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DECOMPOSITION OF $(R\alpha* - \mathrm{H}, \lambda)$ - CONTINUITY

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Abstract:

In this paper we introduce and study the notions of $R\pi*$ - H -open sets, $R\sigma*$ - H -open sets, $R\alpha*$ - H -open sets, $R\beta*$ - H -open sets in hereditary generalized topological spaces. Also we obtained decompositions of $(R\alpha*$ - H, $\lambda)$ -continuity.

1 Introduction

In the year 2002, Csaszar [1] introduced very usefull notions of generalized topology $(G.T.)$ and generalized continuity. A subset A of a space (Z, μ) is μ - α *-open*[2] (resp. μ - σ *-open*[2], μ - π *-open*[2], μ - β *-open*[2]) , if $A \subset i_\mu c_\mu i_\mu(A)$ (resp. $A \subset c_\mu i_\mu(A)$, $A \subset i_\mu c_\mu(A)$, $A \subset c_\mu i_\mu c_\mu(A)$). A subset A of X is said to be μ -regular open, if $A = i_\mu c_\mu(A)$ [4]. A space X is called a C_0 -space [17], if $C_0 = Z$, where C_0 is the set of all representative elements of sets of μ . A nonempty family H of subsets of Z is said to be a *hereditary class* [3], if $A \in$ H and $M \subset A$, then $M \in$ H. A $G.T.S.$ (Z, μ) with a hereditary class H is hereditary generalized topological space $(H.G.T.S.)$ and denoted by (Z, μ, H). For each $A \subseteq X$, $A^*(\mathrm{H}, \mu) = \{z \in X : A \cap M \notin \mathrm{H}$ for every $M \in \mu$ such that $z \in M\}$ [3]. For $A \subset Z$, define $c^*\mu(A) = A \cup A^*(\mathrm{H}, \mu)$ and $\mu^* = \{A \subset Z : Z - A = c^*\mu(Z - A)\}$. Let A be a subset of $H.G.T.S.$ (Z, μ, H) is α - H -open [3](resp. σ - H -open [3],

π - H -open [3], β - H -open [3]), if $A \subseteq i_\mu c^*\mu i_\mu(A)$ (resp. $A \subseteq c^*\mu i_\mu(A)$, $A \subseteq i_\mu c^*\mu(A)$, $A \subseteq c_\mu i_\mu c^*\mu(A)$.)

2 $R\pi*$ - H -open sets

Definition 2.1. *Finite union of μ -regular open sets in (X, μ) is*

called R_π -open in (X, μ). The complement of R_π -open in (X, μ) is R_π -closed in

(X, μ).

Definition 2.2. *Let A be a subset of a hereditary generalized topological space*(X, μ, H). *Then* $A^{*\pi}(H, \mu) = \{x \in X : A \cap U$

define $c^{*\pi}(A) = A \cup A^{*\pi}$.

$\notin$ H, $\forall$ $U \in R_\pi(\mu)\}$. *For* $A \subset X$

Definition 2.3. *Let* (X, μ, H) *be a hereditary generalized topological space. A subset*

A of X is said to be $R\pi*$ - H -open set, if $A \subseteq i_\mu c^{*\pi}(A)$.

Theorem 2.4. *Let* (X, μ, H) *be a hereditary generalized topological space. Then*

1. *Every μ -open is $R\pi*$ - H -open set*
2. *Every π - H -open is $R\pi*$ - H -open set*

Proof. (1). Let a subset A of a hereditary generalized topological space (X, μ, H) is μ -open. Then $A = i_\mu(A)$. Now $A \subset i_\mu(A) \subset i_\mu c^{*\pi}(A)$. Hence A is $R\pi*$ - H -open set.

(2) Let a subset A of a hereditary generalized topological space (X, μ, H) is π - H - open. Then $A \subset i_\mu c\mu* (A)$. Now $A \subset i_\mu c^*\mu(A) \subset i_\mu c^{*\pi}(A)$. Hence A is $R\pi*$ - H -open set.

Example 2.5. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\},$

$X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}\}$. *Then* $A = \{a, b, c\}$ *is* $R\pi*$ - H *-open but not* μ *-open.*

Example 2.6. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}, \{b, c, d\},$

$X\}$ *and* $H = \{\emptyset, \{a\}, \{b\}\}$. *Then* $A = \{b\}$ *is* $R\pi*$ - H *-open but not* π - H *-open.*

Theorem 2.7. *If* $H = \emptyset$, *then every* $R\pi*$ - H *-open set is* μ *-open.*

Proof. Let a subset A of X is $R\pi*$ - H -open set and $H = \emptyset$. Now $A \subset i_\mu c^{*\pi}(A) =$ $i_\mu(A)$, which implies $A \subset i_\mu(A)$. Hence A is μ -open.

Theorem 2.8. *If* $H = P(X)$, *then every* $R\pi*$ - H *-open set is* μ *-open.*

Proof. Let a subset A of X is $R\pi*$ - H -open set and $H = P(X)$. Now $A \subset$ $i_\mu c^{*\pi}(A) = i_\mu(A)$, which implies $A \subset i_\mu(A)$. Hence A is μ -open.

Theorem 2.9. *If* $R_\pi(\mu) = \mu$, *then every* $R\pi*$ - H *-open set is* π - H *-open.*

Proof. Let a subset A of X is $R\pi*$ - H -open set and $R_\pi(\mu) = \mu$. Now $A \subset i_\mu c^{*\pi}(A) = i_\mu(A \cup A^{*\pi}) = i_\mu(A \cup A^*) = i_\mu c^*\mu(A)$, which implies $A \subset i_\mu c^*\mu(A)$. Hence A is π - H -open set.

Theorem 2.10. *Let* (X, μ, H) *be a hereditary generalized topological space and X is a* C_0 *-space. If* A *is* $R\pi*$ - H *-open set and* U *is a* μ *-open. Then* $A \cap U$ *is* $R\pi*$ - H *-open set.*

Proof. Let A be a $R\pi*$ - H -open set and U is a μ -open. Then $A \subset i_\mu c^{*\pi}(A)$ and $U = i_\mu(U)$. Now, $A \cap U \subset i_\mu c^{*\pi}(A) \cap i_\mu(U) \subset i_\mu(c^{*\pi}(A) \cap U) \subset i_\mu((A \cup A^{*\pi}) \cap U)$
$\subset i_\mu((A \cap U) \cup (A^{*\pi} \cap U)) \subset i_\mu((A \cap U) \cup (A \cap U)^{*\pi}) \subset i_\mu((A \cap U) \cup (A \cap U)^{*\pi})$
$\subset i_\mu c^{*\pi}(A \cap U)$.

Hence $A \cap U$ is $R\pi*$ - H -open set.

3 $R\sigma*$ - H -open sets

Definition 3.1. *Let* (X, μ, H) *be a hereditary generalized topological space. A subset* A *of* X *is said to be* $R\sigma*$ - H *-open set, if* $A \subseteq c^{*\pi} i_\mu(A)$.

Theorem 3.2. *Let* (X, μ, H) *be a hereditary generalized topological space. Then*

1. *Every* μ *-open is* $R\sigma*$ - H *-open set*
2. *Every* σ - H *-open is* $R\sigma*$ - H *-open set*

Proof. (1). Let a subset A of a hereditary generalized topological space (X, μ, H) is μ -open. Then $A = i_\mu(A)$. Now $A \subset i_\mu(A) \Rightarrow$ $c^{*\pi}(A) \subset c^{*\pi} i_\mu(A) \Rightarrow A \subset c^{*\pi}(A) \subset c^{*\pi} i_\mu(A)$. Hence A is $R\sigma*$ - H -open set.

(2) Let a subset A of a hereditary generalized topological space (X, μ, H) is σ - H - open. Then $A \subset c\mu* \, i_\mu(A)$. Now $A \subset c^*\mu i_\mu(A)$ $\subset c^{*\pi} i_\mu(A)$. Hence A is $R\pi*$ - H -open set.

Example 3.3. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\},$ $X\}$ *and* $H = \{\emptyset, \{a\}\}$. *Then* $A = \{b, d\}$ *is* $R\sigma*$ - H *-open but not* μ *-open.*

Theorem 3.4. *If* $R_\pi(\mu) = \mu$, *then every* $R\sigma*$ - H *-open set is* σ - H *-open.*

Proof. Let a subset A of X is $R\sigma*$ - H -open set and $R_\pi(\mu) = \mu$. Now $A \subset c^{*\pi} i_\mu(A) = (i_\mu(A)) \cup ((i_\mu(A))^{*\pi}) = (i_\mu(A)) \cup$ $((i_\mu(A))^*) = c^*\mu(i_\mu(A))$, which implies $A \subset c^*\mu i_\mu(A)$. Hence A is σ - H -open set.

Theorem 3.5. *A subset A of a hereditary generalized topological space* (X, μ, H) *is* $R\sigma*$ - H *-open set if and only if* $c^{*\pi}(A) =$ $c^{*\pi} i_\mu(A)$.

Proof. Let $A \subset X$ is $R\sigma*$ - H -open set. Then $A \subset c^{*\pi} i_\mu(A)$, which implies $c^{*\pi}(A) \subset c^{*\pi} c^{*\pi} i_\mu(A) = c^{*\pi} i_\mu(A)$. Therefore $c^{*\pi}(A) \subset c^{*\pi} i_\mu(A)$. For any $A \subseteq X$, $c^{*\pi} i_\mu(A) \subset c^{*\pi}(A)$. Hence $c^{*\pi}(A) = c^{*\pi} i_\mu(A)$.

Converse part: Assume that $c^{*\pi} i_\mu(A) = c^{*\pi}(A)$. Clearly $A \subset c^{*\pi}(A) = c^{*\pi} i_\mu(A)$, which implies $A \subset c^{*\pi} i_\mu(A)$. Hence A is $R\sigma*$ - H -open set.

Theorem 3.6. *subset A of a hereditary generalized topological space* (X, μ, H) *is* $R\sigma*$ - H *-open set if and only if there exist a* μ *-open set such that* $U \subseteq A \subseteq c^{*\pi}(U)$.

Proof. Let subset A of a hereditary generalized topological space (X, μ, H) is $R\sigma*$ - H -open set. Then $A \subset c^{*\pi} i_\mu(A)$. Now we consider the μ -open set $U = i_\mu(A)$, which implies $U \subseteq A \subseteq c^{*\pi}(U)$.

Converse part: Let U be μ -open set such that $U \subseteq A \subseteq c^{*\pi}(U)$. Now, $U \subseteq i_\mu(U) \subseteq i_\mu(A)$, which implies $c^{*\pi}(U) \subseteq c^{*\pi}i_\mu(A)$. Hence $A \subseteq c^{*\pi}i_\mu(A)$. Therefore A is $R\sigma*$ - H -open set.

Theorem 3.7. *A subset A of a hereditary generalized topological space* (X, μ, H) $R\sigma*$ - H *-open set and if* $A \subseteq B$, *then* B *is* $R\sigma*$ - H *-open set.*

Proof. A subset A of a hereditary generalized topological space (X, μ, H) $R\sigma*$ - H -open set and $A \subseteq B$. Then $U \subseteq A \subseteq B \subseteq c^{*\pi}(A) \subseteq c^{*\pi}c^{*\pi}(U) = c^{*\pi}(U)$ by Theorem 5.2.4. Hence B is $R\sigma*$ - H -open set by Theorem 5.2.4.

Theorem 3.8. *Let* (X, μ, H) *be a hereditary generalized topological spaces such that if* $U_i \in R\sigma* \mathrm{HO}(X)$ *for each* $i \in \Delta$, *then* $\{U_i : i \in \Delta\} \in R\sigma* \mathrm{HO}(X)$.

Proof. Let $U_i \in R\sigma* \mathrm{HO}(X)$ for each $i \in \Delta$. Then $U_i \subset c^{*\pi}i_\mu(U_i)$. Now

$_{i\ \Delta}(U_i) \subseteq {}_{i\ \Delta}(c^{*\pi}(i_\mu(U_i))) \subseteq {}_{i\ \Delta}((i_\mu(U_i))^{*\pi}) \cup {}_{i\ \Delta}((i_\mu(U_i))) \subseteq$

$(\ i \ \ \Delta((i_\mu(U_i))))^{*\pi} \cup (\ i \ \ \Delta((i_\mu(U_i)))) = c^{*\pi}(i_\mu(\ i \ \ \Delta\, U_i))$.

Hence $\{U_i : i \in \Delta\} \in R\sigma* \mathrm{HO}(X)$. ∈ ∈ ∈ ∈ ∈ ∈ ∈

Theorem 3.9. *Let* (X, μ, H) *be a hereditary generalized topological space and X is a* C_0 *-space. If* A *is* $R\sigma*$ - H *-open set and* U *is a* μ *-open. Then* $A \cap U$ *is* $R\pi*$ - H *-open set.*

Proof. Let A be a $R\sigma*$ - H -open set and U is a μ -open. Then $A \subset c^{*\pi}i_\mu(A)$ and $U = i_\mu(U)$. Now, $A \cap U \subset c^{*\pi}i_\mu(A) \cap i_\mu(U) \subset (i^{*\pi}(A) \cup i_\mu(A)) \cap i_\mu(U) \subset (i^{*\pi}(A) \cap i_\mu(U)) \cup (i_\mu(A) \cap i_\mu(U)) \subset (i^{*\pi}(A) \cap U) \cup (i_\mu(A \cap U)) \subset (i_\mu(A \cap U))^{*\pi} \cap (i_\mu(A \cap U)) \subset c^{*\pi}i_\mu(A \cap U)$. Hence $A \cap U$ is $R\sigma*$ - H -open set.

4 $R\alpha*$ - H **-open sets**

Definition 4.1. *Let* (X, μ, H) *be a hereditary generalized topological space. A subset* A *of* X *is said to be* $R\alpha*$ - H *-open set, if* $A \subseteq i_\mu c^{*\pi}i_\mu(A)$.

Theorem 4.2. *Let* (X, μ, H) *be a hereditary generalized topological space. Then*

1. *Every* μ *-open is* $R\alpha*$ - H *-open set*
2. *Every* α - H *-open is* $R\alpha*$ - H *-open set*
3. *Every* $R\alpha*$ - H *-open set is* $R\pi*$ - H *-open set*
4. *Every* $R\alpha*$ - H *-open set is* $R\sigma*$ - H *-open set*

Proof. (1). Let a subset A of a hereditary generalized topological space (X, μ, H) is μ -open. Then $A = i_\mu(A)$. Now $A \subset i_\mu(A) \Rightarrow c^{*\pi}(A) \subset c^{*\pi}i_\mu(A) \Rightarrow A \subset c^{*\pi}(A) \subset c^{*\pi}i_\mu(A) \subset i_\mu c^{*\pi}i_\mu(A)$. Hence A is $R\alpha*$ - H -open set.

(2)Let a subset A of a hereditary generalized topological space (X, μ, H) is α - H -open. Then $A \subset i_\mu c^{*}\mu i_\mu(A)$. Now $A \subset i_\mu c^{*}\mu i_\mu(A) \subset i_\mu c^{*\pi}i_\mu(A)$. Hence A is $R\alpha*$ - H -open set.

(3)Let a subset A of a hereditary generalized topological space (X, μ, H) is $R\alpha*$ - H -open set. Then $A \subseteq i_\mu c^{*\pi}i_\mu(A) \subset i_\mu c^{*\pi}(A)$. Hence A is $R\pi*$ - H -open set.

(4)Let a subset A of a hereditary generalized topological space (X, μ, H) is $R\alpha*$ - H -open set. Then $A \subseteq i_\mu c^{*\pi}(A)i_\mu(A) \subset c^{*\pi}i_\mu(A)$. Hence A is $R\sigma*$ - H -open set.

Example 4.3. *Let* $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{a, e\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, e\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}\}$. *Then* $A = \{a, c\}$ *is* $R\alpha*$ - H *-open set but not* μ *-open and* $B = \{a, c, d, e\}$ *is* $R\alpha*$ - H *-open set but not* α - H *-open.*

Example 4.4. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}\}$. *Then* $A = \{b, d\}$ *is* $R\sigma*$ - H *-open but not* $R\alpha*$ - H *-open.*

Example 4.5. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}, \{b, c, d\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}, \{b\}\}$. *Then* $A = \{b\}$ *is* $R\pi*$ - H *-open but not* $R\alpha*$ - H *-open.*

Theorem 4.6. *Let* (X, μ, H) *be a hereditary generalized topological spaces such that if* $U_i \in R\alpha* \mathrm{HO}(X)$ *for each* $i \in \Delta$, *then* $\{U_i : i \in \Delta\} \in R\alpha* \mathrm{HO}(X)$.

Proof. Let $U_i \in R\alpha*\,\mathrm{H}O(X)$ for each $i \in \Delta$. Then $U_i \subset i_\mu c^{*\pi} i_\mu(U_i)$.
Now, $\bigcup_{i\in\Delta}(U_i) \subseteq \bigcup_{i\in\Delta}(i_\mu c^{*\pi}(i_\mu(U_i))) \subseteq i_\mu(c^{*\pi}(i_\mu(U_i))) \subseteq i_\mu(\bigcup_{i\in\Delta}((i_\mu(U_i))^{*\pi}) \cup \bigcup_{i\in\Delta}((i_\mu(U_i)))) \subseteq i_\mu((\bigcup_{i\in\Delta}((i_\mu(U_i))))^{*\pi} \cup (\bigcup_{i\in\Delta}((i_\mu(U_i))))) = i_\mu c^{*\pi}(i_\mu(\bigcup_{i\in\Delta} U_i))$.
Hence $\{U_i : i \in \Delta\} \in R\alpha*\,\mathrm{H}O(X)$.

Theorem 4.7. *Let* (X, μ, H) *be a hereditary genralized topological space. If* $A \in R\alpha*\,\mathrm{H}O(X)$ *and* $B \in R\sigma*\,\mathrm{H}O(X)$. *Then* $A \cap B \in R\sigma*\,\mathrm{H}O(X)$.

Proof. Let $A \in R\alpha*\,\mathrm{H}O(X)$ and $B \in R\sigma*\,\mathrm{H}O(X)$. Then $A \subseteq i_\mu c^{*\pi} i_\mu(A)$ and $B \subseteq c^{*\pi} i_\mu(B)$. Now, $A \cap B \subseteq i_\mu c^{*\pi} i_\mu(A) \cap c^{*\pi} i_\mu(B) \subseteq c^{*\pi} i_\mu(A) \cap c^{*\pi} i_\mu(B) \subseteq c^{*\pi}(i_\mu(A) \cap i_\mu(B)) \subseteq c^{*\pi}(i_\mu(A \cap B))$. Therefore $A \cap B \subseteq c^{*\pi}(i_\mu(A \cap B))$. Hence $A \cap B \in R\sigma*\,\mathrm{H}O(X)$.

Theorem 4.8. *Let* (X, μ, H) *be a hereditary genralized topological space. If* $A \in R\alpha*\,\mathrm{H}O(X)$ *and* $B \in R\pi*\,\mathrm{H}O(X)$. *Then* $A \cap B \in R\pi*\,\mathrm{H}O(X)$.

Proof. Let $A \in R\alpha*\,\mathrm{H}O(X)$ and $B \in R\sigma*\,\mathrm{H}O(X)$. Then $A \subseteq i_\mu c^{*\pi} i_\mu(A)$ and $B \subseteq i_\mu c^{*\pi}(B)$. Now, $A \cap B \subseteq i_\mu(c^{*\pi} i_\mu(A) \cap c^{*\pi}(B)) \subseteq i_\mu(c^{*\pi}(A) \cap c^{*\pi}(B)) \subseteq i_\mu(c^{*\pi}(A \cap B))$. Therefore $A \cap B \subseteq i_\mu c^{*\pi}(A \cap B))$. Hence $A \cap B \in R\pi*\,\mathrm{H}O(X)$.

Theorem 4.9. *Let* (X, μ, H) *be a hereditary genralized topological space. If* $A \in R\alpha*\,\mathrm{H}O(X)$ *and* $B \in R\pi*\,\mathrm{H}O(X)$. *Then* $A \cap B \in R\pi*\,\mathrm{H}O(X)$.

Proof. Let $A \in R\alpha*\,\mathrm{H}O(X)$ and $B \in \mu$ Then $A \subseteq i_\mu c^{*\pi} i_\mu(A)$ and $B \subseteq i_\mu c^{*\pi}(B)$. Now, $A \cap B \subseteq i_\mu(c^{*\pi} i_\mu(A) \cap c^{*\pi}(B)) \subseteq i_\mu(c^{*\pi}(A) \cap c^{*\pi}(B)) \subseteq i_\mu(c^{*\pi}(A \cap B))$. Therefore $A \cap B \subseteq i_\mu c^{*\pi}(A \cap B)$. Hence $A \cap B \in R\pi*\,\mathrm{H}O(X)$.

Theorem 4.10. *Let A be a hereditary genralized topological space* (X, μ, H). *Then the following are equivalent.*

1. *A is* $R\alpha*$ - H *-open set*
2. *A is* $R\sigma*$ - H *-open set and* $R\pi*$ - H *-open set*

Proof. (1) ⇒ (2). Let a subset A of hereditary genralized topological space (X, μ, H) is $R\alpha*$ - H -open set. Then its both $R\sigma*$ - H -open set and $R\pi*$ - H -open set by Theorem 5.3.1.

(2) ⇒ (1). Let a subset A of hereditary genralized topological space (X, μ, H) is both $R\sigma*$ - H -open set and $R\pi*$ - H -open set. Then $A \subset c^{*\pi} i_\mu(A)$ and $A \subset i_\mu c^{*\pi}(A)$. Now $A \subset i_\mu c^{*\pi}(A) \subset i_\mu c^{*\pi} c^{*\pi} i_\mu(A) \subset i_\mu c^{*\pi} i_\mu(A)$. Therefore $A \subset i_\mu c^{*\pi} i_\mu(A)$. Hence A is $R\alpha*$ - H -open set.

Remark 4.11. *The notions of* $R\sigma*$ - H *-open set and* $R\pi*$ - H *-open set are independent.*

Example 4.12. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}\}$. *Then* $A = \{b, d\}$ *is* $R\sigma*$ - H *-open but not* $R\pi*$ - H *-open.*

Example 4.13. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}, \{b, c, d\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}, \{b\}\}$. *Then* $A = \{b\}$ *is* $R\pi*$ - H *-open set but not* $R\sigma*$ - H *-open set.*

5 $R\beta*$ - H **-open sets**

Definition 5.1. *Let* (X, μ, H) *be a hereditary generalized topological space. A subset* A *of* X *is said to be* $R\beta*$ - H *-open set, if* $A \subseteq c_\mu i_\mu c^{*\pi}(A)$.

Theorem 5.2. *Let* (X, μ, H) *be a hereditary generalized topological space. Then*

1. *Every* μ *-open is* $R\beta*$ - H *-open set*
2. *Every* β - H *-open is* $R\beta*$ - H *-open set*
3. *Every* $R\sigma*$ - H *-open set is* $R\beta*$ - H *-open set*
4. *Every* $R\pi*$ - H *-open set is* $R\beta*$ - H *-open set*
5. *Every* $R\alpha*$ - H *-open set is* $R\beta*$ - H *-open set*

Proof. (1). Let a subset A of a hereditary generalized topological space (X, μ, H) is μ -open. Then $A = i_\mu(A)$. Now $A \subset i_\mu(A) \subset c_\mu i_\mu c^{*\pi}(A)$. Hence A is $R\beta*$ - H - open set.

(2)Let a subset A of a hereditary generalized topological space (X, μ, H) is π - H -open. Then $A \subset c_\mu i_\mu c^{*}\mu(A)$. Now $A \subset c_\mu i_\mu c^{*}\mu(A) \subset c_\mu i_\mu c^{*\pi}(A)$. Hence A is $R\beta*$ - H -open set.

(3)Let a subset A of a hereditary generalized topological space (X, μ, H) is

$R\sigma*$ - H -open set. Then $A \subset c^{*\pi}i_\mu(A) \subset c^{*\pi}i_\mu c^{*\pi}(A) \subset c_\mu i_\mu c^{*\pi}(A)$. Therefore

$\mu \quad \mu \quad \mu \quad \mu$

$A \subset c_\mu i_\mu c^{*\pi}(A)$. Hence A is R^* - H -open set.

$\mu \quad \beta$

(4)Let a subset A of a hereditary generalized topological space (X, μ, H) is $R\pi*$ -

H -open set. Then $A \subset i_\mu c^{*\pi}(A) \subset c_\mu i_\mu c^{*\pi}(A)$. Therefore $A \subset c_\mu i_\mu c^{*\pi}(A)$. Hence

$\mu \quad \mu \quad \mu$

A is $R\beta*$ - H -open set.

(5)Let a subset A of a hereditary generalized topological space (X, μ, H) is

$R\alpha*$ - H -open set. Then $A \subset i_\mu c^{*\pi}i_\mu(A) \subset i_\mu c^{*\pi}(A) \subset c_\mu i_\mu c^{*\pi}(A)$. Therefore

$\mu \quad \mu \quad \mu$

$A \subset c_\mu i_\mu c^{*\pi}(A)$. Hence A is R^* - H -open set.

$\mu \quad \beta$

Example 5.3. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}\}$. *Then* $A = \{a, d\}$ *is* $R\beta*$ - H *-open but not* μ *-open (resp.* $R\alpha*$ - H *-open,* $R\sigma*$ - H *-open,* $R\pi*$ - H *-open).*

Example 5.4. *Let* $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{a, e\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, e\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{a\}\}$. *Then* $A = \{e\}$ *is* $R\beta*$ - H *-open set but not* β - H *-open.*

Theorem 5.5. *If* $\mathrm{H} = \emptyset$, *then every* $R\beta*$ - H *-open set is* μ - σ *-open.*

Proof. Let a subset A of X is $R\beta*$ - H -open set and $\mathrm{H} = \emptyset$. Now $A \subset c_\mu i_\mu c^{*\pi}(A) = c_\mu i_\mu(A)$, which implies $A \subset c_\mu i_\mu(A)$. Hence A is μ - σ -open.

Theorem 5.6. *If* $\mathrm{H} = P(X)$, *then every* $R\pi*$ - H *-open set is* μ - σ *-open..*

Proof. Let a subset A of X is $R\beta*$ - H -open set and $\mathrm{H} = \emptyset$. Now $A \subset c_\mu i_\mu c^{*\pi}(A) = c_\mu i_\mu(A)$, which implies $A \subset c_\mu i_\mu(A)$. Hence A is μ - σ -open.

Theorem 5.7. *If* $R_\beta(\mu) = \mu$, *then every* $R\beta*$ - H *-open set is* β - H *open.*

Proof. Let a subset A of X is $R\beta*$ - H -open set and $R_\beta(\mu) = \mu$. Now $A \subset c_\mu i_\mu c^{*\pi}(A) = c_\mu i_\mu(A \cup A^{*\pi}) = c_\mu i_\mu(A \cup A^*) = c_\mu i_\mu c^*\mu(A)$, which implies $A \subset c_\mu i_\mu c^*\mu(A)$. Hence A is β - H -open set.

Theorem 5.8. *Let* (X, μ, H) *be a hereditary generalized topological space and X is a* C_0 *-space. If* A *is* $R\beta*$ - H *-open set and* U *is a* μ *-open. Then* $A \cap U$ *is* $R\beta*$ - H *-open set.*

Proof. Let A be a $R\beta*$ - H -open set and U is a μ -open. Then $A \subset c_\mu i_\mu c^{*\pi}(A)$ and $U = i_\mu(U)$. Now, $A \cap U \subset c_\mu i_\mu c^{*\pi}(A) \cap i_\mu(U) \subset c_\mu i_\mu(c^{*\pi}(A) \cap c_\mu i_\mu(U)) \subset c_\mu(i_\mu(c^{*\pi}(A) \cap i_\mu(U))) \subset c_\mu i_\mu(c^{*\pi}(A) \cap U) \subset c_\mu i_\mu((A \cup A^{*\pi}) \cap U) \subset c_\mu i_\mu((A \cap U) \cup (A^{*\pi} \cap U)) \subset c_\mu i_\mu((A \cap U) \cup (A \cap U)^{*\pi}) \subset c_\mu i_\mu((A \cap U) \cup (A \cap U)^{*\pi}) \subset c_\mu i_\mu c^{*\pi}(A \cap U)$. Hence $A \cap U$ is $R\beta*$ - H -open set.

6 Decomposition of $(R\alpha*$ - H, $\lambda)$ -continuity

Definition 6.1. *A function* $f : (X, \mu, \mathrm{H}) \to (Y, \lambda)$ *is said to be* $(R\alpha*$ - H, $\lambda)$ *- continuous (resp.* $(R\pi*$ - H, $\lambda)$ *- continuous,* $(R\sigma*$ - H, $\lambda)$ *-continuous,* $(R\beta*$ - H, $\lambda)$ *- continuous) if* $f^{-1}(V)$ *is* $R\alpha*$ - H *-open set (* $R\pi*$ - H *-open set,* $R\sigma*$ - H *-open set,* $R\beta*$ - H *-open set) for each open set* $V \in \lambda$.

Propositon 6.2. *Every* $(R\alpha*$ - H, $\lambda)$ *-continuous is* $(R\pi*$ - H, $\lambda)$ *-continuous but not conversely.*

Proof. This is obvious from Theorem 4.2.

Propositon 6.3. *Every* $(R\alpha*$ - H, $\lambda)$ *-continuous is* $(R\sigma*$ - H, $\lambda)$ *-continuous but not conversely.*

Proof. This is obvious from Theorem 4.2.

Propositon 6.4. *Every* $(R\alpha*$ - H, $\lambda)$ *-continuous is* $(R\beta*$ - H, $\lambda)$ *-continuous but not conversely.*

Proof. This is obvious from Theorem 5.2.

Propositon 6.5. *Every* $(R\sigma*$ - H, $\lambda)$ *-continuous is* $(R\beta*$ - H, $\lambda)$ *-continuous but not conversely.*

Proof. This is obvious from Theorem 5.2.

Propositon 6.6. *Every* $(R\pi*$ - H, $\lambda)$ *-continuous is* $(R\beta*$ - H, $\lambda)$ *-continuous.*

Proof. This is obvious from Theorem 5.2.

Theorem 6.7. *For a function* $f : (X, \mu, \text{H}) \to (Y, \lambda)$, *the following are equivalent.*

1. $(R\alpha*$ - H, $\lambda)$ *-continuous*
2. $(R\sigma*$ - H, $\lambda)$ *-continuous and* $(R\pi*$ - H, $\lambda)$ *-continuous*

Proof. This is obvious from Theorem 4.10.

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