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PSEUDO IRREGULAR GRAPHS

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Abstract

In this paper, we have introduced pseudo irregular graphs and have also discussed about various types of pseudo irregular graphs. Here, we have also studied about various types of pseudo irregular graphs containing a given graph as an induced subgraph.

Keywords: **H**ighly irregular graph, support highly irregular graphs, pseudo irregular graphs, induced subgraph.

AMS subject classification: Primary: 05C12.

1 Introduction

The concept of Highly irregular graph was introduced in 1987 by Yousef Alavi, Gary Chartrand, F.R.K.Chung, Paul Erdos, R.L. Graham, Ortrud R. Oellermann in [2]. Gary Chartrand, Paul Erdos, OrtrudR.Oellermann discussed how to define an irregular graph [6]. The concept of Neighbourly irregular graphs was introduced and studied by S. Gnana Bhragasam and S.K. Ayyaswamy [5]. N.R. Santhi Maheswari and C. Sekar introduced the concept of semi neighbourly irregular graphs [10]. Likewise many types of irregular graphs have been identified. Dasong Cao called $2-degree$ of v [4] as the sum of the degrees of the vertices adjacent to v, for $v \in V(G)$ and it is denoted by t(v). Aimei Yu, Mei Lu and Feng Tian introduced average degree(pseudo degree) of a vertex v[1]. N.R. SanthiMaheswari and C. Sekar defined 2-degree and pseudo degree of a vertex in pseudo regular fuzzy graphs [19]. N.R. SanthiMaheswari and M. Sudha introduced pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs[17]. N.R. SanthiMaheswari and Rajeswari introduced strongly pseudo irregular fuzzy graph and N.R.SanthiMaheswari and Karpagavalli introduced pseudo regularity on some fuzzy graphs[15, 16]. N.R.SanthiMaheswari and C. Sekar introduced pseudo degree and total pseudo degree in bipolar fuzzy graphs, pseudo regular bipolar fuzzy graph, pseudo irregular bipolar fuzzy graphs, neighbourly pseudo and strongly pseudo irregular bipolar fuzzy graphs and discussed some of its properties[20, 22]. N.R. SanthiMaheswari and K. Amutha introduced pseudo edge regular, pseudo neighbourly edge irregular graph[12].

N.R. SanthiMaheswari and K. Amutha introduced support neighbourly edge irregular graphs[11]. K. Priyadharshini and N.R. SanthiMaheswari introduced support highly irregular graphs[13]. K. Priyadharshini and N.R. SanthiMaheswari also introduced support highly irregular graph containing a given graph [14].These ideas motivate us to introduce pseudo irregular graphs and various types of pseudo irregular graphs containing a given graph as an induced subgraph.

2 Preliminaries

Definition 2.1 *The degree of a vertex v in a graph G is the number of lines incident with v. It is denoted by $d_G(v)$ or $d(v)$ in a graph G.*

Definition 2.2 *In a graph* $G(V,E)$, *for any vertex* $v \in V$, *the open neighbourhood of* v *is the set of all vertices adjacent to* $v.(i.e)$ $N(v) = \{u \in V(G)/uv \in E(G)\}$. *The closed neighbourhood of* v *is defined by* $N[v] = N(v) \cup \{v\}$.

Definition 2.3 *A graph* G *is said to be* r- *regular if all the vertices of* G *have the same degree* r. $(i.e)$ $\delta(G) = \Delta(G) = r$.

Definition 2.4 *A graph* G *is said to be neighbourly irregular if no two adjacent vertices of* G *have the same degree[5].*

Definition 2.5 *A graph* G *is said to be highly irregular if each of its vertices is adjacent only to vertices with distinct degree[2].*

Definition 2.6 *The (2-degree) support s(v) of a vertex v is the sum of degrees of its neighbours in the graph* G. *That is,* $s(v) = \sum_{u \in N(v)} d(u)$*[1].*

Definition 2.7 *The support* $s_G(e)$ *or simply* $s(e)$ *of an edge* e *is the sum of edge degrees of its neighbour edges in the graph* $G.(i.e)$,$s(e) = \sum_{e_i \in N(e)} d(e_i)$*[11].*

Definition 2.8 *The average degree(pseudo degree) of a vertex is defined as* $t(v)/d(v)$, *where* $t(v)$ *is 2-degree of* v *and* $d(v)$ *is the degree of* v.*[1]*

3 Pseudo Irregular Graphs

Definition 3.1 *A connected graph G is said to be pseudo irregular graph if there exist a vertex which is adjacent to the vertices with distinct pseudo degrees.*

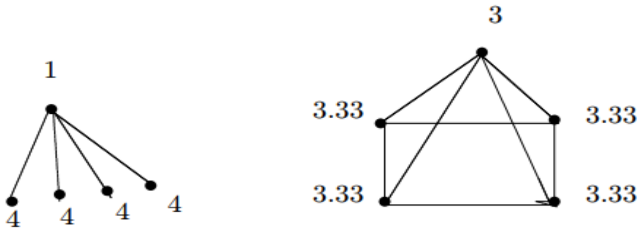


Figure 1

Here, we have discussed about some types of irregularity in pseudo degrees.

4 Pseudo Neighbourly Irregular Graphs Containing a Given Graph

In this section we have discussed about pseudo neighbourly irregular graphs and some of its properties.

Definition 4.1 *If any two adjacent vertices of a connected graph* G *have distinct pseudo degree, then* G *is called a pseudo neighbourly irregular graph[13]". The graph given below is a* PNI *graph of order* 5.

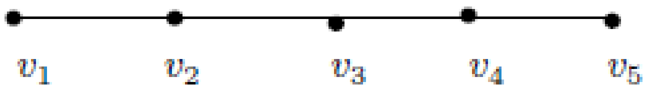


Figure 2

Theorem 4.2 *Every path* P_n, $n \geq 6$ *is an induced subgraph of* PNI *graph of order* $n + [\frac{n}{4}] - 1$ *if* n *is even and* $n = 9 + 4k$, $k \geq 0$ *and of order* $n + [\frac{n}{4}]$ *if* $n = 7 + 4k$,$k \geq 0$.

Proof. Case: 1

Consider P_n, $n \geq 6$ where n is even.

Attach a pendant vertex at each v_{5+4k},$k \geq 0$.

Case: 2

Consider P_n, where $n = 7 + 4k$, $k \geq 0$. Attach a pendant vertex at v_{5+4k}, $k \geq 0$ and attach another pendant vertex at v_{n-1}. Hence we get the required PNI graph of order $n + [\frac{n}{4}]$. Case: 3

Consider P_n where $n = 9 + 4k$, $k \geq 0$. It is same as Case-1.

Hence we get the required PNI graphs containing the path P_n, $n \geq 6$.

Example 4.3 *The graphs shown below are the examples of* PNI *graph containing a given graph as constructed in the above proof.*

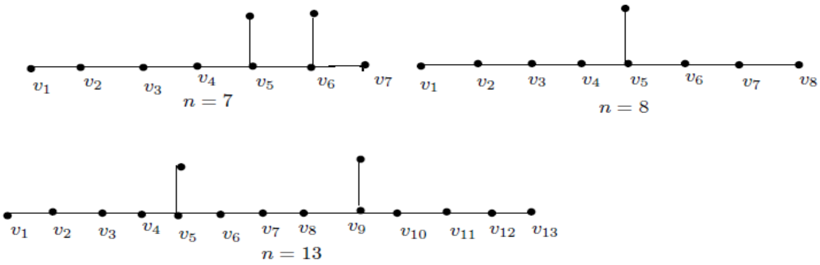


Figure 3

Theorem 4.4 *Every cycle* C_n *is an induced subgraph of* PNI *of order* $2n - 1$.

Proof. Consider a cycle C_n, $n \geq 3$. Now introduce $n - 1$ new vertices $u_1, u_2, u_3, \ldots, u_{n-1}$. Attach v_i to each u_j for j. Hence we get the required PNI graph of order $2n - 1$.

Example 4.5 *The following graph is an example for* PNI *graph containing a cycle* C_5 *of order* 9.

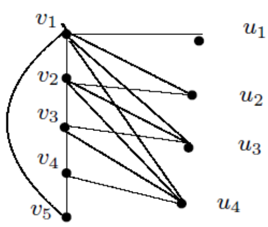


Figure 4

Theorem 4.6 *Every Complete graph* K_n *is an induced subgraph of* PNI *graph of order* $3n$.

Proof. Consider two copies of G, G_1 and G_2. Let $V_1 = \{u_1, u_2, \ldots, u_n\}$ be the vertices of G_1 and $V_2 =$

$\{v_1, v_2, \dots, v_n\}$ be the vertices of G_2. Introduce n vertices corresponding to v_n. Now join u_i to w_j $\forall$ $j > i$ and join w_j to each v_i where $j > i$. Hence we get the required PNI graph of order $3n$.

Example 4.7 *The following is an example of* PNI *graph containing a complete graph* K_5 *as constructed in the above proof.*

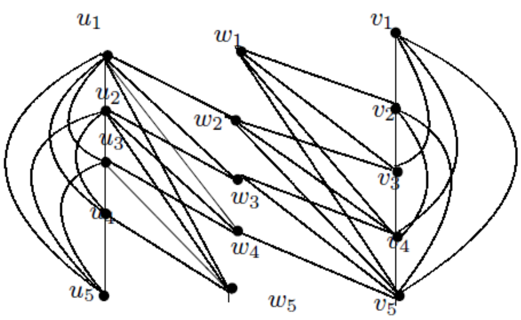


Figure 5

Theorem 4.8 *Every Path* P_n, $n \geq 6$ *is an induced subgraph of* PNI *graph of order* $n + \lfloor \frac{n}{4} \rfloor$ *if* n *is odd and* $n + \lfloor \frac{n}{4} \rfloor + 1$ *if* n *is even.*

Proof. Case: 1, n is odd

Consider a path $P_n, n \geq 6$. Attach a pendant vertices at every v_{4i}, $i \geq 1$. Hence we get the required PNI graph containing a path P_n of order $n + \lfloor \frac{n}{4} \rfloor$.

Case: 2, n is even

Attach the pendant vertices at every v_{4i}, $i \geq 1$. Now attach 2 pendant vertices if v_{4i} is v_n or v_{n-3}. Hence we get the required PNI graph containing a path P_n of order $n + \lfloor \frac{n}{4} \rfloor + 1$.

Example 4.9 *The following is an example of* PNI *graph containing the path* P_7 *and* P_{12} *as constructed in the above proof.*

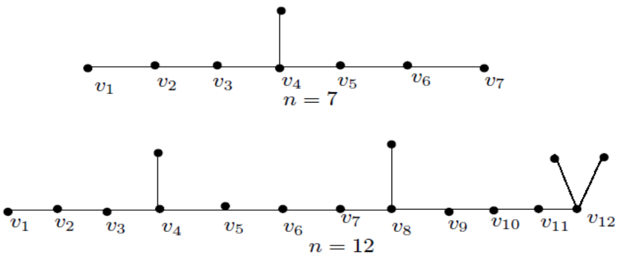


Figure 6

Theorem 4.10 *Every Cycle* C_n, $n \geq 6$ *is an induced subgraph of* PNI *graph of order* $n + \lceil \frac{n}{4} \rceil$ *if* n *is even and* $n + \lfloor \frac{n}{3} \rfloor + 1$ *if* n *is odd.*

Proof. Case: 1, where n is even.

By adding a pendant vertex at every v_{4i+1}, we get the required PNI graph containing C_n, $n \geq 6$ of order $n + \lceil \frac{n}{4} \rceil$.

Case: 2, where n is odd.

Add a pendant vertex at every v_{4i+1} and attach two pendant vertex if v_{4i+1} is v_n or v_{n-2}. Hence we get the required PNI graph containing a C_n, $n \geq 6$ of order $n + \lfloor \frac{n}{3} \rfloor + 1$.

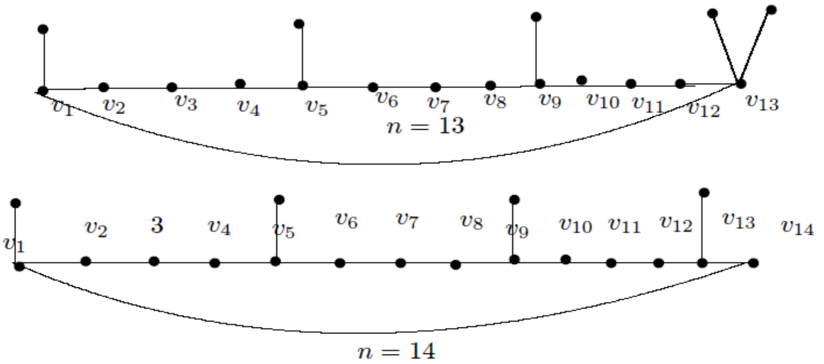


Figure 7

5 Pseudo Highly Irregular Graphs Containing a Given Graph

Definition 5.1 *" If every vertex of a connected graph G is adjacent only to the vertices with distinct pseudo degree, then G is called a pseudo highly irregular graph[13]". The graph given below is a PHI graph of order* 4.

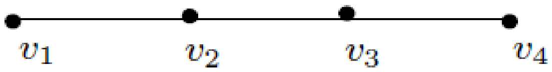


Figure 8

Theorem 5.2 *Every path P_n of order $n \geq 6$ is an induced subgraph of a PHI graph of order $2n$.*

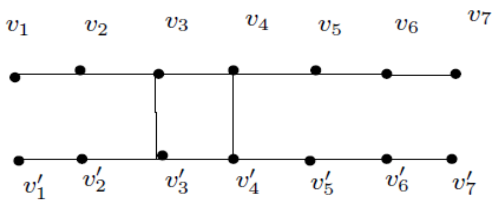
Proof. Let P_n be any path of order $n \geq 6$. Take two copies of P_n as S_1 and S'_1. Let the vertices of S_1 and S'_1 be $\{v_1, v_2, \ldots, v_n\}$ and $\{v'_1, v'_2, \ldots, v'_n\}$. Then we shall add some edges to complete the construction of PHI graph.

(i) Join v_{3+4i} and v'_{3+4i} for all $i \geq 0$ and stop if v_{3+4i} is v_n.

(ii) Join v_{4+4i} and v'_{4+4i} for all $i \geq 0$ and stop if v_{4+4i} is v_{n-1} or v_n.

Hence we get the required PHI graph of order $2n$.

Example 5.3 *The PHI graph containing the path of order 7 and 8 as given in the above construction is given below:*



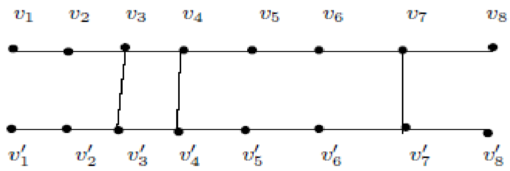


Figure 9

Theorem 5.4 *Every cycle C_n,$n \geq 5$ is an induced subgraph of a PHI graph of order $2n-4$. Proof.* Consider any cycle C_n of order $n \geq 5$.

Introduce $n-4$ new vertices $u_1, u_2, u_3, \ldots, u_{n-4}$. Join $v_i u_i$, for all i correspondingly. Next join $v_i u_j$ for i $\leq j$. Hence we get a required PHI graph containing C_n,$n \geq 5$ as an induced subgraph of order 2n-4.

Example 5.5 *The PHI graph containing the cycle of order 5 and 7 as given in the above construction is given below:*

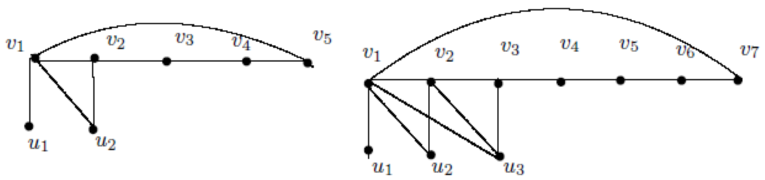


Figure 10

Theorem 5.6 *Every complete graph K_n,$n \geq 2$ is an induced subgraph of a PHI graph of order $4n$.*

Proof. Consider G_1 and G'_1 as two copies of K_n. Introduce $2n$ new vertices $u_1, u_2, u_3, \ldots, u_n$ and $u'_1, u'_2, u'_3, \ldots, u'_n$. The construction is as follows:

(i) Join $v_i u_j$, $1 \leq$ i $\leq j$, where 1$\leq$ i, j $\leq n$,

(ii) Join $v'_i u'_j$, j $\leq i$, *where* $j = n+1-i$ and

(iii) Join u_i u'_{n+1-i} respectively. Hence we get the desired pseudo highly irregular graph of order $4n$.

Example 5.7 *The PHI graph containing the cycle of order 5 as given in the proof of the above construction is given below:*

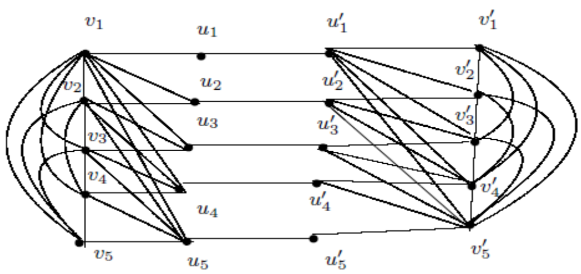


Figure 11

6 Pseudo Strongly Irregular Graphs Containing a given graph

Definition 6.1 *A connected Graph G is called pseudo strongly irregular graph if all the vertices in the graph have distinct pseudo degrees.*

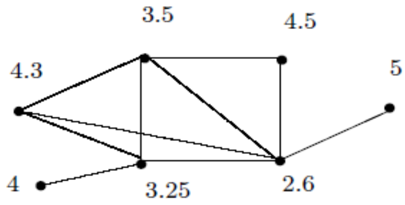


Figure 12

Theorem 6.2 *Every complete graph $K_n,\ n \geq 4$ is an induced subgraph of PSI graph of order $2n$*

Proof. Consider a complete graph $K_n,\ n \geq 4$. Let $V_1 = \{v_1, v_2, \dots, v_n\}$ be the vertices of K_n. Introduce $n - 1$ vertices $\{u_1, u_2, \dots, u_{n-1}\}$. Now join v_i to u_j for $i \leq j$ and attach a pendant vertex at u_{n-1}. Now we get a PSI graph containing a complete graph K_n of order $2n$.

Example 6.3 *The following diagram is an example of PSI graph containing a complete graph K_4 of order* 8

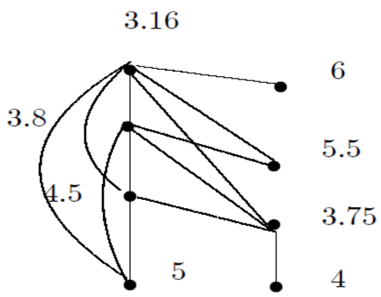


Figure 13

Theorem 6.4 *A graph which is pseudo strongly irregular is both pseudo highly irregular and pseudo neighbourly irregular.*

Proof. Consider a connected graph G which is pseudo strongly irregular. Then each of the vertices of G have distinct pseudo degrees. Therefore G is both pseudo neighbourly irregular and pseudo highly irregular.

Remark 6.5 *Converse of the above theorem need not be true.*

Example 6.6 *The following is an example of both pseudo neighbourly irregular and pseudo highly irregular graphs but not pseudo strongly irregular graph.*

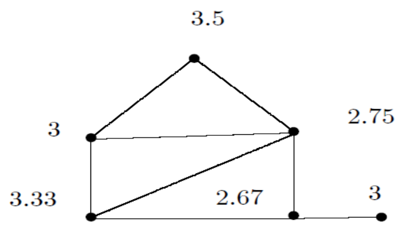


Figure 14

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