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FEA Approach for structural and wear analysis of Araldite LY556 and DNR Composite Gear

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ABSTRACT

Gears are one of the most used elements in the field of mechanical engineering for the transition of power. Gears are designed and manufactured with various materials like nylon, polyacetal, Derlin and Many more. In this research, the spur gear is designed according to the required dimension and is manufactured by composite material made from Araldite LY556 and DNR base material for power transmission. FEM analysis is done on the gear for Dynamic load to find the stress developed in it even the wear analysis to measure wear loss by using LS-Dyna software. Similarly bending strength of a single tooth of gear is being determined for different loads by Ansys software and these results are compared to the analytical results.

***Keyword**: Composite Gear, Ls-Dyna Wear analysis, Epoxy and DNR, Dynamic analysis.*

1.0 INTRODUCTION

Gears are the most common machine elements that are required by almost all machine designers from time to time. Most commonly the gears are used to transmit the angular velocity and torque from one end to another end. Due to gears' compactness and reliability, it is appropriate to say that gears will be more predominant in upcoming industrial machines. It becomes necessary to further refine gear application as there is shift from the heavy industry to the small scale industries and from heavy equipment to office instruments like printers.

The gears are generally manufactured from cast iron, plastic or polymer material. The gear used in copying machines is generally manufactured with polyacetylene, Nylon, and many more materials, which are expensive due to the cost of material. As there is demand for the enhanced service life of the gear, which is more reliable, lightweight and more efficient are needed to design and manufacture. For this, gears are tried to produce new material or composite material without affecting the functionality of the gears. Here, a composite prepared by blending Epoxy and Deprotenised Natural Rubber is used for fabricating Gears. The most suitable proportion for mixing composite materials is predetermined. This is done by preparing different proportions of material and performing various mechanical and tribological tests to get its respective properties and identify the best proportion which gives optimum results. The feasibility of gear production by composite by Epoxy and DNR materials can be seen.

In FEA analysis of gears, the gears are imported from other modeling software such as solid works and CATIA. The properties of the material used here will be the properties obtained by performing various mechanical and tribological tests as the gears are made by the composite of epoxy and DNR material. Gear can be analyzed by both static and dynamic analysis. The torque specifications and dimensions of the gear of the copying machine are taken as references in this study.

2.0 MATERIAL USED

The material used in the manufacturing of gears is Araldite LY556 epoxy resin and Deproteinized Natural rubber. These materials are mixed in the proportion of 90% of epoxy and 10% of DNR in the presence of a solvent. By conducting various mechanical and tribological tests the respective properties of the material are as mentioned below.

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Sl. No.	Parameter	Value
1	Young's modulus	2600Mpa
2	Poisons ratio	0.4
3	Coefficient of friction	0.36

3.0 FEM

The Finite Element Method uses numerical analysis to provide a rough answer to a variety of engineering issues. Due to its versatility and diversity, it is gaining a lot of interest as an analysis tool in more and more industries and engineering schools. In the current engineering environment, we have discovered that it is vital to obtain a rough answer rather than an exact answer to an issue, as there is no way to get analytical results for many engineering problems through mathematical solutions. A mathematical formula that yields an analytical solution of an unknown quantity in any location of the desired body, hence applied to an infinite number of areas of the body, by FEM method provides an approximate solution.

3.0 MANUFACTURE OF GEARS USING COMPOSITE MATERIAL

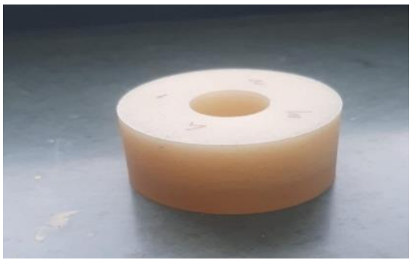
The epoxy composite can be cast by open moulding in which two methods can be followed mainly the hot method and the cold method. Casting using the hot method has led to various defects and difficulties in manufacturing. So the cold method is employed over hot moulding for epoxy composites. Process and steps followed in preparing epoxy composite are shown in fig.1 and 2. Before the gear is machined the epoxy composite is cast by preparing a mixture of Araldite LY556 epoxy and DNR in presence of solvent and after adding hardener stirred properly. Then the mix is poured into the mould prepared. After pouring the mix it is left to cool for 24 hours. After the cast is hardened it is removed from the mould and all the burrs are cleaned. Then it is taken to conventional machines like lathe and hobbing machine where the gear is machined as the required gear standard.

Steps followed in composite casting and manufacturing of gears:

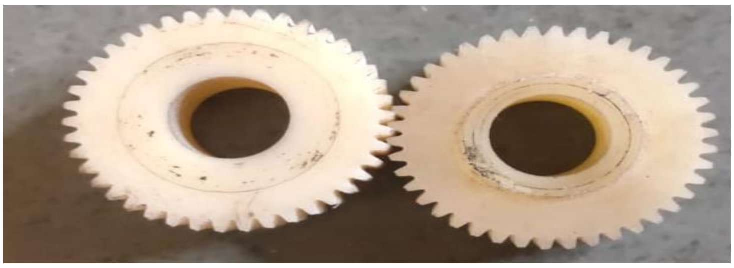
(a) Measuring of epoxy

(b)blending of DNR

(c)Stirring the mixture



(d) Cast obtained



(e) gears machined through conventional machines

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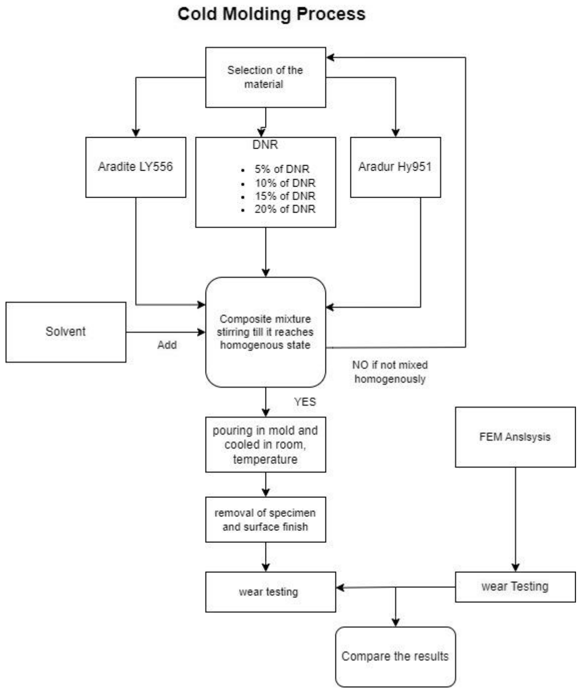


The standard proportion of Gear systems

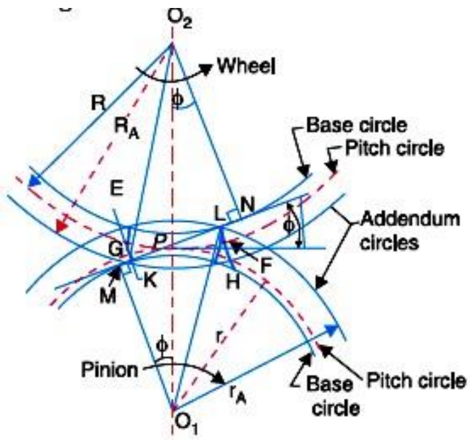
As there are 3 types of gears can be manufactured, In these types use of a 14.5° pressure angle system has less frequent due to problems in inter changeability. Whereas 20° pressure angle is most commonly used because of its various benefits like reduction in the risk of undercutting, greater length of contact and most importantly due to increased pressure angle the tooth becomes slightly stringer at the root leading to a stronger tooth and increases the load carrying capacity. So, 20° pressure angle is considered for preparing the gears.

Table.2 Standard proportions of Gear systems

S. No.	Particulars	$14\frac{1}{2}°$ composite or full depth involute system	20° full depth involute system	20° stub involute system
1.	Addenddm	1 m	1 m	0.8 m
2.	Dedendum	1.25 m	1.25 m	1 m
3.	Working depth	2 m	2 m	1.60 m
4.	Minimum total depth	2.25 m	2.25 m	1.80 m
5.	Tooth thickness	1.5708 m	1.5708 m	1.5708 m
6.	Minimum clearance	0.25 m	0.25 m	0.2 m
7.	Fillet radius at root	0.4 m	0.4 m	0.4 m

Arc of Contact:

It is the path traced by a point on the pitch on the pitch circle from the beginning to the end of the engagements of a given pair of teeth.



Path of contact

Length of the arc of contact = $\frac{\text{length of path of contact}}{\cos\emptyset}$

DESIGN SPECIFICATION:

Bending stress, contact stress and total deformation are calculate din the gears specification given below, to test the material the spur gear are selected and 20˚ involue gear are designed for upto 3000 RPM and 10 N-m

Table 2. Specification of Spur Gear Tooth

Description	Units	symbol	Value	Value
Number of teeth on pinion		Np		42
Number of teeth on gear		Ng		42
Pressure angle	Degree	Ø		20°
Gear Module	MM	M		0.95
Addendum	MM	h_a	m	1.25
Dedendum	MM	h_d	1.25m	1.875
PCD	MM	d_p	mN	39.9
PCR	MM	r_p		19.95
BCD	MM	d_b	$d_p \cos\emptyset$	37.49
ACD	MM	d_a	$d_p + 2m$	41.8
DCD	MM	d_d	$d_p - \left(2 + \frac{\pi}{N}\right) m$	37.92
Face width	MM	b		11.4
Tooth thickness	MM	t	1.5808	15.1

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Calculation of Contact stress using Hertz Equation for GEAR

According to Hertz Theory the contact stress for the gears are calculated as below equations [20]

$$\sigma_c = \frac{2F}{\pi BL} \quad \text{(i)}$$

$$B = \sqrt{\left[\frac{2F\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}{\pi L\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]} \quad \text{(ii)}$$

Where, σ_c = Max contact Stress in n/mm^2.

F= Force applied on the gears in N

B = Half width deformation in mm.

L= Axial length in mm.

d1 & d2 = Diameter of cylinder in mm

E1 & E2 = Modules of Elasticity of cylinder in N/mm^2

μ1,μ2 = Poisson's ratio of materials.

Substituting the value of half width of deformation B in equation (i) and squaring both sides,

$$\sigma_c^2 = \frac{F}{\pi L}\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu_1}{E_1}+\frac{1-\mu_2}{E_2}\right)}\right] \quad \text{(iii)}$$

If the material of both cylinders is the same, then the module of elasticity and Poisson's ratio will be equal. Substituting E1 = E2 = E and μ1 = μ2 = μ in eq. (iii),

$$\sigma_c^2 = \frac{F}{2\pi L}\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu}{E}\right)}\right] \quad \text{(iv)}$$

replacing the radii r_1 and r_2 by the radii of curvature at the pitch point

$$r_1 = \frac{d_{pp}\sin\emptyset}{2} \text{ and } r_2 = \frac{d_{pg}\sin\emptyset}{2}$$

Where d_{pp} = PCD of pinion

d_{pg} = PCD of gear

Since both have equal geometry in all respects as given in table 2, therefore

$d_{pp} = d_{pg} = d_p$

$$r_1 = \frac{d_p\sin\emptyset}{2} \quad \text{and} \quad r_2 = \frac{d_p\sin\emptyset}{2}$$

$$\Rightarrow \quad r_1 = r_1 = r = \frac{d_p\sin\emptyset}{2} \quad \text{(v)}$$

Substitute in eq.(iv) we get

$$\sigma_c^2 = \frac{1}{\pi(1-\mu}\left[\frac{FE}{Lr}\right] \quad \text{(vi)}$$

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For both materials, Poisson's ratio μ = 0.31 in eq.(vi) and solving;

$$\sigma_c = 0.6841 \left[\frac{FE}{Lr}\right]^{\frac{1}{2}} \qquad \text{(vii)}$$

Modulus of elasticity of composite E = 2600MPa and substitute in (vii),

$$\sigma_c = 0.6841 \left[\frac{FX\ 2600}{Lr}\right]^{\frac{1}{2}}$$

$$\sigma_c = 34.88 \left[\frac{F}{Lr}\right]^{\frac{1}{2}} \qquad \text{(viii)}$$

From (v) $r = \frac{d_p sin\emptyset}{2} = r_p sin\emptyset$

$$\sigma_c = 34.88 \left[\frac{F}{Lr_p\ sin\emptyset}\right]^{\frac{1}{2}} \qquad \text{(ix)}$$

Now, F = $\frac{F_t}{cos\emptyset}$, Substitute this value in (ix),

$$\sigma_c = 34.88 \left[\frac{F}{Lr_p\ sin\emptyset cos\emptyset}\right]^{\frac{1}{2}} \qquad \text{(x)}$$

Also axial length L is equal to face width of spur gears b, there for replacing L by b in (x)

$$\sigma_c = 34.88 \left[\frac{F}{b\ r_p\ sin\emptyset cos\emptyset}\right]^{\frac{1}{2}} \qquad \text{(xi)}$$

Power transmitted at torque force 10N at 3000rpm

$$P = \frac{2\pi n_p T}{60\times10^6}$$

Where T= r X F =20 X 10= 200Nmm

Also $F_t = \frac{2T}{d_p}$ where d_p= 39.9mm

F_t= 10.025 N

Again b=11.4 mm pressure angle =20' r_p= 19.95 mm

$$\sigma_c = 34.88 \left[\frac{10.025}{11.4X19.95\ sin20cos20}\right]^{\frac{1}{2}} = 4.715\ Mpa$$

Bending strength of gear tooth by Lewis Equation:

The gear is tooth is fixed to the base and tooth end is applied with the pressure by the other gear tooth, in radial and tangential directions, the force exerted by the tooth is given by F, force [21], to analyze the gear, some of the assumption made which is as given below:

1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.
3. The load is distributed uniformly across the full face width.
4. Forces due to tooth sliding friction are negligible.
5. Stress concentration in the tooth fillet is negligible.

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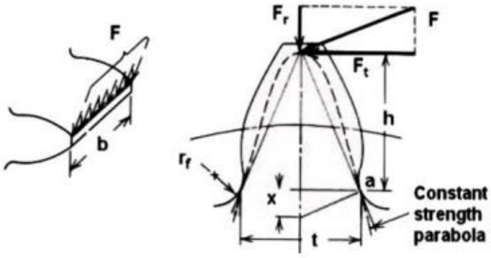

The loading on done in the gear as shown in the figure above, the F is full loading, F_r & F_t are the Radial and tangential loading on the gear. In the gear h is height, t is thickness of the gear and b is face width of the gear which is as shown in the figure above [22]

At section

Mb = Pt X h

The bending stress is given by

$$\sigma_b = \frac{M_b \times y}{I} = \frac{P_t \times h \times \frac{t}{2}}{\frac{b \times t^3}{12}}$$

Calculations:

Analytical calculating banding stress with a different loads such as 0.5kg,1.0kg ,1.5 kg and 2.0kg.

(a) For 0.5kg.

Bending stress of gear tooth at load 0.5kg is F=P_t =4.905 N

d =40mm

Mb = $P_t \, X \, h$ = 13.856

$\sigma_b = \left(\frac{M_b \, y}{I}\right) = \frac{P_t \, h \, \frac{t}{2}}{\left(\frac{b \, t^3}{12}\right)} =$ 3.21 MPa. (by substituting value of b =11.4 and t= 1.5017)

Dynamic Analysis of gear:

For dynamic analysis a model of spur gear is designed according to given specification in any CAD software and save in IGES format. Spur gears are in same module and designed such that two gears mesh each other at tooth. Material properties of composite are entered in the software before applying properties to gears.

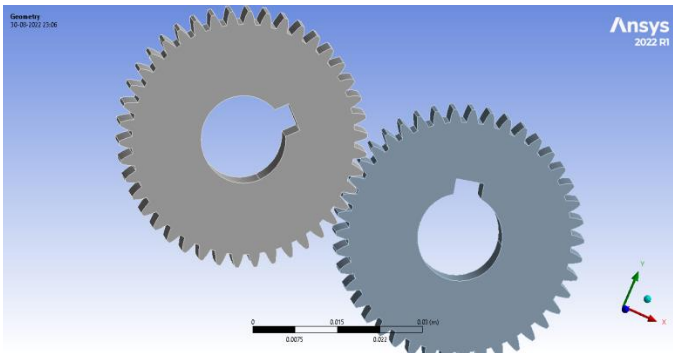

Fig : Gears engaging at the simulation software

Meshing

Meshing basically entails breaking up the entire model into smaller cells in order to solve the equations at each individual cell. It provides a precise solution and raises the standard of that solution. Here, mesh

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generation is done with a 1 mm element size and medium smoothing. Quality of mesh done on the gears is fine so that the solution time can be saved and can get the correct nearby answer of the gears.

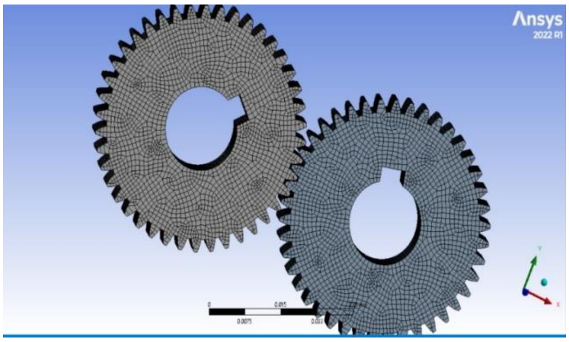

Fig: Meshing of Gears simulation software

Boundary Requirements

As shown in the figure below the boundary condition is applied in ANSYS Workbench. Boundary conditions are the gear to be driven is given free rotation along Z-axis and velocity of 31.41rad/s along z-axis is given which drives other gear. After the boundary conditions are given solution for the problem is requested by clicking solve.

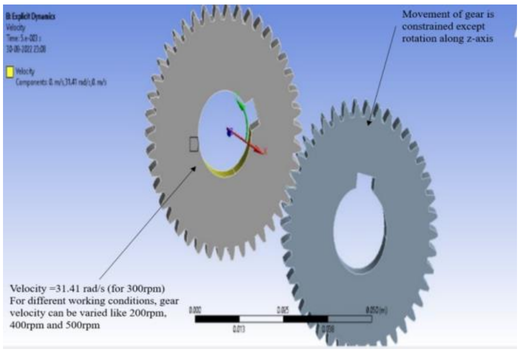

Fig: Boundary condition on Gears.

Post processing: after solving problem the von-misses stress is obtained which is equivalent to stress developed due meshing gear operating given velocity.

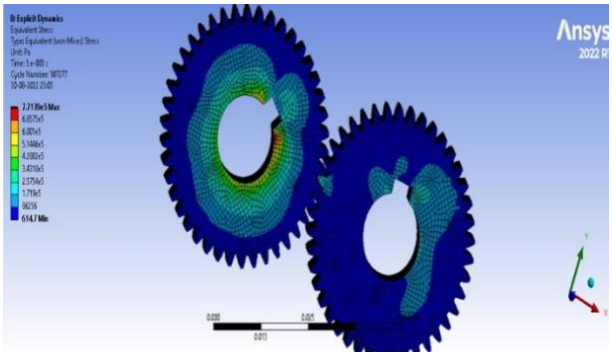

Fig: Stress analysis on Gears.

Analysis of single gear tooth

Bending stress of gear tooth can be obtained by simulating through ANSYS by considering it as cantilever beam and analyzing it statically. Gear tooth is split from the complete gear and imported in workbench. Properties of material is applied to gear tooth and meshing is done for better results fine mesh is considered.

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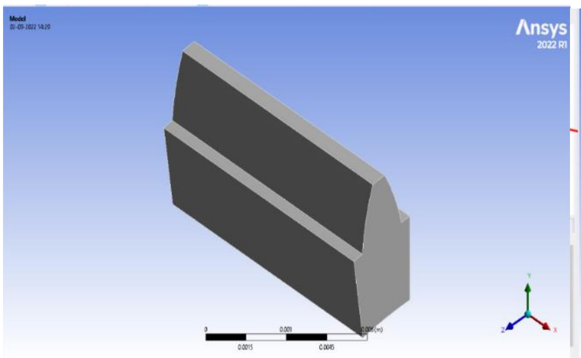
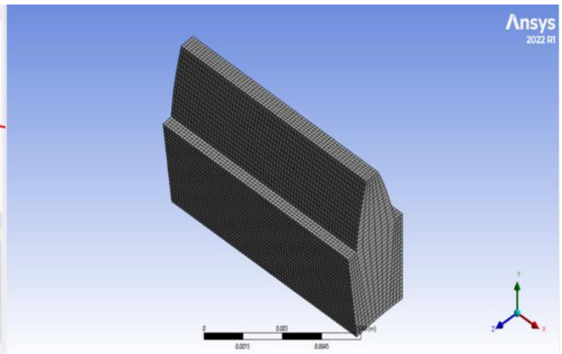
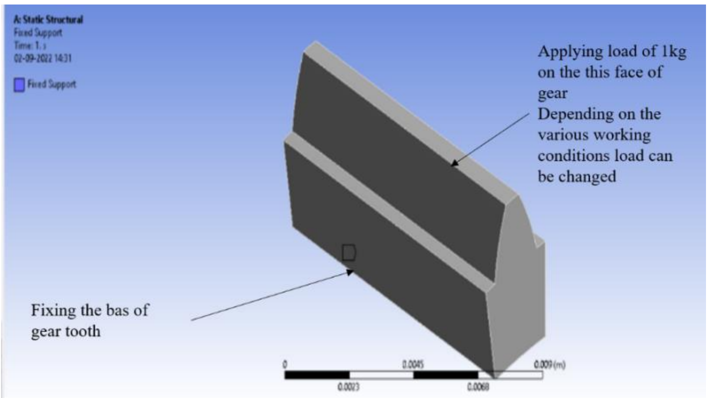


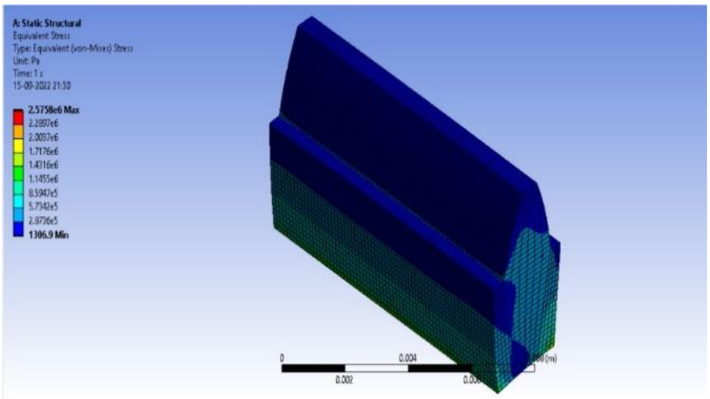


Bounding condition

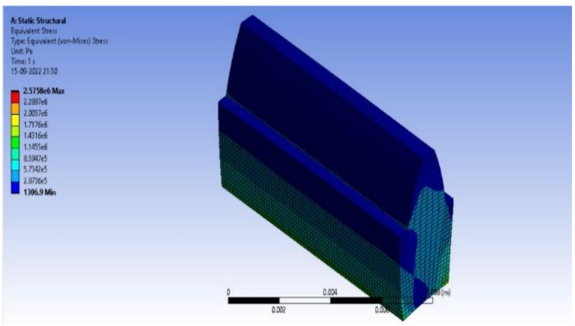
The bottom face of the tooth is fixed which acts as cantilever support and torque force is applied on the face tooth on which meshing occurs.



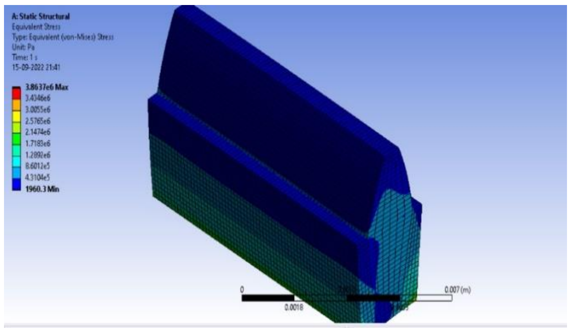
Solution is obtained by solving the problem and in post process the bending stress is requested.



Simillar to the above case the Bending stress of single toothof gear at different torque forces is found by solving through ANSYS workbench



0.5kg



0.75kg

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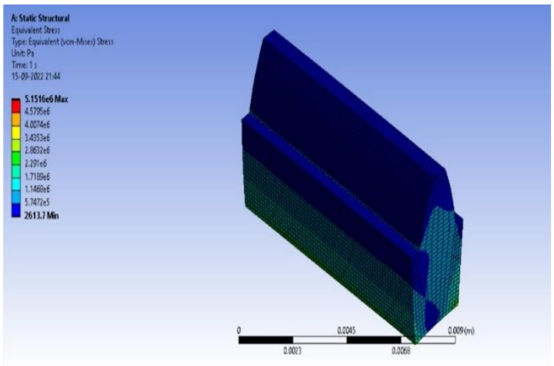
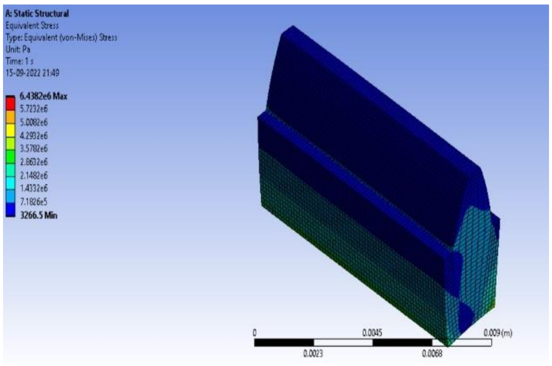
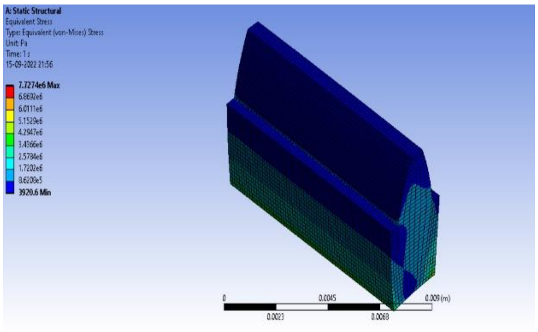


1.0kg



1.25kg



1.5kg

Wear rate:

As gears are meshed with each other and transmit the power from driver to driven because of torque transmission the gears get worm out. The worn gears are measured by the Archard law method where the sliding distance and load applied is directly proportional to the coefficient of the friction of the material. Here in this experimentation the gears are undergone the various load from 0 to 1.5Kg load at varying speeds.

$$Q = \frac{K W L}{H}$$ Archard Law of wear.

Where:

Q = Volume of Debris Produced

K = Wear constant

W= Normal Load

L = Sliding Distance

H = Hardness of the softest contacting surface

The ERC gear is tested in the LS-dyna post processing and the conditions are set as per the Archard method. The dynamic analysis is done on the software and results are time dependent, as the one rotation to complete verses wear depth, so that the number of the rotation is directly in proportion to the wear of the gear.

In this testing, the wear depth of the gears can be shown in the figure below. The value of debris produced is 10-9, meaning the wear and debris produced in the meshing process is very minimal or negligible. So the overall life of the gear is many, varying from the loading conditions to the

speed of the rotation.

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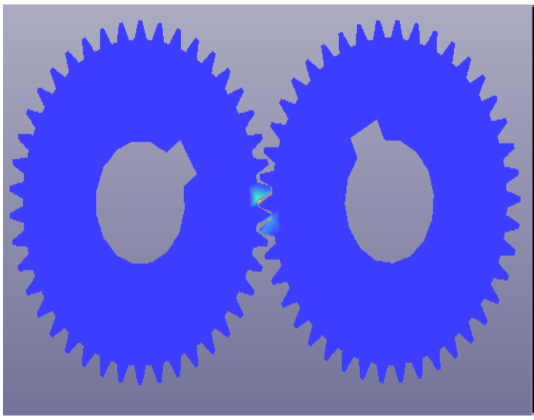


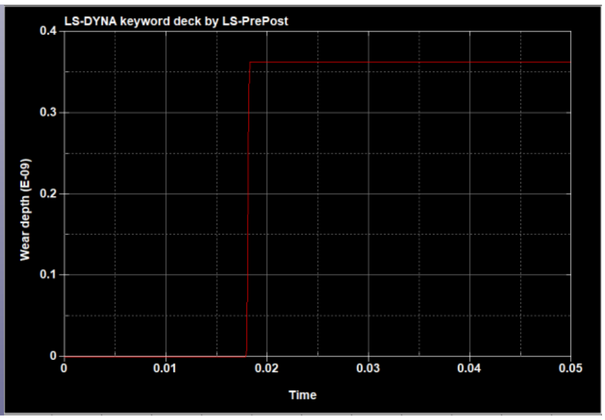


Figure: Gear teeth depth wears   Individual Gear wear Depth v/s the time of each tooth

Results:

As the gears transmit the power from one gear to another by receiving the power from the motor. In this power, transmission gears undergo multiple stresses on it. As shown in the above, results are found from the various analyses. These tests are dynamic analysis, teeth stress and the wear rate on the individual tooth. These results are discussed below:

Dynamic Loading:

The driver gear is given a velocity of 1000 RPM, and torque of 15N is applied to the driving gear. The torque loading is given to the center of the shaft. The dynamic analysis is done using Ansys software and applied boundary conditions on the gears. After simulation, the test results are equivalent stress at a maximum of 2.71 MPa. The loading is minimal compared to the max allowable stress on the material.

Bending tests:

From the resulst obatain from the ansys simulation that the laod applied on the gear tooth on tangetila to the teeth. The loading increases the stress concentration at the gear incraeses. By using the above formulae and given data the bending strees is calculated for remaining loads and compared to stress obtained by FEA simulation results obtained from workbench.

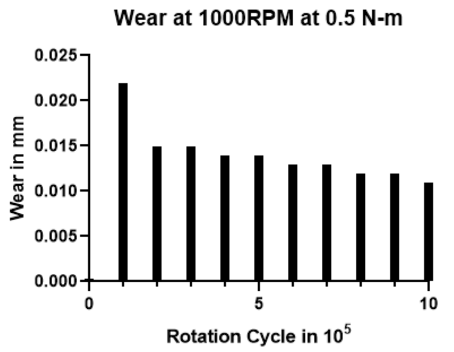
Load	**Bending stress in MPa (Theoretical)**	**Bending Stress MPa (workbench)**
0.5Kg (4.905N)	3.22	2.62
0.75Kg (7.3575N)	3.67	3.86
1.0Kg(9.81N)	5.04	5.15
1.25kg (12.26N)	6.12	6.43
1.5Kg(14.7N)	7.85	7.727

As the testing results are shown in the above table, the applied load is minimum conditions. As the polymer gears are used at low speeds and the minimum torque applied. The gear goes under little amount of loading in the teeth's. The failure stress on teeth is lower than the results from the testing and the mathematics results. This way, the gears can be used for low speed applications as the gear won't get failures with much less torque.

Wear rate:

In the Ls-Dyna simulation, the wear depth of the gear is analysed by the Archard Law method were the tooth meshes with each tooth and transmits the power from one end to the other end. In this, the gear is analyzed by the Archard method. There the wear depth of each tooth is collected. If multiplied by the rotations, the wear rate of the gear can be analyzed.

In the Ls-Dyna pre-post there is provision to check the wear depth in cycles in an application called wear. Where SEQ is given and the material property is given in the wear application. By calculation, it will show the wear of that material after these many rotations or the depth of wear to rotation required. In this way, the gear is analysed and gathers the wear rate of the material as given below calculation.



Conclusion:

The Epoxy Rubber composites material can be used for gear applications as it is simulated in the software to test the behavior of the material. It is tested in the simulation model in various testing conditions. The gears showed the best results. The Araldite LY556 combined with 10 % Deprotenised natural rubber and the composite prepared are simulated in the test and the conclusion of the results are as follows:

- The bending test on the ER Composites can stand the max loading in the bending test as compared with mathematics modeling. As the conditions for the testing are kept lighter. It can be tested for a higher load application requirements.
- The teeth undergo cantilever loading. As in the testing, it shows that one is fixed to the base of the gear and the other end is undergoing the loading of the other gear. In this testing it is found that the gears can withstand the max loading as compared to higher loading conditions.
- In software simulation, gear wear can be found in minimum time spent. In this simulation, the gear is tested on the 10 lakh rotation of gear with torque. it shows that the gear can with stand the 10 lakhs rotation with a wear depth of 0.17mm in total. As the overall teeth width is 2.03mm. by this the gear can stand up to multiple lakh rotations. So it can be used in the various applications. As per the requirements.

Checking all these testing in the simulation software, the time to test these gears is reduced to minimum time and the results are appropriate to test in a real life scenario and utilize the gears for the higher end methods. These simulations gave the idea of gear capacity to undergo the various loading or not and whether to manufacture the gear or not.

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