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The transmuted inverse xgamma distribution and its statistical properties.

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Abstract:

In this article, we develop a new transmuted distribution named as transmuted inverse xgamma distribution (TIXGD). Various statistical properties of TIXGD are studied systemically and found out the expressions for moments, moment generating function (MGF) and quantile function. Hazard rate function (HRF) and Survival function (SF) are plotted on the graphs for the same. Maximum likelihood estimation (MLE) of estimation of parameters is discussed.

Keywords: *Transmuted family, Reliability characteristics, Moments, Moment Generating Function.*

1. Introduction:

The development of a new probability model is critical because it has more flexibility, allowing it to explain a considerably broader range of real-life situations. The information about the probability distribution is the most important and necessary need for analysing the given data. Before we analyses the survivability and reliability properties of the given data, we need to know which probability distribution is best suited to the data set. So, in this article, we present a transmuted version of inverse xgamma distribution.

A random variable X is said to have a transmuted distribution if its cumulative distribution function (cdf) is given by

$$G(x) = (1+\lambda)F(x) - \lambda F^2(x), \qquad |\lambda| \leq 1,$$

Where F(x) is called cdf of the base distribution.

Notice that at $\lambda = 0$ we have the base distribution of the random variable X.

The quadratic rank transmutation map (QRTM) was studied by Shaw and Buckley (2007) and many other authors have used generated several other known theoretical models by the understanding of this study. Many authors have developed several distributions and we mentioned few names here: Aryal and Tsokos (2009, 2011), Merovci (2013, 2013a), Ashour and Eltehiwy (2013), Khan and King (2013), Elbatal (2013), Johnson (1994), Yadav et al. (2021), Balaswamy (2018) and Bhatti (2019) have done work on several distributions.

Rest of article organized as follows: TIXGD is introduced in section 2. Reliability characteristics of TIXGD are discussed along with graph in section 3. In section 4, we derived the statistical properties viz., moment, MGF and quantile function. Maximum likelihood estimation procedure is also discussed in section 5. Conclusive words are given in section 6.

2. TIXGD

In this section we describe the TIXGD. Proposed distribution is transmuted version inverse xgamma distribution [see, Yadav et al. (2021a)]. The probability density function (PDF) of TIXGD is given below;

$$f(x) = \frac{\theta^2}{1+\theta}.\frac{1}{x^2}\left(1+\frac{\theta}{2}\times\frac{1}{x^2}\right) e^{-\frac{\theta}{x}}\left[1+\lambda-2\lambda\left\{\left(1+\frac{1}{1+\theta}\times\frac{1}{x}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x^2}\right)e^{-\frac{\theta}{x}}\right\}\right]$$

$;|\lambda| \leq 1, \ \theta > 0, \ x > 0$

(1)

and the cumulative density function (CDF) is given as

$$F(x) = \left[\left(1+\frac{1}{(1+\theta)}\times\frac{1}{x}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x^2}\right)\right]e^{-\frac{\theta}{x}}\left[1+\lambda-\lambda\left\{\left(1+\frac{1}{1+\theta}\times\frac{1}{x}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x^2}\right)e^{-\frac{\theta}{x}}\right\}\right]$$

(2)

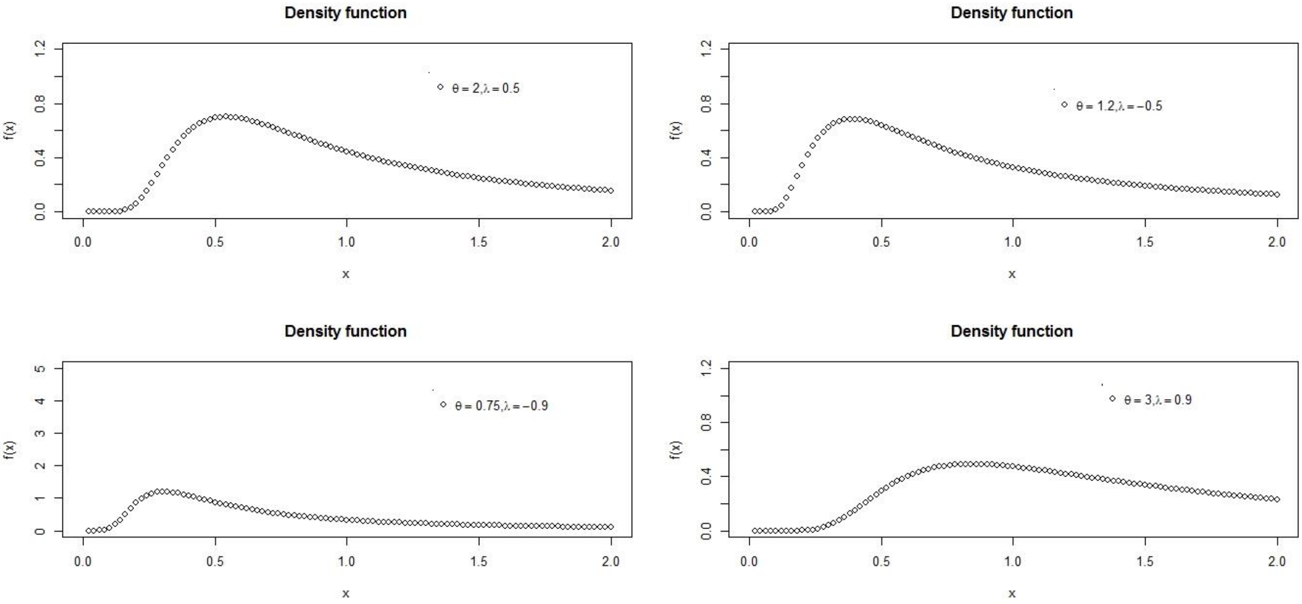

Figure 1: Probability density function

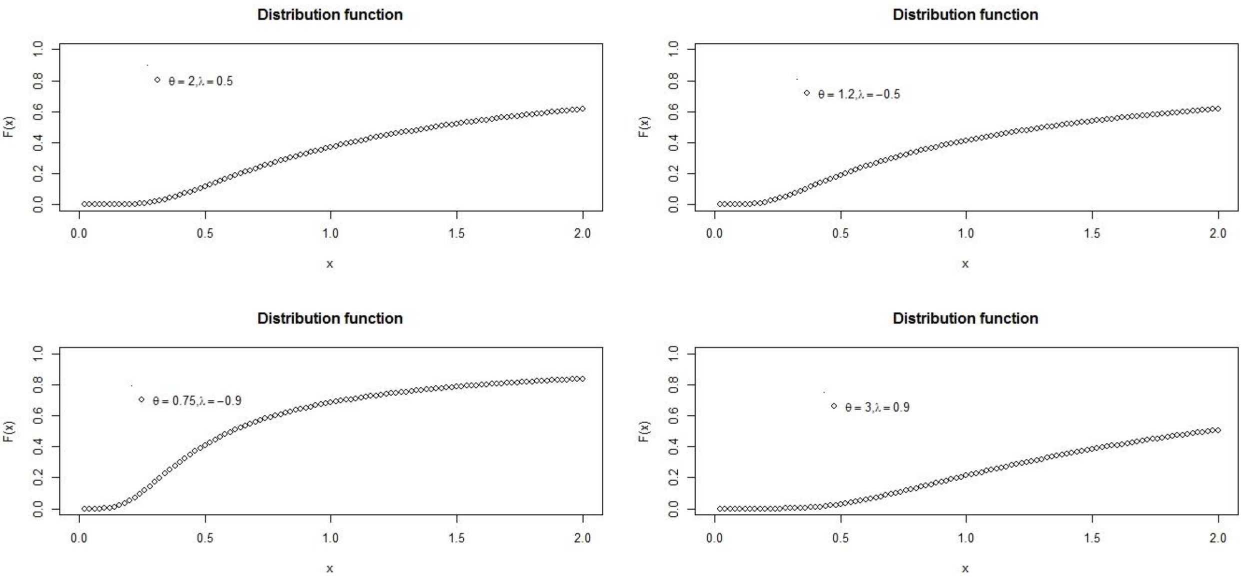

Figure 2: Cumulative density function

3. Reliability Characteristics:

The survival function is given below and it is denoted by $S(x)$,

$$S(x) = 1 - F(x)$$

$$S(x) = 1 - \left[\left(1 + \frac{1}{(1+\theta)} \times \frac{1}{x} + \frac{\theta^2}{2(1+\theta)} \times \frac{1}{x^2}\right)\right] e^{-\frac{\theta}{x}} \left[1 + \lambda - \lambda \left\{\left(1 + \frac{1}{1+\theta} \times \frac{1}{x} + \frac{\theta^2}{2(1+\theta)} \times \frac{1}{x^2}\right) e^{-\frac{\theta}{x}}\right\}\right]$$

(3)

The Hazard rate function is given below and it is denoted by $H(x)$,

$$H(x) = \frac{f(x)}{S(x)}$$

$$H(x) = \frac{\left(\frac{\theta^2}{1+\theta} \cdot \frac{1}{x^2}\left(1+\frac{\theta}{2} \times \frac{1}{x^2}\right) e^{-\frac{\theta}{x}}\left[1+\lambda-2\lambda\left\{\left(1+\frac{1}{1+\theta} \times \frac{1}{x}+\frac{\theta^2}{2(1+\theta)} \times \frac{1}{x^2}\right) e^{-\frac{\theta}{x}}\right\}\right]\right)}{1-\left[\left(1+\frac{1}{(1+\theta)} \times \frac{1}{x}+\frac{\theta^2}{2(1+\theta)} \times \frac{1}{x^2}\right)\right] e^{-\frac{\theta}{x}}\left[1+\lambda-\lambda\left\{\left(1+\frac{1}{1+\theta} \times \frac{1}{x}+\frac{\theta^2}{2(1+\theta)} \times \frac{1}{x^2}\right) e^{-\frac{\theta}{x}}\right\}\right]} \tag{4}$$

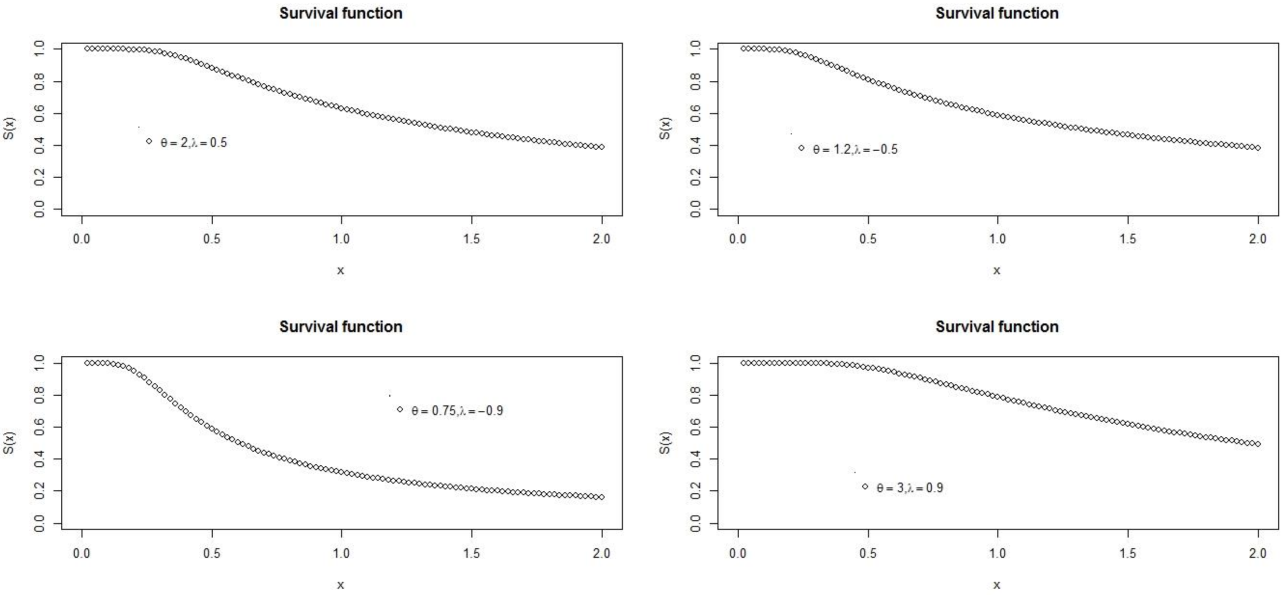

Figure 3: Survival function

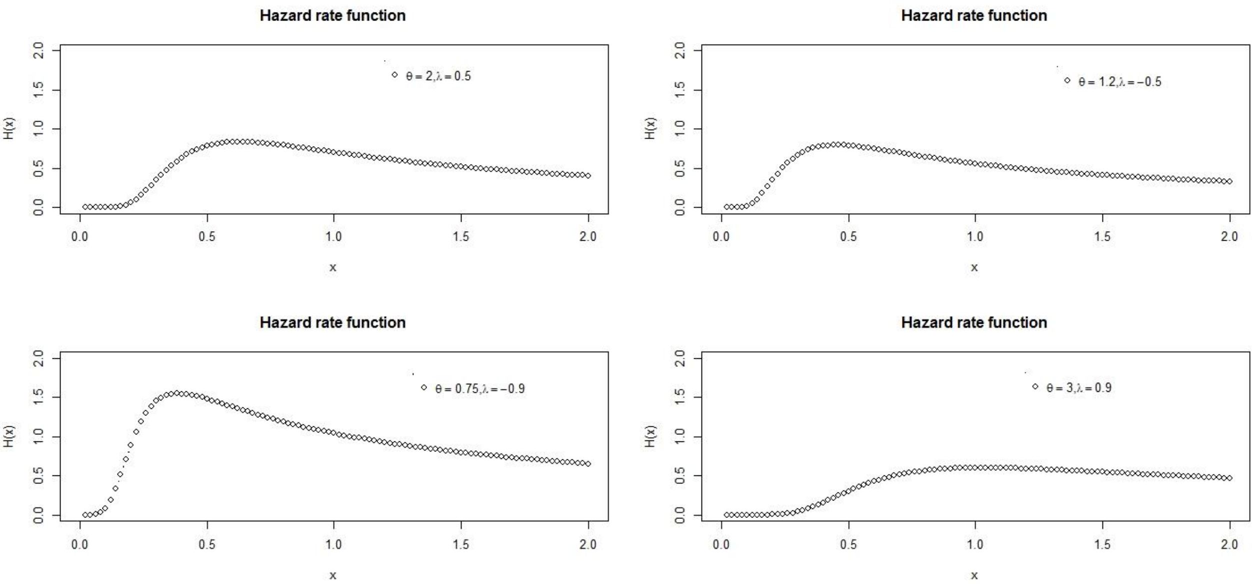

Figure 4: Hazard rate function

4. <u>Some Properties of the TIXGD distribution:</u>

In this section we covered some of the statistical properties of the TIXGD viz., moments, quantile function and MGF.

4.1 Moments

The r^{th} moment of a random variable X, is given by;

$$E[X^r] = \int_0^\infty x^r \ f\ (x)dx$$

$$E[X^r] = \int_0^\infty x^r \frac{\theta^2}{1+\theta} \times \frac{1}{x^2}\left(1+\frac{\theta}{2x^2}\right) e^{-\frac{\theta}{x}}(1+\lambda) - 2\lambda \int_0^\infty x^r \frac{\theta^2}{1+\theta} \times \frac{1}{x^2}\left(1+\frac{\theta}{2x^2}\right) e^{-\frac{\theta}{x}} \left[\left(1+\frac{1}{1+\theta}\times\frac{1}{x}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x^2}\right)e^{-\frac{\theta}{x}}\right]$$

$$E[X^r] = (1+\lambda)\frac{\theta^2}{1+\theta}\left[\frac{\Gamma(1-r)}{\theta^{1-r}}+\frac{\theta}{2}\frac{\Gamma(3-r)}{\theta^{3-r}}\right] - 2\lambda\frac{\theta^2}{1+\theta}\left[\frac{\Gamma(1-r)}{(2\theta)^{1-r}}+\frac{1}{1+\theta}.\frac{\Gamma(2-r)}{(2\theta)^{2-r}}+\frac{\theta^2}{2(1+\theta)}.\frac{\Gamma(3-r)}{(2\theta)^{3-r}}+\frac{\theta}{2}.\frac{\Gamma(3-r)}{(2\theta)^{3-r}}+\frac{\theta}{2(1+\theta)}.\frac{\Gamma(4-r)}{(2\theta)^{4-r}} + \frac{\theta^3}{4(1+\theta)}.\frac{\Gamma(5-r)}{(2\theta)^{5-r}}\right]$$

(5)

First four raw moments can be obtained by replacing r = 1; 2; 3; 4 in equation (5).

4.2 MGF

Let X have a TIXGD. Then, the moment generating function of X, is given as $M_X(t)$, i.e.,

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x)\ dx$$

$$E(e^{tx}) = \int_0^\infty e^{tx} \frac{\theta}{1+\theta} \times \frac{1}{x^2}\left(1+\frac{\theta}{2x^2}\right) e^{-\frac{\theta}{x}}(1+\lambda) - 2\lambda \int_0^\infty e^{tx} \frac{\theta}{1+\theta} \times \frac{1}{x^2}\left(1+\frac{\theta}{2x^2}\right) e^{-\frac{\theta}{x}} \left[\left(1+\frac{1}{1+\theta}\times\frac{1}{x}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x^2}\right)e^{-\frac{\theta}{x}}\right]$$

Final expression of MGF is given in equation (6).

$$E(e^{tx}) = \frac{(1+\lambda)\,\theta^2}{1+\theta}\sum_{i=0}^{\infty}\frac{t^i}{i\,!}\left[\frac{\Gamma(1-i)}{\theta^{1-i}}+\frac{\theta}{2}\frac{\Gamma(3-i)}{\theta^{3-i}}\right] - \frac{\theta^2}{1+\theta}\,2\lambda\left[\sum_{i=0}^{\infty}\frac{t^i}{i\,!}\left\{\frac{\Gamma(1-i)}{(2\theta)^{1-i}}+\frac{\Gamma(2-i)}{(2\theta)^{2-i}}\frac{1}{1+\theta}+\frac{\theta^2}{2(1+\theta)}\frac{\Gamma(3-i)}{(2\Theta)^{3-i}}+\frac{\theta}{2}\frac{\Gamma(3-i)}{(2\Theta)^{3-i}}+\frac{\theta}{2(1+\theta)}\frac{\Gamma(4-i)}{(2\Theta)^{4-i}} + \frac{\theta^3}{4(1+\theta)}\frac{\Gamma(5-i)}{(2\Theta)^{5-i}}\right\}\right]$$

(6)

4.3 Quantile Function:

The Quantile function of TIXGD is given as

$$p = F\big(Q(p)\big)$$

$$p = \left[\left(1+\frac{1}{(1+\theta)}\times\frac{1}{Q(p)}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{Q(p)^2}\right)\right]e^{-\frac{\theta}{Q(p)}}\left[1+\lambda-\lambda\left\{\left(1+\frac{1}{1+\theta}\times\frac{1}{Q(p)}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{Q(p)^2}\right)e^{-\frac{\theta}{Q(p)}}\right\}\right]$$

(7)

5. Estimation procedure

Here we discussed the maximum likelihood estimator (MLE) of the parameters which is used estimate the parameters of TIXGD. Maximizing the likelihood function by using parameters of model is the principle of MLE. The sample likelihood function under this is shown below

$$L=\prod_{i=1}^{n} f(x_i)$$

$$L=\prod_{i=1}^{n} \frac{\theta^2}{1+\theta} \cdot \frac{1}{x_i^2}\left(1+\frac{\theta}{2}\times\frac{1}{x_i^2}\right) e^{-\frac{\theta}{x_i}}\left[1+\lambda-2\lambda\left\{\left(1+\frac{1}{1+\theta}\times\frac{1}{x_i}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x_i^2}\right)e^{-\frac{\theta}{x_i}}\right\}\right]$$

Now, sample log-likelihood function is

$$\text{Log } L=\log\left(\frac{\theta^2}{1+\theta}\right)^n+\sum_{i=1}^{n} log\left(\frac{1}{x_i^2}\left(1+\frac{\theta}{2}\times\frac{1}{x_i^2}\right) e^{-\frac{\theta}{x_i}}\right)+\sum_{i=1}^{n}\log\left[1+\lambda-2\lambda\left\{\left(1+\frac{1}{1+\theta}\times\frac{1}{x_i}+\frac{\theta^2}{2(1+\theta)}\times\frac{1}{x_i^2}\right)e^{-\frac{\theta}{x_i}}\right\}\right] \quad \textbf{(8)}$$

Do the derivative of equation (8) with respect to λ and θ, and equate them equal to zero to get the estimate of λ and θ.

$$\frac{\partial \log L}{\partial \theta}=0$$

And

$$\frac{\partial \log L}{\partial \lambda}=0$$

It is not possible to get the tractable form of estimate of parameters λ and θ. For estimation of parameter, we use R programming.

6. Conclusions

In this paper we propose a new model that named as Transmuted Inverse X-Gamma Distribution (TIXGD) which is a new version of inverse xgamma distribution. We have obtained the expression of probability distribution function, cumulative distribution function, survival function and hazard rate function respectively. Some of the statistical properties like moment, moment generating function and quantile function. We discussed the estimation procedure for the estimation of parameters of TIXGD, named as MLE.

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