ISSN: 0974-5823 Vol. 7 (Special Issue, Jan.-Mar. 2022)

International Journal of Mechanical Engineering

INTUITIONISTIC FUZZY CONTRA $\hat{\beta}$ GENERALIZED CONTINUOUS MAPPING

ARUN PRAKASH K[1], UMADEVI M[2], VENGATAASALAM S[3]

[1,3]Kongu Engineering College, Erode, Tamil Nadu, India.

[2] PSNA College of Engineering and Technology, Dindigul, Tamil Nadu, India.

Abstract: The intent of this paper is to introduce and study the concepts of intuitionistic fuzzy contra $\hat{\beta}$ generalized continuous functions in intuitionistic fuzzy topological space.

Keywords and Phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy $\hat{\beta}$ generalized closed sets, Intuitionistic fuzzy $\hat{\beta}$ generalized continuous mapping, intuitionistic fuzzy almost $\hat{\beta}$ generalized continuous mapping, intuitionistic fuzzy contra $\hat{\beta}$ generalized continuous mapping.

Introduction

The fuzzy concept has wide application in all real life problems such as control system and information sciences. Especially, in mathematics fuzzy set is introduced by L. A. Zadeh [15]. The theory of fuzzy topological space was introduced and developed by C. L. Chang [3]. The various notions in classical topology have been extended to fuzzy topological space. In 1986, the "intuitionistic fuzzy set" was first initiated by Atanassov [2]. The concept of intuitionistic fuzzy topological spaces was defined by Coker [4] in 1997. This concept yields a wide field for working in the area of fuzzy topology and its application. One of the specification is associated to the properties of intuitionistic fuzzy sets introduced by Gurcay [6] in 1997. in 2013, M.Umadevi K.Arun Prakash and S.Vengataasalam developed $IF\hat{\beta}GCS$ in the topological space[12] and to study the application of $IF\hat{\beta}GCS$, $IF_{\hat{\beta}g}T_{1/2}$ space introduced. Furthermore, these authors introduced the concepts of $IF\hat{\beta}$ generalized irresolute mapping and its characterizations are also discussed in 2016[13]. In this paper, intuitionistic fuzzy contra $\hat{\beta}$ generalized continuous mapping introduced and defined several theorems. The characterizations of the functions discussed.

1. Preliminaries

Definition 2.1 [2] An intuitionistic fuzzy set (IFS for short) P in X is an object having the form $P=\{\langle x,\mu_P(x),\gamma_P(x)\rangle / x\in X\}$ where the functions $\mu_P : X\to[0,1]$ and $\gamma_P : X\to[0,1]$ denote the degree of the membership (namely $\mu_P(x)$) and the degree of non- membership (namely $\gamma_P(x)$) of each element $x\in X$ to the set A respectively, $0\le\mu_p(x)+\gamma_p(x)\le 1$ for each $x\in X$.

Definition 2.2 [2] Let P and Q be IFS's of the forms $P=\{\langle x,\mu_P(x),\gamma_P(x)\rangle / x\in X\}$ and $Q=\{\langle x,\mu_Q(x),\gamma_Q(x)\rangle / x\in X\}$. Then,

(a) $P\subseteq Q$ if and only if $\mu_P(x)\le \mu_Q(x)$ and $\gamma_P(x)\le \gamma_Q(x)$ for all $x\in X$,

(b) $P=Q$ if and only if $P\subseteq Q$ and $Q\subseteq P$,

(c) $\overline{P}=\{\langle x,\gamma_P(x),\mu_P(x)\rangle / x\in X\}$,

(d) $P\cap Q=\{\langle x,\mu_P(x)\wedge \mu_Q(x),\gamma_P(x)\vee \gamma_Q(x)\rangle / x\in X\}$

(e) $P\cup Q=\{\langle x,\mu_P(x)\vee \mu_Q(x),\gamma_P(x)\wedge \gamma_Q(x)\rangle / x\in X\}$

(f) $0_\sim=\{\langle x,0,1\rangle / x\in X\}$ and $1_\sim=\{\langle x,1,0\rangle / x\in X\}$

(g) $\overline{\overline{P}}=P$, $\overline{1_\sim}=0_\sim$, $\overline{0_\sim}=1_\sim$.

Definition 2.3 [4] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFS's in X satisfying the following axioms:

(i) $0_\sim$, $1_\sim\in\tau$,

(ii) $S_1\cap S_2\in\tau$ for any $S_1,S_2\in\tau$,

(iii) $\cup S_i\in\tau$ for any family $\{S_i \mid i\in J\}\subseteq\tau$.

Here (X,τ) is said to be an intuitionistic fuzzy topological space($IFTS$ for short) and any IFS in τ is known as an intuitionistic fuzzy open set ($IFOS$ for short) in X. The complement $\overline{P}$ of an $IFOS$ P in $IFTS$ (X,τ) is known as intuitionistic fuzzy closed set ($IFCS$ for short) in X.

Definition 2.4 [4] Let X and Y are two non-empty sets and $k:X\to Y$ be a function. If $Q=\{\langle x,\mu_Q(x),\gamma_Q(x)\rangle / x\in X\}$ is an IFS in Y, then the pre image of Q under k, denoted by $k^{-1}(Q)$, is the IFS in X defined by $k^{-1}(Q)=\{\langle x,k^{-1}\mu_Q(x),\gamma_Q(x)\rangle / x\in X\}$.

Definition 2.5 [4] Let (X,τ) be an $IFTS$ and $P=\{\langle x,\mu_P(x),\gamma_P(x)\rangle / x\in X\}$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

$$\operatorname{int}(P)=\cup\{G \mid G\ is\ an\ IFOS\ in\ X\ and\ G\subseteq P\}$$

$$cl(P)=\cap\{K \mid K\ is\ an\ IFOCS\ in\ X\ and\ P\subseteq K\}$$

Note that, for any $IFS\ P$ in (X,τ), we have $cl(\overline{P})=\overline{\operatorname{int}(P)}$ and int $\overline{(P)}$ = $\overline{cl(P)}$

Definition 2.7 [10] An IFS P of an $IFTS$ (X,τ) is called an intuitionistic fuzzy $\hat{\beta}$ -generalized closed set if $cl(\operatorname{int}(cl(P)))\subseteq U$, whenever $P\subseteq U$ and U is an $IFOS$.

The complement $\overline{P}$ of an intuitionistic fuzzy $\hat{\beta}$ generalized closed set P is called an intuitionistic fuzzy $\hat{\beta}$ generalized open set.

Definition 2.9 [9] A function $\psi:(X,\tau)\rightarrow(Y,\kappa)$ from an $IFTS$ (X,τ) into an (Y,κ) is called an intuitionistic fuzzy $\hat{\beta}$ generalized closed function[$IF\hat{\beta}G$ closed function in short], if $\psi(Q)$ is an intuitionistic fuzzy $\hat{\beta}$ generalized closed set in Y for every $IFCS$ Q in X.

Definition 2.10 [5] An IFS P is said to be an intuitionistic dense (IFD for short) in another IFS Q in an $IFTS$ (X,τ) if $cl(P)=Q$.

Definition 2.11 A function $\psi:(X,\tau)\rightarrow(Y,\kappa)$ from an $IFTS$ (X,τ) into an (Y,κ) said to be an

(a) intuitionistic fuzzy contra continuous function (IF contra continuous function in short) if $\psi^{-1}(P)$ is an $IFCS$ in Y for every $IFOS$ P in X.

(b) intuitionistic fuzzy contra α - continuous function ($IFc\alpha$ continuous function in short) if $\psi^{-1}(P)$ is an $IF\alpha OS$ in Y for every $IFOS$ P in X.

(c) intuitionistic fuzzy contra generalized continuous function ($IFcG$ continuous function in short) if $\psi^{-1}(P)$ is an $IFGCS$ in Y for every $IFOS$ P in X.

(d) intuitionistic fuzzy contra generalized semi continuous function ($IFcGS$ continuous function in short) if $\psi^{-1}(P)$ is an $IFGSCS$ in Y for every $IFOS$ P in X.

INTUITIONISTIC FUZZY CONTRA $\hat{\beta}$ GENERALIZED CONTINUOUS MAPPINGS

Intuitionistic fuzzy contra $\hat{\beta}$ generalized continuous mapping is introduced and their characteristics are studied in this section.

Definition 3.1: A mapping $\phi:(X,\tau)\rightarrow(Y,\kappa)$ is called an Intuitionistic Fuzzy Contra $\hat{\beta}$ Generalized continuous mapping ($IFC\hat{\beta}G$continuous mapping) if $\phi^{-1}(T)$ is an IF$\hat{\beta}$GCS in Y for every IFOS T in Y.

Example 3.2: Assume that $X=\{x_1,x_2\}$and $Y=\{y_1,y_2\}$. Let $S=\langle x,\left(\frac{x_1}{0.1},\frac{x_2}{0.2}\right),\left(\frac{x_1}{0.1},\frac{x_2}{0.7}\right)\rangle$ and $T=\langle y,\left(\frac{y_1}{0.3},\frac{y_2}{0.2}\right),\left(\frac{y_1}{0.5},\frac{y_2}{0.7}\right)\rangle$. Then $\tau=\{0_\sim,1_\sim,S\}$ and $\kappa=\{0_\sim,1_\sim,T\}$ are $IFTSs$ on X and Y correspondingly. Construct a function $\phi:(X,\tau)\rightarrow(Y,\kappa)$ by $\phi(x_1)=y_1,\ \phi(x_2)=y_2$. Then $\phi^{-1}(T)=\langle x,\left(\frac{x_1}{0.3},\frac{x_2}{0.2}\right),\left(\frac{x_1}{0.5},\frac{x_2}{0.7}\right)\rangle$, $cl(\phi^{-1}(T))=1_\sim,\ cl(int(cl(\phi^{-1}(T))))=1_\sim$ and $\phi^{-1}(T)\subseteq 1_\sim$. Thus $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X. Hence ϕ is an $IFC\hat{\beta}G$ continuous mapping.

Theorem 3.3: In IFTS (X,τ) every intuitionistic fuzzy contra continuous mapping is an IFC$\hat{\beta}$G continuous mapping, but converse implication does not hold.

Proof: **C**onsider an intuitionistic fuzzy contra continuous mapping $\phi:(X,\tau)\rightarrow(Y,\kappa)$ and an IFOS T in Y. By assumption, $\phi^{-1}(T)$ is an IFCS in X. As by Theorem 2.2.4 each IFCS is an $IF\hat{\beta}GCS$, $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X for each IFOS T in Y. Therefore ϕ is an $IFC\hat{\beta}G$ continuous mapping.

Example 3.4: Assume that $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $S = \langle x, \left(\frac{x_1}{0.3}, \frac{x_2}{0.2}\right), \left(\frac{x_1}{0.4}, \frac{x_2}{0.3}\right)\rangle$ and $T = \langle y, \left(\frac{y_1}{05}, \frac{y_2}{0.7}\right), \left(\frac{y_1}{0.3}, \frac{y_2}{0.2}\right)\rangle$. Then $\tau = \{0_\sim, 1_\sim, S\}$ and $\kappa = \{0_\sim, 1_\sim, T\}$ are $IFTS$ on X and Y correspondingly. Construct a function $\phi: (X, \tau) \to (Y, \kappa)$ by $\phi(x_1) = y_1$, $\phi(x_2) = y_2$. Now $\phi^{-1}(T) = \langle x, \left(\frac{x_1}{0.5}, \frac{x_2}{0.7}\right), \left(\frac{x_1}{0.3}, \frac{x_2}{0.2}\right)\rangle$, $cl(\phi^{-1}(T)) = 1_\sim$, $cl(int(cl(\phi^{-1}(T))) = 1_\sim$ and $\phi^{-1}(T) \subseteq 1_\sim$ only. Thus $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X. Therefore ϕ is an $IFC\hat{\beta}G$continuous mapping. Then $cl(\phi^{-1}(T)) = 1_\sim \neq \phi^{-1}(T)$, implies $\phi^{-1}(T)$ is an IFCS in X. Hence ϕ is not an intuitionistic fuzzy contra continuous mapping.

Theorem 3.5: In IFTS(X, τ) every intuitionistic fuzzy contra α continuous mapping is an IFC$\hat{\beta}$G continuous mapping, but converse implication does not hold.

Proof: Consider an intuitionistic fuzzy contra α continuous mapping $\phi: (X, \tau) \to (Y, \kappa)$ and an IFOS T in Y. By assumption $\phi^{-1}(T)$ is an $IF\alpha CS$ in X. As by Theorem 2.2.8 [14] every $IF\alpha CS$ is an $IF\hat{\beta}GCS$, $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X. Therefore ϕ is an $IFC\hat{\beta}G$ continuous mapping.

Example 6.2.6: Assume that $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $S = \langle x, \left(\frac{x_1}{0.1}, \frac{x_2}{0.3}\right), \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\rangle$ and $T = \langle y, \left(\frac{y_1}{0.3}, \frac{y_2}{0.4}\right), \left(\frac{y_1}{0.2}, \frac{y_2}{0.1}\right)\rangle$. Then $\tau = \{0_\sim, 1_\sim, S\}$ and $\kappa = \{0_\sim, 1_\sim, T\}$ are IFTS on X and Y correspondingly. Construct a mapping $\phi: (X, \tau) \to (Y, \kappa)$ by $\phi(x_1) = y_1, \phi(x_2) = y_2$. Now $\phi^{-1}(T) = \langle x, \left(\frac{x_1}{03}, \frac{x_2}{0.4}\right), \left(\frac{x_1}{0.2}, \frac{x_2}{0.1}\right)\rangle$, $cl(\phi^{-1}(T)) = 1_\sim$, $cl(int(cl(\phi^{-1}(T))) = 1_\sim$,$\phi^{-1}(T) \subseteq 1_\sim$. Then $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X. Thus ϕ is an $IFC\hat{\beta}G$ continuous mapping. Now $cl(int(cl(\phi^{-1}(T))) = 1_\sim \nsubseteq \phi^{-1}(T)$. Then $\phi^{-1}(T)$ is not an $IF\alpha CS$ in X. Therefore ϕ is not an intuitionistic fuzzy contra α continuous mapping.

The following diagram shows the relationships between $IFC\hat{\beta}G$ continuous mapping with other existing intuitionistic fuzzy contra continuous mappings.

$IFC\ cts\ M$ ⟶ $\boldsymbol{IFC\widehat{\beta}G\ cts\ M}$ ⟵ $IFC\alpha\ cts\ M$

Figure 3.1 Relation between $\boldsymbol{IFC\widehat{\beta}G\ cts\ M}$ and existing $\boldsymbol{IFC\ cts\ M}$

The reverse implication in the diagram is not true in general as seen from the above illustrated examples.

Theorem 3.7: If $\phi: (X, \tau) \to (Y, \kappa)$ is an $IFC\hat{\beta}G$ continuous mapping and (X, τ) is an $IF_{\hat{\beta}g}T_{1/2}$ space then ϕ is an intuitionistic fuzzy contra continuous mapping.

Proof: Consider $IFOS$ T in Y. By assumption $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X. As (X, τ) is an $IF_{\hat{\beta}g}T_{1/2}$space, $\phi^{-1}(T)$is an $IFCS$ in X. Therefore ϕ is an intuitionistic fuzzy contra continuous mapping.

Theorem 3.8: If $\phi: (X, \tau) \to (Y, \kappa)$is a mapping and (X, τ) is an $IF_{\hat{\beta}g}T_{1/2}$space, then the statements below will equivalent:

(i) ϕ is an $IFC\hat{\beta}G$ continuous mapping and

(ii) ϕ is an intuitionistic fuzzy contra continuous mapping.

Proof: **(i) ⇒ (ii):** Since from Theorem 3.7 the proof is obvious.

(ii) ⇒ (i): Proof is obvious from Theorem 3.3.

Theorem 3.9: If $\phi: (X, \tau) \to (Y, \kappa)$ be a mapping then the statements below will equivalent:

(i) ϕ is an $IFC\hat{\beta}G$ continuous mapping and

(ii) $\phi^{-1}(S)$ is an $IF\hat{\beta}GOS$ in X for each $IFCS$S in Y.

Proof: (i)⇒(ii): Consider an $IFCS$S in Y. Thus $\bar{S}$ is an $IFOS$ in Y. By assumption $\phi^{-1}(\bar{S}) = (\overline{\phi^{-1}(S)})$ is an $IF\hat{\beta}GCS$ in X. Hence $\phi^{-1}(S)$ is an $IF\hat{\beta}GOS$ in X.
(ii) ⇒ (i): Consider an $IFOS$S in Y. Thus$\bar{S}$ is an $IFCS$ in Y. By assumption $\phi^{-1}(\bar{S}) = (\overline{\phi^{-1}(S)})$ is an $IF\hat{\beta}GOS$ in X. Then $\phi^{-1}(S)$ is an $IF\hat{\beta}GCS$ in X. Hence ϕ is an $IFC\hat{\beta}G$ continuous mapping.

Theorem 3.10: If $\phi:(X,\tau)\to(Y,\kappa)$ be a mapping and $\phi^{-1}(T)$ be an $IFRCS$ in X for each $IFOS$T in Y, then ϕ is an $IFC\hat{\beta}G$continuous mapping.

Proof: Consider an $IFOS$ T be in Y. By assumption $\phi^{-1}(T)$ is an $IFRCS$ in X. As from Theorem 2.2.6[14], it has been prove that each IFRCS is an IF$\hat{\beta}$GCS, $\phi^{-1}(T)$ is an IF$\hat{\beta}$GCS in X. Therefore ϕ is an IFC$\hat{\beta}$G continuous mapping.

Theorem 3.11: In a mapping $\phi:(X,\tau)\to(Y,\kappa)$ if one of the subsequent properties is held:

(i) $\phi\big(cl(S)\big)\subseteq int(\phi(S))$ for every IFS S in X,
(ii) $cl\big(\phi^{-1}(T)\big)\subseteq \phi^{-1}(int(T))$ for every IFS T in Y and
(iii) $\phi^{-1}(cl(T))\subseteq int(\phi^{-1}(T))$ for every IFST in Y.

Then ϕ is an $IFC\hat{\beta}G$continuous mapping.

Proof: (i) ⇒ (ii): Consider an IFST in Y. Put $S=\phi^{-1}(T)$. By assumption $\phi\big(cl(\phi^{-1}(T))\big)\subseteq int\Big(\phi\big(\phi^{-1}(T)\big)\Big)=int(T)$. Then $\phi^{-1}(\phi(cl(\phi^{-1}(T))))\subseteq\phi^{-1}(int(T))$. Hence $cl\big(\phi^{-1}(T)\big)\subseteq \phi^{-1}(int(T))$.

(ii) ⇒ (iii): Taking complement for the result (ii) will implies (iii).

Assume (iii) holds. Consider an $IFCS$T in Y. Thus $cl(T)=T$. By assumption $\phi^{-1}(T)=\phi^{-1}(cl(T))\subseteq int(\phi^{-1}(T))$. Therefore $\phi^{-1}(T)\subseteq int(\phi^{-1}(T))$. But $int\big(\phi^{-1}(T)\big)\subseteq\phi^{-1}(T)$. Therefore $\phi^{-1}(T)$ is an $IFOS$ in X. As by the Theorem 2.2.4[14], $\phi^{-1}(T)$is an $IF\hat{\beta}GOS$ in X. Therefore ϕ is an $IFC\hat{\beta}G$continuous mapping.

Theorem 3.12: Let $\phi:(X,\tau)\to(Y,\kappa)$ is a bijective mapping. Then ϕ is an $IFC\hat{\beta}G$ continuous mapping if $cl(\phi(S))\subseteq\phi(int(S))$ for every IFSS in X.

Proof: Consider an $IFCS$ S in Y. Thus $cl(S)=S$ and $\phi^{-1}(S)$ is an IFS in X. By assumption $cl\Big(\phi\big(\phi^{-1}(S)\big)\Big)\subseteq\phi(int\big(\phi^{-1}(S)\big))$. As ϕ is bijective mapping, $\phi(\phi^{-1}(S))=S$. Then $S=cl(S)=cl(\phi(\phi^{-1}(S)))\subseteq\phi(int\big(\phi^{-1}(S)\big))$. Now $\phi^{-1}(S)\subseteq\phi^{-1}(\phi(int(\phi^{-1}(S))))=int\big(\phi^{-1}(S)\big)\subseteq\phi^{-1}(S)$. Therefore $\phi^{-1}(S)$ is an $IFOS$ in X and $\phi^{-1}(S)$ is an $IF\hat{\beta}GOS$ in X. Hence ϕ is an $IFC\hat{\beta}G$ continuous mapping.

Theorem 3.13: If $\phi:(X,\tau)\to(Y,\kappa)$is an $IFC\hat{\beta}G$ continuous mapping and (X,τ) is an $IF_{\hat{\beta}g}T_{1/2}$ space then the conditions below will hold:

(i) $cl\big(\phi^{-1}(T)\big)\subseteq \phi^{-1}(int(cl(T)))$ for each $IFOS$ T in Y and

(ii) $\phi^{-1}\Big(cl\big(int(T)\big)\Big)\subseteq int\ (\phi^{-1}(T))$ for each $IFCS$T in Y.

Proof: (i) ⇒ (ii): Consider an $IFOS$T in Y. By assumption $\phi^{-1}(T)$ is an $IF\hat{\beta}GCS$ in X. As X is an $IF_{\hat{\beta}g}T_{1/2}$ space, $\phi^{-1}(T)$ is an $IFCS$ in X. Then $cl\big(\phi^{-1}(T)\big)=\phi^{-1}(T)=\phi^{-1}(int(T))\subseteq\phi^{-1}(int(cl(T)))$. Therefore $cl\big(\phi^{-1}(T)\big)\subseteq\phi^{-1}(int(cl(T)))$.

(i) ⇒ (ii): Taking complement of (i) we get (ii).

Theorem 3.14: If $\phi:(X,\tau)\to(Y,\kappa)$ is a mapping and (X,τ) is an $IF_{\hat{\beta}g}T_{1/2}$space. Then the conditions below will equivalent:

(i) ϕ is an $IFC\hat{\beta}G$continuous mapping,

(ii) for every $p_{(\alpha,\beta)}$ in X and $IFCS$T containing $\phi(p_{(\alpha,\beta)})$, there exists an $IFOS$ S in X containing $p_{(\alpha,\beta)}\in S\subseteq\phi^{-1}(T)$ and

(iii) for every $p_{(\alpha,\beta)}$ in X and $IFCS$T containing $\phi(p_{(\alpha,\beta)})$, there exists an $IFOS$S in X containing $p_{(\alpha,\beta)}\in\phi(S)\subseteq$ T.

Proof: (i) ⇒ (ii): Consider an $IFC\hat{\beta}G$ continuous mapping ϕ and an $IFCS$T in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X, such that $\phi\big(p_{(\alpha,\beta)}\big)\in T$ then $p_{(\alpha,\beta)}\in\phi^{-1}(T)$. By assumption $\phi^{-1}(T)$ is an $IF\hat{\beta}GOS$ in X. As X is an

$IF_{\hat{\beta}g}T_{1/2}$space, $\phi^{-1}(T)$ is an $IFOS$ in X. For any IFOS S in Y, $S = int(\phi^{-1}(T)) \subseteq \phi^{-1}(T)$. Therefore $p_{(\alpha,\beta)} \in S \subseteq \phi^{-1}(T)$.

(ii) ⇒(iii): The result follows from the relations $\phi(S) \subseteq \phi(\phi^{-1}(T)) \subseteq T$.
(iii) ⇒ (i): Consider an $IFCS$T in Y and an IFP $p_{(\alpha,\beta)}$ in X, such that $\phi(p_{(\alpha,\beta)}) \in T$. By assumption there exists an $IFOS$S in X. Such that $p_{(\alpha,\beta)} \in$ S and $\phi(S) \subseteq T$implies $p_{(\alpha,\beta)} \in S \subseteq \phi^{-1}(g(S)) \subseteq \phi^{-1}(T)$. That is $p_{(\alpha,\beta)} \in \phi^{-1}(T)$. As S is an $IFOS$, $S = int(S) \subseteq int(\phi^{-1}(T))$. Thus $p_{(\alpha,\beta)} \in int(\phi^{-1}(T))$. But $\phi^{-1}(T) = \cup_{p_{(\alpha,\beta)\in\phi^{-1}(T)}} p_{(\alpha,\beta)} \subseteq int(\phi^{-1}(T)) \subseteq \phi^{-1}(T)$. Therefore $\phi^{-1}(T)$ is an $IFOS$ in X. Hence $\phi^{-1}(T)$ is an $IF\hat{\beta}GOS$ in X. Hence ϕ is an $IFC\hat{\beta}G$ continuous mapping.

Theorem 3.15: Let $\phi_1: (X,\tau) \to (Y,\kappa)$ and $\phi_2: (Y,\kappa) \to (Z,\eta)$ be any two mappings. If ϕ_1 is an $IFC\hat{\beta}G$continuous mapping and ϕ_2 is an IF continuous mapping, then $\phi_2 \circ \phi_1$ is an $IFC\hat{\beta}G$ continuous mapping.

Proof: Consider an $IFOS$ S in Z. As $\phi_2$is an IF continuous mapping, ${\phi_2}^{-1}(S)$ is an $IFOS$ in Y. Further, as ϕ_1 is an $IFC\hat{\beta}G$ continuous mapping, ${\phi_1}^{-1}({\phi_2}^{-1}(S))$ = $(\phi_2 \circ \phi_1)^{-1}(S)$ is an $IF\hat{\beta}GCS$ in X. Therefore $\phi_2 \circ \phi_1$ is an $IFC\hat{\beta}G$continuous mapping.

Theorem 3.16: Let $\phi_1: (X,\tau) \to (Y,\kappa)$ and $\phi_2: (Y,\kappa) \to (Z,\eta)$ be any two mappings. If ϕ_1 is an $IFC\hat{\beta}G$continuous mapping and ϕ_2 is an IF contra continuous mapping, then $\phi_2 \circ \phi_1$ is an $IF\hat{\beta}G$continuous mapping.

Proof: Consider an IFOS T be in Z. As ϕ_2 is an IF contra continuous mapping, ${\phi_2}^{-1}(T)$ is an IFCS in Y. Moreover, as ϕ_1 is an IFC$\hat{\beta}$G continuous mapping, ${\phi_1}^{-1}({\phi_2}^{-1}(S))$ = $(\phi_2 \circ \phi_1)^{-1}(S)$ is an IF$\hat{\beta}$GOS in X. Therefore $\phi_2 \circ \phi_1$ is an IF$\hat{\beta}$G continuous mapping.

Theorem 3.17: Let $\phi_1: (X,\tau) \to (Y,\kappa)$ and $\phi_2: (Y,\kappa) \to (Z,\eta)$ be any two mappings. If ϕ_1 is an$IF\hat{\beta}G$ irresolute mapping and ϕ_2 is an $IFC\hat{\beta}G$continuous mapping, then $\phi_2 \circ \phi_1$ is an $IFC\hat{\beta}G$ continuous mapping.

Proof: Consider an $IFOS$T in Z. As ϕ_2 is an $IFC\hat{\beta}G$continuous mapping, ${\phi_2}^{-1}(T)$ is an $IF\hat{\beta}GCS$ in Y. Moreover, as ϕ_1 is an $IF\hat{\beta}G$ irresolute mapping, ${\phi_1}^{-1}({\phi_2}^{-1}(S))$ = $(\phi_2 \circ \phi_1)^{-1}(S)$ is an $IF\hat{\beta}GCS$ in X. Hence $\phi_2 \circ \phi_1$ is an $IFC\hat{\beta}G$continuous mapping.

Theorem 3.18: If $\phi: (X,\tau) \to (Y,\kappa)$ be any mapping and $\phi^{-1}(cl(T)) \subseteq int(\phi^{-1}(T))$ for each IFS T in Y then ϕ is an IFC$\hat{\beta}$G continuous mapping.

Proof: Consider an IFCST in Y. Then $cl(T) = T$. By assumption $\phi^{-1}(T) = \phi^{-1}(cl(T)) \subseteq int(\phi^{-1}(T)) \subseteq \phi^{-1}(T)$. Hence $\phi^{-1}(T)$ is an IFOS in X. Therefore ϕ is an intuitionistic fuzzy contra continuous mapping. Then by Theorem 3.3, ϕ is an IFC$\hat{\beta}$G continuous mapping.

Theorem 3.19: In an IFC$\hat{\beta}$G continuous mapping $\phi: (X,\tau) \to (Y,\kappa)$, $(X,\tau)$is an $IF_{\hat{\beta}g}T_{1/2}$space implied and implies $\phi^{-1}(scl(T)) \subseteq int(\phi^{-1}(cl(T)))$ for each IFS T in Y.

Proof: Necessity: Consider an IFST in Y. Then $cl(T)$ is an IFCS in Y. By assumption $\phi^{-1}(cl(B)$ is an IF$\hat{\beta}$GOS in X. As(X,τ) is an $IF_{\hat{\beta}g}T_{1/2}$space, $\phi^{-1}(cl(T))$ is an IF$\hat{\beta}$GOS in X. Hence $\phi^{-1}(scl(T)) \subseteq \phi^{-1}(cl(T)) = int(\phi^{-1}(cl(T)))$. Thus IFOS $\phi^{-1}(scl(T)) \subseteq int(\phi^{-1}(cl(T)))$.

Sufficiency: Consider an IFCS T in Y. Then $cl(T) = T$, and by assumption $\phi^{-1}(scl(T)) \subseteq int(\phi^{-1}(cl(T))) = int(\phi^{-1}(T))$. Since every IFCS is an IFSCS, $scl(T) = T$. Therefore $\phi^{-1}(T) = \phi^{-1}(scl(T)) \subseteq int(\phi^{-1}(T)) \subseteq \phi^{-1}(T)$ implies $\phi^{-1}(T)$ is an IFOS in X. Therefore $\phi^{-1}(T)$ is an $IF\hat{\beta}GOS$ in X. Hence ϕ is an IFC$\hat{\beta}$G continuous mapping.

Theorem 3.20: An IF continuous mapping$\phi: (X,\tau) \to (Y,\kappa)$ is an $IFC\hat{\beta}G$continuous mapping if $IF\hat{\beta}GO(X) = IF\hat{\beta}GC(X)$.

Proof: Consider an IFOS S in Y. By assumption,$\phi^{-1}(S)$ is an $IFOS$ in X and hence $\phi^{-1}(S)$ is an $IF\hat{\beta}GOS$ in X. Since IF$\hat{\beta}$GO(X) = IF$\hat{\beta}$GC(X), $\phi^{-1}(S)$ is an $IF\hat{\beta}GCS$ in X. Henceϕ is an $IFC\hat{\beta}G$ continuous.

4. Conclusion:

In this research article, we have introduced a new kind of closed function called IFC$\hat{\beta}$G continuous mapping. The properties, equivalent conditions and some characterizations of the new mapping we established via theorems and converse parts are illustrated by suitable examples. As a future work, we like to extend the same concept to contra closed mapping, which is opposite to continuous mapping.

References:

[1] K.Arun Prakash, R. Santhi, *Intuitionistic fuzzy semi generalized closed functions,* International journal of Mathematics and soft computing, 2(2), 85-94, 2012.

[2] K.T.Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20, 87-96, 1986.

[3] C.L.Chang, *Fuzzy topological spaces,* J.Math.Anal.Appl. 24, 182-190, 1968.

[4] D.Coker, *An introduction to intuitionistic fuzzy topological spaces,* Fuzzy sets and systems, 88, 81-89, 1997.

[5] Coker and M. Demirci, on intuitionistic fuzzy points, Notes on *IFS,* 79-84,1995.

[6] H.Gurcay, D.Coker, *On fuzzy continuity in intuitionistic fuzzy topological spaces,*J.Fuzzy.Math, 5, 365-378, 1997.

[7] A Manimaran, K.Arun Prakash and P.Thangaraj, *Intuitionistic fuzzy almost open functions in intuitionistic fuzzy topological spaces*, International journal of Mathematical Archive, 3, 373 – 379, 2012.

[8] R. Santhi, K.Arun Prakash, *On intuitioistic fuzzy semi-generalized closed sets and its applications,* Int. J. Contemp. Math.Sciences,Vol.5, No .34,1677 -1688, 2010.

[9] M. Umadevi, K. Arun Prakash and S.Vengataasalam, *Intuitionistic fuzzy Almost $\hat{\beta}$ generalized closed mapping, VSRD InternationalJournal of Technical & Non-Technical Research, Vol. IX Issue III March 2018-PAGE-141*.

[10] M.Umadevi, K. Arun Prakash and S.Vengataasalam,*Intuitionistic fuzzy Almost $\hat{\beta}$ generalized continuous mapping, VSRD InternationalJournal of Technical & Non-Technical Research, Vol. IX Issue III March 2018-PAGE-141*.

[11] M.Umadevi, K. Arun Prakash and S.Vengataasalam, *Intuitionistic fuzzy $\hat{\beta}$ generalized closed mapping, submitted to* IJMA.

[12] M.Umadevi, K.Arun Prakash and S.Vengataasalam, *Intuitionistic fuzzy $\hat{\beta}$ generalized closed sets and its applications*, IJMA, 4(12), 2229-5046, 2013.

[13] M.Umadevi, S. Vengataasalam and K. Arun Prakash, *Intuitionistic fuzzy $\hat{\beta}$ generalized irresolute mapping,* Asian Journal of Research in Social Sciences and Humanities, 6(6), 1135-1146, 2016.

[14] M.Umadevi, S.Vengataasalam, *Intuitionistic fuzzy $\hat{\beta}$ Generalized Closed and continuous mapping***,** submitted as a thesis at Anna University, Chennai**.** (UMADEVI -1323769189).doc (D21438217), 1323769189-TS(UMADEVI M).pdf (D27758475)

[15] L.A.Zadeh, *Fuzzy sets*, Information Control, 8, 338-353, 1965.