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On $N\lambda\Psi g$- Closed Sets In Nano Toplogical Spaces

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Abstract

In this paper, we introduce nano $(\boldsymbol{\Lambda},\boldsymbol{\Psi})$ -open, nano $(\boldsymbol{\Lambda},\boldsymbol{\Psi})$ -closed sets, nano $\lambda\Psi g$ -closed sets in nano topological spaces and investigate some of their properties.

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Key words : $N(\boldsymbol{\Lambda},\boldsymbol{\Psi})$ –open sets, $N(\boldsymbol{\Lambda},\boldsymbol{\Psi})$ -closed sets and $N\lambda\Psi g$ -closed sets.

1 Introduction

M.K.R.S.Veerakumar [6] was introduced the notion of Ψ closed sets in topological spaces. Maki [3] introduced the notion of Λ-sets in topological spaces in 1986. Λ-set is a set A which is equal to its kernel, i.e., to the intersection of all open supersets of A. Lellis Thivagar introduced [2] nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X. The elements of nano topological space are called nano open sets. He has also defined nano closed sets, nano interior and nano closure of a set. He also introduced the weak forms of nano open sets.N.R.Santhi Maheswari and P.Subbulakshmi [5] nano $N\Lambda_\Psi(A)$ sets, nano $N\Lambda^*_\Psi(A)$ sets, nano Λ_Ψ -set and nano Λ^*_Ψ -set in nano topological spaces and investigate some of their properties . In this paper, we introduce Nano $(\boldsymbol{\Lambda},\boldsymbol{\Psi})$ -Closed sets , Nano $(\boldsymbol{\Lambda},\boldsymbol{\Psi})$ -Open sets and Nano $\lambda\Psi$ generalized Closed sets. Also we introduced their characterizations and also established their properties and relationships with other classes of early defined forms.

2 Preliminaries

Definition 2.1. [2] Let U be the Universe and R be an equivalence relation U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where X ⊆ U. $\tau_R(X)$ satisfies the following axioms:

(1) U and φ ∈ $\tau_R(X)$.

(2) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$. We call $(U, \tau_R(X))$ is a nano topological space. The elements of $\tau_R(X)$ are called a open sets and the complement of a nano open set is called nano closed sets.

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U , U/R denotes the family of equivalence classes of U by R.

Definition 2.2. [2] If $(U, \tau_R(X))$ is a nano topological space with respect to X . Where X ⊆ G and if A ⊆ G,then

- The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint(A) , Nint(A) is the largest nano open subset of A.
- The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A) . Ncl(A) is the smallest nano closed set containing A.

Definition 2.3. [2] Let $(U, \tau_R(X))$ be a nano topological space and A ⊆ G.Then A is said to be

(i) Nano semi-open if A ⊆ Ncl(Nint(A)

(ii) Nano α-open if A ⊆ Nint(Ncl(Nint(A))

(iii) Nano regular-open (briefly Nr) if A = Nint(Ncl(A)

Definition 2.4. [5] Let A be a subset of a nano topological space $(U, \tau_R(X))$. A subset $N\Lambda_\psi(A)$ is defined as $N\Lambda_\psi = \cap\{H/A \subseteq H \text{ and } H \in N\psi\, O(U, \tau_R(X))\}$.

Definition 2.5. [5] A subset A of a nano topological space $(U, \tau_R(X))$ is called a $N\Lambda_\psi(A)$ -set if $A = N\Lambda_\psi(A)$. The set of all $N\Lambda_\psi(A)$ -sets is denoted by NΛψ$(U, \tau_R(X))$.

Definition 2.6. Let $(U, \tau_R(X))$ be a nano topological space and A ⊆ G. Then A is said to be

1. Nano generalized closed (briefly Ng) [1] if Ncl(A) ⊆ G whenever A ⊆ G and G is nano-open in U .
2. Nano semi generalized closed (briefly Nsg) [1] if Nscl(A) ⊆ G whenever A ⊆ G and G is nano-semi open in U .
3. Nano Ψ -closed [6] if Nscl(A) ⊆ G whenever A ⊆ G and G is nano-sg open in U .

Lemma 2.7. [5] The intersection of any two $N\Lambda_\psi$-sets are $N\Lambda_\psi$-set.

3 Nano (Λ, Ψ) -closed sets and Nano (Λ, Ψ)- open sets.

Definition 3.1. Let A be a subset of a nano topological space $(U, \tau_R(X))$. A subset $N(\Lambda, \Psi)$ closed if A = B ∩ C , where B is $N\Lambda_\psi$ set and C is a Nψ closed set. The family of all $N(\Lambda, \Psi)$-closed sets is denoted by $N\Lambda_\psi C(U, \tau_R(X))$.

Remark 3.2. The complement of $N(\Lambda, \Psi)$-closed set is called the $N(\Lambda, \Psi)$-open set.

Theorem 3.3. If A is $N\Psi$ -closed then it is $N(\Lambda, \Psi)$-closed.

Proof. Let A be a $N\Psi$- closed set. Then $A = U \cap A$, where U is $N\Lambda_\psi$-set and A is a $N\Psi$-closed set. Hence A is a $N(\Lambda, \Psi)$ - closed set.

Remark 3.4. The converse of the above theorem need not be true as shown in the following example.

Example 3.5. Let U={a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X ={a, d} . Then $\tau_R(X)$ = {U, φ, {a}, {b, d}, {a, b, d}} . Then $N\Psi C(U, \tau_R(X)) = \{\varphi, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{b, c, d\}, U\}$ and$N\Lambda_\psi C(U, \tau_R(X)) = \{\varphi, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, U\}$. Here {a,b,d} is $N(\Lambda, \Psi)$- closed but is not $N\Psi$-closed.

Theorem 3.6. If A is $N\Lambda_\psi$-closed then it is $N(\Lambda, \Psi)$-closed.

Proof. Let A be a $N\Lambda_\psi$- closed set. Then $A = U \cap A$, where U is $N\Psi$-closed and A is a $N\Lambda_\psi$-closed set. Hence A is a $N(\Lambda, \Psi)$ - closed set.

Remark 3.7. The converse of the above theorem need not be true as shown in the following example.

Example 3.8. Let U= {a, b, c, d} with U/R = {{a}, {b,c, d}} and X ={b,c} . Then $\tau_R(X)$ = {U, φ, {b, c, d} } . Then $N\Lambda_\psi C(U, \tau_R(X)) = \{\varphi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{b, c, d\}, U\}$ and$N(\Lambda, \Psi)C(U, \tau_R(X)) = P(U)$. Here {a,b,c} is $N(\Lambda, \Psi)$- closed but is not $N\Lambda_\psi$-closed.

Theorem 3.9. For a subset A of a nano topological space $(U, \tau_R(X))$ the following statements are equivalent

1. A is a $N(\Lambda, \Psi)$-closed set
2. $A = B \cap N\Psi cl(A)$, where B is a $N\Lambda_\Psi$-set
3. $A = N\Lambda_\Psi(A) \cap N\Psi cl(A)$

Proof. (1) $\Rightarrow$ (2) Let $A = B \cap C$, where B is a $N\Lambda_\Psi$-set and C is a $N\Psi$-closed set. Now, $A \subseteq C$ and C is $N\Psi$-closed which implies $N\Psi cl(A) \subseteq N\Psi cl(C) = C$. That implies, $N\Psi cl(A) \subseteq C$ and $A = B \cap C \supseteq B \cap N\Psi cl(A) \supseteq A$. Hence $A = B \cap N\Psi cl(A)$, where B is a $N\Lambda_\Psi$-set.

(2) $\Rightarrow$ (3) Let $A = B \cap N\Psi cl(A)$, where B is a $N\Lambda_\Psi$-set. Now, $A \subseteq B$ and B is a $N\Lambda_\Psi$-set which impiles $N\Lambda_\Psi(A) \subseteq B$ and $A \subseteq N\Lambda_\Psi(A) \cap N\Psi cl(A) \subseteq B \cap N\Psi cl(A) = A$. Hence $A = N\Lambda_\Psi(A) \cap N\Psi cl(A)$.

(3) $\Rightarrow$ (1) Since $N\Lambda_\Psi(A)$ is a $N\Lambda_\Psi$-set, $N\Psi cl(A)$ is $N\Psi$-closed and $A = N\Lambda_\Psi(A) \cap N\Psi cl(A)$, by definition 3.1, we have A is a $N(\Lambda, \Psi)$-closed set.

Theorem 3.10. For a subset $A_i (i \in I)$ of a nano topological space $(U, \tau_R(X))$ the following properties hold:

1. U and ϕ are both $N(\Lambda, \Psi)$-closed and also $N(\Lambda, \Psi)$-open.
2. If A_i is $N(\Lambda, \Psi)$-closed for each $i \in I$, then $\bigcap\{A_i / i \in I\}$ is $N(\Lambda, \Psi)$-closed.

Proof. (1) It is obvious.

(2) Let A_i be a $N(\Lambda, \Psi)$- closed set for each $i \in I$. Therefore, for each $i \in I$, there exist a $N\Lambda_\Psi$-set B_i and a $N\Psi$-closed set C_i such that $A_i = B_i \cap C_i$. $\bigcap_{i\in I} A_i = \bigcap_{i\in I} (B_i \cap C_i) = (\bigcap_{i\in I} B_i) \cap (\bigcap_{i\in I} C_i)$. By lemma 2.7, $\bigcap_{i\in I} B_i$ is a $N\Lambda_\Psi$-set and $\bigcap_{i\in I} C_i$ is a $N\Psi$-closed set. Therefore, $\bigcap_{i\in I} A_i$ is the intersection of a $N\Lambda_\Psi$- set and a $N\Psi$-closed set. Hence $\bigcap_{i\in I} A_i$ is $N(\Lambda, \Psi)$-closed.

4 Nano λΨ generalized closed set

Definition 4.1. A subset A of a nano topological space $(U, \tau_R(X))$ is called Nano λΨ generalized closed set (briefly $N\lambda\Psi g$-closed) if $N\Psi cl(A) \subseteq H$, whenever $A \subseteq H$ and H is $N(\Lambda, \Psi)$-open in U.

The family of all $N\lambda\Psi g$-closed set of $(U, \tau_R(X))$ called by $N\lambda\Psi GC((U, \tau_R(X))$.

Theorem 4.2. Every nano closed set is $N\lambda\Psi g$ - closed.

Proof. Let A be a nano closed and H be a $N(\Lambda, \Psi)$-open set containing A. Since A is nano-closed, we have Ncl(A) = A. But $N\Psi cl(A) \subseteq Ncl(A) = A \subseteq H$. Hence A is $N\lambda\Psi g$ -closed.

Remark 4.3. The converse of the above theorem need not be true as shown in the following example.

Example 4.4. Let U= {a, b, c, d} with U/R = {{a}, {b},{c, d}} and X ={a,c} . Then $\tau_R(X)$ = {U, φ,{a},{c,d}, {a, c, d} } . Then $N\lambda\Psi GC(U, \tau_R(X)) = \{ U, \varphi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{b, c, d\}$. Here {c} is $N\lambda\Psi g$ - closed but is not nano closed.

Theorem 4.5. Every $N\Psi$- closed set is $N\lambda\Psi g$ - closed.

Proof. Let A be a $N\Psi$ -closed and H be a $N(\Lambda, \Psi)$-open set containing A. Since A is $N\Psi$ -closed, we have $N\Psi$cl(A) = A. Therefore $N\Psi cl(A) = A \subseteq H$. Hence A is $N\lambda\Psi g$ -closed.

Remark 4.6. The converse of the above theorem need not be true as shown in the following example.

Example 4.7. Let U={a, b, c, d} with U/R = {{b}, {c},{a, d}} and X ={a,b} . Then $\tau_R(X)$ = {U, φ,{b},{a,d}, {a, b, d} } . Then $N\Psi C(U, \tau_R(X))$ = {U,ϕ,{b},{c},{a,d},{b,c},{a,c,d}} and $N\lambda\Psi GC(U, \tau_R(X))$ = { U,ϕ,{a},{b},{c},{d},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}. Here {a} is $N\lambda\Psi g$ - closed but is not $N\Psi$- closed.

Theorem 4.8. Every Nr- closed set is $N\lambda\Psi g$ - closed.

Proof. Let A be a Nr -closed and H be a $N(\Lambda, \Psi)$-open set containing A. Since A is Nr -closed, we have Nrcl(A) = A. But $N\Psi cl(A) \subseteq Nrcl(A) = A \subseteq H$. Hence A is $N\lambda\Psi g$ -closed.

Remark 4.9. The converse of the above theorem need not be true as shown in the following example.

Example 4.10. Let U={a, b, c, d} with U/R = {{a},{b,c, d}} and X ={a,c} . Then $\tau_R(X)$ = {U, φ,{a},{a,d},{b,c, d} } . Then $NRC(U,\tau_R(X))$ ={U,ϕ,{a},{b,c,d}} and $N\lambda\Psi GC(U,\tau_R(X))$ = P(U). Here {b} is $N\lambda\Psi g$ - closed but is not Nr- closed.

Theorem 4.11. Every Ns- closed set is $N\lambda\Psi g$ - closed.

Proof. Let A be a Ns -closed and H be a $N(\Lambda,\Psi)$-open set containing A. Since A is Ns -closed, we have Nscl(A) = A. But $N\Psi cl(A) \subseteq Nscl(A) = A \subseteq H$. Hence A is $N\lambda\Psi g$ -closed.

Remark 4.12. The converse of the above theorem need not be true as shown in the following example.

Example 4.13. Let U={a, b, c, d} with U/R = {{a},{b,c, d}} and X ={b,c} . Then $\tau_R(X)$ = {U, φ,{b,c, d} } . Then $NSC(U,\tau_R(X))$ ={U,ϕ,{a}} and $N\lambda\Psi GC(U,\tau_R(X))$ = P(U). Here {c,d} is $N\lambda\Psi g$ - closed but is not Ns- closed.

Theorem 4.14. Every $N\alpha$- closed set is $N\lambda\Psi g$ - closed.

Proof. Let A be a $N\alpha$ -closed and H be a $N(\Lambda,\Psi)$-open set containing A. Since A is $N\alpha$ -closed, we have $N\alpha$cl(A) = A. But $N\Psi cl(A) \subseteq N\alpha cl(A) = A \subseteq H$. Hence A is $N\lambda\Psi g$ -closed.

Remark 4.15. The converse of the above theorem need not be true as shown in the following example.

Example 4.16. Let U={a, b, c, d} with U/R = {{a},{b},{c, d}} and X ={a,c} . Then $\tau_R(X)$ = {U, φ,{a},{c,d},{a,c, d} } . Then $N\alpha C(U,\tau_R(X))$ = {U,ϕ,{b}{a,b},{b,c,d}} and $N\lambda\Psi GC(U,\tau_R(X))$ = {U,ϕ, {a},{b},{c},{d},{a,b},{b,c}{c,d},{b,c,d}} . Here {d} is $N\lambda\Psi g$ - closed but is not $N\alpha$- closed.

Theorem 4.17. Every Ng- closed set is $N\lambda\Psi g$ - closed.

Proof. Let A be a Ng –closed in U. Let H be a nano open set in U such that $A \subseteq H$. Since every open set is $N(\Lambda,\Psi)$-open, we have $N\Psi cl(A) \subseteq Ncl(A) \subseteq H$, where H is $N(\Lambda,\Psi)$-open. Hence A is $N\lambda\Psi g$ -closed.

Remark 4.18. The converse of the above theorem need not be true as shown in the following example.

Example 4.19. Let U={a, b, c, d} with U/R = {{b},{c},{a, d}} and X ={a,b} . Then $\tau_R(X)$ = {U, φ,{b},{a,d},{a,b, d} } . Then $NGC(U,\tau_R(X))$ = {U,ϕ,{c}{a,c},{b,c},{c,d},{a,b,c}, {b,c,d},{a,c,d}} and $N\lambda\Psi GC(U,\tau_R(X))$ = {U, ϕ, {a}, {b}, {c},{d},{a,c},{a,d},{b,c}, {c,d}, {a,b,c}, {a,c,d},{b,c,d}} . Here {a} is $N\lambda\Psi g$ - closed but is not Ng- closed.

Theorem 4.20. The intersection of any two subsets of $N\lambda\Psi g$-closed sets in $(U,\tau_R(X))$ is $N\lambda\Psi g$-closed.

Proof. If $A \cap B \subseteq G$ and G is $N(\Lambda,\Psi)$-open sets. Since A and B are $N\lambda\Psi g$-closed, $N\Psi cl(A) \subseteq G$ and $N\Psi cl(A) \subseteq G$, whenever $A \subseteq G$ and $B \subseteq G$ and G is $N(\Lambda,\Psi)$-open and hence $N\Psi cl(A \cap B) = N\Psi cl(A) \cap N\Psi cl(B) \subseteq G$. Hence $N\Psi cl(A \cap B) \subseteq G$. Thus $A \cap B$ is $N\lambda\Psi g$-closed.

Remark 4.21. If the subsets A and B are $N\lambda\Psi g$ -closed sets, their union need not be $N\lambda\Psi g$ -closed set.

Example 4.22. Let U={a, b, c, d} with U/R = {{a},{b},{c, d}} and X ={a,c} . Then $\tau_R(X)$ = {U, φ,{a},{c,d},{a,c, d} } . Then $N\lambda\Psi GC(U,\tau_R(X))$ = {U,ϕ,{a},{b},{c},{d}, {a,b},{b,c},{c,d}, {b,c,d}}. Here {a} and {d} are $N\lambda\Psi g$ - closed but there union {a,d} is not $N\lambda\Psi g$- closed.

Theorem 4.23. Let A be a $N\lambda\Psi g$-closed set in $(U,\tau_R(X))$. Then $N\Psi cl(A) - A$ does not contain a non empty $N(\Lambda,\Psi)$-closed set.

Proof. Suppose that A is $N\lambda\Psi g$-closed . Let H be a $N(\Lambda,\Psi)$-closed set contained in $N\Psi cl(A) - A$. Now H^c is a $N(\Lambda,\Psi)$-open set in U such that $A \subseteq H^c$. Since A is a $N\lambda\Psi g$-closed, $N\Psi cl(A) \subseteq H^c$. Thus

$H\subseteq(N\Psi cl(A))^c$. Also $H\subseteq N\Psi cl(A) - A$. Therefore $H\subseteq(N\Psi cl(A))^c \cap N\Psi cl(A) = \phi$. Hence $N\Psi cl(A) - A$ does not contain a non empty $N(\Lambda,\Psi)$-closed set.

Theorem 4.24. If A is a $N(\Lambda,\Psi)$ -open set and $N\lambda\Psi g$ -closed set of $(U,\tau_R(X))$ then A is a $N\Psi$ -closed set of U .

Proof. Let A be $N(\Lambda,\Psi)$-open and $N\lambda\Psi g$ -closed then $N\Psi cl(A) - A$. Hence A is $N\Psi$ -closed.

Theorem 4.25. If A is a $N\lambda\Psi g$ -closed set and $N(\Lambda,\Psi)$ -open and H is $N\Psi$ - closed in $(U,\tau_R(X))$, then $A \cap H$ is $N\Psi$ -closed.

Proof. Let A be $N\lambda\Psi g$ -closed and $N(\Lambda,\Psi)$-open. By theorem 4.24, A is $N\Psi$ -closed. Since H is $N\Psi$ -closed in U , $A \cap H$ is $N\Psi$ -closed in U .

Theorem 4.26. If A is a $N\lambda\Psi g$ -closed set in $(U,\tau_R(X))$ and $A \subseteq B\subseteq N\Psi cl(A))$, then B is also a $N\lambda\Psi g$ -closed set.

Proof. Let G be a $N(\Lambda,\Psi)$-open set of U containing B . Then$A\subseteq G$. Since A is $N\lambda\Psi g$ -closed, $\mathrm{N\Psi cl(A)} \subseteq$G. Also since $\mathrm{B \subseteq N\Psi cl(A), N\Psi cl(B)\subseteq N\Psi cl(N\Psi cl(A)) = N\Psi cl(A)}$. Hence $\mathrm{N\Psi cl(B) \subseteq G}$ and therefore B is $N\lambda\Psi g$ - closed.

Theorem 4.27. Let A be a $N\lambda\Psi g$ -closed set of$(U,\tau_R(X))$. Then A is NΨ- closed iff NΨcl(A)– A is $N(\Lambda,\Psi)$-closed.

Proof. **Necessity:** Let A be a NNΨ -closed subset of $(U,\tau_R(X))$. Then $\mathrm{N\Psi cl(A) = A}$and so $\mathrm{N\Psi cl(A) - A = \varphi}$, which is $N(\Lambda,\Psi)$-closed.

Sufficiency: Let $\mathrm{N\Psi cl(A) - A}$ be $N(\Lambda,\Psi)$ -closed. Since A is $N\lambda\Psi g$ -closed, by theorem 4.23, $\mathrm{N\Psi cl(A) - A}$ does not contain a non-empty $N(\Lambda,\Psi)$-closed set which implies $\mathrm{N\Psi cl(A) - A = \varphi}$. Therefore $\mathrm{N\Psi cl(A) = A}$ and hence A is $N\Psi$ -closed.

Definition 4.28. A subset A of a nano topological space $(U,\tau_R(X))$ is called $N\lambda\Psi g$ -open if its complement A^c is $N\lambda\Psi g$ -closed in $(U,\tau_R(X))$. The family of all $N\lambda\Psi g$ -open sets in $(U,\tau_R(X))$ is denoted by $\mathrm{N\lambda\Psi GO}(U,\tau_R(X))$.

Lemma 4.29. For a subset A of $(U,\tau_R(X))$, $\mathrm{N\Psi int(U - A) = U - N\Psi cl(A)}$.

Proof. Now, $\mathrm{N\Psi int(A) \subseteq A \subseteq N\Psi cl(A)}$. Hence $\mathrm{U - N\Psi cl(A)\subseteq U - A \subseteq U - N\Psi int(A)}$. Therefore $\mathrm{U - N\Psi cl(A)}$ is the $N\Psi$ -open set contained in U − A. But $\mathrm{N\Psi int(U - A)}$ is the largest $N\Psi$ -open set contained in U − A. Thus $\mathrm{U - N\Psi cl(A) \subseteq N\Psi int(U - A)}$. On the other hand, if $\mathrm{x \in N\Psi int(U - A)}$, there exists a $N\Psi$ -open set G containing x such that$\mathrm{G \subseteq U - A}$. Hence $\mathrm{G \cap A = \varphi}$. Therefore, $\mathrm{x \notin N\Psi cl(A)}$ and hence $\mathrm{x \in (U - N\Psi cl(A))}$. Thus $\mathrm{N\Psi int(U - A) \subseteq U - N\Psi cl(A)}$. Hence $\mathrm{N\Psi int(U - A) = U - N\Psi cl(A)}$.

Theorem 4.30. A subset A of a nano topological space $(U,\tau_R(X))$ is $N\lambda\Psi g$ - open if and only if $\mathrm{F \subseteq N\Psi int(A)}$, whenever $\mathrm{F \subseteq A}$ and F is $N(\Lambda,\Psi)$-closed.

Proof. **Necessity:** Assume that A is $N\lambda\Psi g$ -open. Then A^c is $N\lambda\Psi g$ -closed. Let F be a $N(\Lambda,\Psi)$-closed set in $(U,\tau_R(X))$ such that $\mathrm{F \subseteq A}$. Then F^c is $N(\Lambda,\Psi)$-open in $(U,\tau_R(X))$ such that $A^c\subseteq F^c$. Since A^c is $N\lambda\Psi g$ -closed, $\mathrm{N\Psi cl(A^c)\subseteq F^c}$. Also since $\mathrm{N\Psi cl}(A^c) = \{\mathrm{N\Psi int(A)}\}^c \subseteq A^c$. Hence $\mathrm{F \subseteq N\Psi int(A^c)}$.

Sufficiency: Conversely, assume that F ⊆ NΨint(A) , whenever F ⊆ A and F is $N(\Lambda,\Psi)$-closed in $(U,\tau_R(X))$. Then $A^c \subseteq F^c$ and F^c is $N(\Lambda,\Psi)$-open. Take $G = F^c$, since F ⊆ NΨint(A), NΨcl(A^c) ⊆ F^c = G. Also since NΨcl(A^c) = {NΨint(A)}^{c} ⊆ U. Thus A^c is $N\lambda\Psi g$ -closed and hence A is $N\lambda\Psi g$ -open.

Proposition 4.31. If NΨint(A) ⊆ B ⊆ A and A is $N\lambda\Psi g$ -open in $(U,\tau_R(X))$, then B is $N\lambda\Psi g$ -open in $(U,\tau_R(X))$.

Proof. Suppose that A is $N\lambda\Psi g$ -open and NΨint(A) ⊆ B ⊆ A. Let F be a $N(\Lambda,\Psi)$-closed and F ⊆ B. Since F ⊆ B, B ⊆ A . Since A is $N\lambda\Psi g$ -open, F ⊆ NΨint(A) . Since NΨint(A) ⊆ B , NΨint(NΨint(A)) ⊆ NΨint(B). Then NΨint(A) ⊆ NΨint(B). Since F ⊆ NΨint(A), NΨint(F) ⊆ NΨint(B). Therefore B is $N\lambda\Psi g$ –open.

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