps}(G)$ and $\gamma_{supps}(\underline{G})$ and is denoted by $\gamma_{gsupps}(G)$.

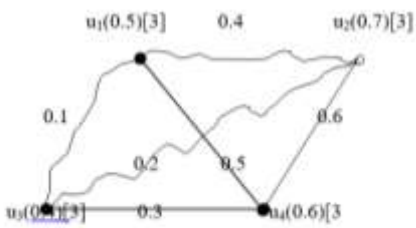
(ie) $\gamma_{gsupps}(G) = \min \{ \gamma_{supps}(G) , \gamma_{supps}(\underline{G}) \}$

Example : 3.2



(G) Fig – 3(a)

Here D(G) = {u_3} ; $\gamma_{supps}(G) = 3.4$; n [$\gamma_{supps}(G)$] = 1



($\underline{G}$) Fig – 3(b)

Here D($\underline{G}$) = {u_4}; $\gamma_{supps}(\underline{G}) = 3.6$; n[$\gamma_{supps}(\underline{G})$] = 1

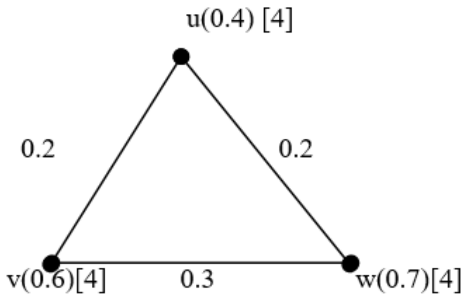
$\gamma_{gsupps}(G) = \min \{ \gamma_{supps}(G) , \gamma_{supps}(\underline{G}) \} = \min \{ 3.4, 3.6 \} = 3.4$ and n[$\gamma_{gsupps}(G)$] = 1

Note : Hereafter we use G - Support domination using strong arc for Global support domination using strong arc.

Definition : 3.3

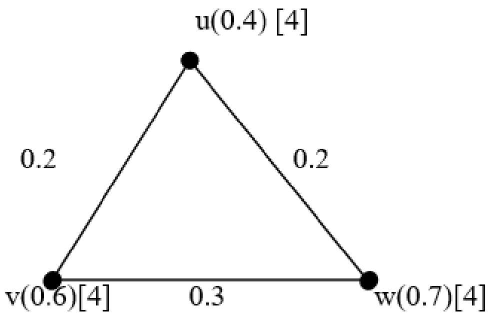
Let G be a fuzzy graph. If D is the support dominating set of both G and $\underline{G}$,then D is said to be Perfect Global Support Dominating Set Using strong arc. The fuzzy cardinality of Perfect global support dominating set using strong arc is called Perfect global support domination number and is denoted by $\gamma_{pgsupps}(G)$.

Example : 3.4



(G) – Fig 4(a)

Here $D_{supps}(G) = \{u\}$. $\therefore \gamma_{supps}(G) = supps(u) + \sigma(u) = 4.4$; $n[\gamma_{supps}(G)] = 1$.



($\underline{G}$) – Fig 4(b)

Here $D_{supps}(\underline{G}) = \{u\}$. $\therefore \gamma_{supps}(\underline{G}) = supps(u) + \sigma(u) = 4.4$; $n[\gamma_{supps}(\underline{G})] = 1$.

Therefore $\gamma_{gsupps}(G) = \min \{0.4,0.4\} = 0.4$. Here {u} is the support dominating set using strong arc for both G and $\underline{G}$.

Therefore the domination is **Perfect Global Support Domination Using Strong Arc** and perfect global support domination number using strong arc is denoted by $\boldsymbol{\gamma_{pgsupps}(G)}$.

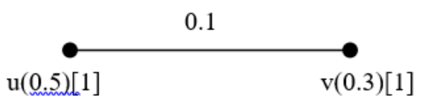
$\gamma_{pgsupps}(G) = 4.4$; $n[\gamma_{pgsupps}(G)] = 1$; And {u} is Perfect Global Support Dominating Set Using strong arc.

4. G – SUPPORT DOMINATION USING STRONG ARCS OF SOME STANDARD GRAPHS.

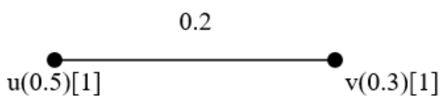
4.1. FUZZY PATHS FP_n

Result : 4.1.1

Let G be FP_2.



(G) –Fig 5(a)



($\underline{G}$)- Fig 5(b)

Here $D_{supps}(G) = \{v\}$; $D_{supps}(\underline{G}) = \{v\}$

$\gamma_{supps}(G) = 1.3$; $\gamma_{supps}(\underline{G}) = 1.3$. $\therefore \gamma_{gsupps}(G) = 1.3$. {v} is the support dominating set for both G and $\underline{G}$. $\therefore$ {v} is a perfect G- support dominating set using strong arc for G. $\therefore \gamma_{pg\,supps}(G) = 1.3$.and $n[\gamma_{pg\,supps}(G)] = 1$.

Results :

Case (i) FP_2

$n[\gamma_{supps}(G)] = 1$; $n[\gamma_{supps}(\underline{G})] = 1$

$n[\gamma_{gsupps}(G)] = 1$.

Case (ii) FP_3

$n[\gamma_{supps}(G)] = 1$; $n[\gamma_{supps}(\underline{G})] = 1$

$n[\gamma_{gsupps}(G)] = 1$.

And $[\gamma_{supps}(G)] < [\gamma_{supps}(\underline{G})]$

Case (iii) FP_4

$n[\gamma_{supps}(G)] =2$; $n[\gamma_{supps}(\underline{G})] = 2$

$n[\gamma_{gsupps}(G)] =2$.

And $[\gamma_{supps}(G)] < [\gamma_{supps}(\underline{G})]$

Case (iv) FP_5

$n[\gamma_{supps}(G)] = 2$; $n[\gamma_{supps}(\underline{G})] = 2$

$n[\gamma_{gsupps}(G)] = 2$.

And $[\gamma_{supps}(G)] < [\gamma_{supps}(\underline{G})]$

Case (v) FP_6

$n[\gamma_{supps}(G)] = 2$; $n[\gamma_{supps}(\underline{G})] = 2$

$n[\gamma_{gsupps}(G)] = 2$.

And $[\gamma_{supps}(G)] < [\gamma_{supps}(\underline{G})]$

Case (vi) FP_7

$n[\gamma_{supps}(G)] = 3$; $n[\gamma_{supps}(\underline{G})] = 2$

$n[\gamma_{gsupps}(G)] = 2$.

And $[\gamma_{supps}(G)] < [\gamma_{supps}(\underline{G})]$

Proceeding like this , the following results are derived.

Conclusion for G = FP_n

1. $n[\gamma_{supps}(G)] = \lfloor \frac{n}{3} \rfloor + 1$, n = 2,4,5,7,…..

   $= \lfloor \frac{n}{3} \rfloor$, n = 3,6,9,…. (where 'n' is the number of vertices)

2. $n[\gamma_{supps}(\underline{G})] = 1$, for n = 2,3

   = 2 for n > 3

……………………………………………………………………………………………………

Result: 4.1.2

In fuzzy path FP_n , $\gamma_{supps}(FP_n) < \gamma_{supps}(\underline{FPn})$

Proof :

Let G = FP_n be a fuzzy path. In fuzzy path every arc is a strong arc. Let $u_1,u_2,u_3,….u_n$ be the vertices of FP_n. The two end vertices u_1 and u_n have degree one and the remaining vertices are of degree two. Therefore support of u_1 and u_n using strong arc are two. Support of u_2 and u_{n-1} using strong arc will be three. And the support of the remaining vertices using strong arc will be four.

But in $\underline{FPn}$, every vertex is adjacent with every other vertex and there will be (n-1) non strong arcs in $\underline{FPn}$. Therefore support of each vertex of $\underline{FPn}$ using strong arc will be greater than that of FP_n.

Therefore $\gamma_{supps}(FP_n) < \gamma_{supps}(\underline{FPn})$.

Result: 4.1.3

The G- Support domination number of FP_n using strong arc is equal to the support domination number of FP_n using strong arc.

(ie) $\gamma_{gsupps}(FP_n) = \gamma_{supps}(FP_n)$.

Result: 4.1.4

The support of two end points u_1, u_n of FP_n are equal and minimum in FP_n whereas the support of the same points u_1, u_n are equal and maximum in $\underline{FPn}$.

4.2. FUZZY CYCLES FC_n

Result 4.2.1

In FC_n, (n>4) (i) $\gamma_{gsupps}(FC_n) = \gamma_{supps}(FC_n)$.

$= 4x \lceil \frac{n}{3} \rceil + \sigma(v_i),\ v_i \in D(FC_n)$

(ii) $n[\gamma_{gsupps}(FC_n)] = \lceil \frac{n}{3} \rceil$.

Proof:

Let FC_n be a fuzzy cycle with all arcs as strong arcs. Since each vertex of FC_n is adjacent to exactly two vertices , the support of each vertex is 4.

Also wkt $\gamma_s(FC_n) = \lceil \frac{n}{3} \rceil$. $\therefore$ $n[\gamma_{supps}(FC_n)] = 4x \lceil \frac{n}{3} \rceil + \sigma(v_i),\ v_i \in D(FC_n)$.

The non strong arcs in $\underline{FCn}$, (n>4), are $(u_1,u_2), (u_2,u_3), \ldots\ldots.(u_n,u_1)$, the number of non strong arcs is n. $\therefore$ Degree of each vertex using strong is (n-3).

Therefore support of each vertex in $\underline{FCn}$, (n>4) is greater than that of in FCn.

$\therefore \quad \gamma_{supps}(FC_n) < \gamma_{supps}(\underline{FCn})$, for n> 4

$\therefore \gamma_{gsupps}(FC_n) = \gamma_{supps}(FC_n)$, (n> 4)

And $n[\gamma_{gsupps}(FC_n)] = n[\gamma_{supps}(FC_n)] = \lceil \frac{n}{3} \rceil$.

4.3. FUZZY STAR $FK_{1,n}$

Result : 4.3.1

Let $FK_{1,n}$ be a fuzzy star with (n+1) vertices.

Then (i) $n[\gamma_{supps}(FK_{1,n})] = 1$

(ii) $\gamma_{supps}(FK_{1,n}) = n + \sigma(u)$, where $u \in D(FK_{1,n})$

(iii) $n[\underline{\gamma\ supps(FK1,n)}] = 1$

(iv) $\gamma_{supps}(FK_{1,n}) < \underline{\gamma\ supps(FK1,n)}$

(v) $\gamma_{gsupps}(FK_{1,n}) = \gamma_{supps}(FK_{1,n})$

(vi) $n[\gamma_{gsupps}(FK_{1,n})] = 1$.

Proof :

(i) Let $FK_{1,n}$ be a fuzzy star with (n+1) vertices. Let u be the vertex which support dominates every other vertices v1,v2,v3,….vn and each vi is adjacent to u only. Therefore support of u using strong arc is 'n'.

$\therefore n[\gamma_{supps}(FK_{1,n})] = 1$ and

(ii) $\gamma_{supps}(FK_{1,n}) = n + \sigma(u)$, where $u \in D(FK_{1,n})$

(iii) In $\underline{FK1,n}$ there are n(n-1)/2 number of arcs not necessarily all arcs are strong. Also in $\underline{FK1,n}$ only one vertex dominates every other vertex . $\therefore$ $n[\underline{\gamma\ supps(FK1,n)}] = 1$.

(iv) Since all vertices dominate more than one vertex in $\underline{FK1,n}$, support of each vertex is more than 'n'. $\therefore$ $\gamma_{supps}(FK_{1,n}) < \underline{\gamma\ supps(FK1,n)}$

(v) From (iv) , it follows that $\gamma_{gsupps}(FK_{1,n}) = \gamma_{supps}(FK_{1,n})$

(vi) From (i) and (iv) , we get $n[\gamma_{gsupps}(FK_{1,n})] = 1$

4.4. FUZZY COMPLETE GRAPH FK_n

Result : 4.4.1

In fuzzy complete graph FK_n with all arcs as strong arcs, $\gamma_{supps}(G) \geq \gamma_{supps}(\underline{G})$, where $G = FK_n$

Proof :

Let G = FK_n be a fuzzy complete graph in which all arcs are strong arcs. Then support of each vertex is $(n-1)^2$.

Therefore $\gamma_{supps}(G) = (n-1)^2 + \sigma(v)$, $v \in D(G)$.

In $\underline{G}$, all arcs may or may not be strong.

Case (i) : All arcs of $\underline{G}$ are strong.

If all the arcs of $\underline{G}$ are strong, then all the vertices of $\underline{G}$ have the same support as that of in G.

$\therefore \gamma_{supps}(\underline{G}) = \gamma_{supps}(G)$.

Case (ii) : $\underline{G}$ with non strong arcs.

If $\underline{G}$ has some non strong arcs, then support of the vertices which are incident non strong arcs are less than that of in G.(ie) if $\underline{G}$ has one non strong arc, then the highest support of a vertex will be $[(n-1)^2 - 2]$. The vertices incident with non strong arc has support (n-1) (n-2).

Similarly, if $\underline{G}$ has more number of non strong arcs, then support of each vertex becomes lesser than that of in G.

$\therefore \gamma_{supps}(\underline{G}) < \gamma_{supps}(G)$. Therefore by case (i) and (ii), $\gamma_{supps}(G) \geq \gamma_{supps}(\underline{G})$.

4.5. FUZZY COMPLETE BIPARTITE GRAPH FKm,m

Result : 4.5.1.

In fuzzy complete bipartite graph $FK_{m,m}$ with all arcs as strong arcs, $\gamma_{supps}(G) > \gamma_{supps}(\underline{G})$, where G= $FK_{m,m}$

Proof:

Let G= $FK_{m,m}$ be a fuzzy complete bipartite graph in which all arcs are strong arcs. Then support of each vertex of G is m^2 and $\gamma_{supps}(G) = 2m^2 + \sum\sigma(u_i)$, $u_i \in D(G)$. But $\underline{G}$ will contain more number of non strong arcs. Therefore support of each vertex of $\underline{G}$ is less than that of in G.

$\therefore \gamma_{supps}(G) > \gamma_{supps}(\underline{G})$.

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