Transmuted extended xgamma-2 distribution and its statistical properties.

Harsh Tripathi [a] and Chitra [a*]

[a]Department of Mtahematics, Lovely Professional University, Punjab, India

Abstract

A new distribution, transmuted extended xgamma-2 Distribution (TEtXGD-2) is developed in this paper. We determined statistical properties of TEtXGD-2 viz., reliability characteristics such as survival function (SF) and hazard rate functions (HRF), moment, quantile function and moment generating function (MGF). Also, we have presented behavioural changes of survival and hazard rate functions through graph for different set up of the parameters λ and β. At last, we introduced the parameter estimation procedure through maximum likelihood estimation (MLE).

Keywords: Transmuted family, reliability characteristics, moment, moment generating function, maximum likelihood estimation.

1 Introduction

In many fields of real life such as engineering, banking, finance etc. amongst these modelling and analysing of certain data is very important. A lot of lifetime case modellings have been used to solve huge problems and to work on. The quality of this procedure used in a statistical analysis depends effectively on the assumed probability model or distribution. However, there is still lot to perform in this field. In this we are dealing with two parameter distribution which may provide reasonably precision in fitting of data.

There are so many techniques for generalization of distributions exist in literature and the quadratic rank transmutation map (QRTM) is one of the technique, was studied by Shaw and Buckley (2007). Cumulative distribution function (CDF) of the transmuted distribution is:

$$\eta(x) = (1+\lambda)F(x) - \lambda F^2(x); \qquad |\lambda| \le 1$$

where F(x) is CDF of baseline distribution. Many authors have developed several distributions and we mentioned few names here: Aryal and Tsokos (2009,2011), Gradshteyn and Ryzhik (2000), Johnson et al. (1994), Khan and King (2013), Balaswamy (2018), Bhatti et al. (2019), Yadav et al. (2021), Kundu and Raqab (2005) and Merovci (2013) have introduced the different probability distributions in several years.

Rest of article is organized follows: TEtXGD-2 is introduced in section 2. Surviaval properties of TEtXGD-2 are discussed along with graph in section 3. In section 4, we derived the statistical properties viz., moment, MGF and quantile function. MLE procedure is also discussed in section 5. Conclusive words are given in section 6.

2 TEtXGD-2

In this section we introduced transmuted version of extended xgamma distribution [see, Saha et al. (2022)]. The probability distribution function (PDF) of TEtXGD-2 is

$$f(x) = \frac{2^3}{(2+\beta)\Gamma(4)}(2^2 + 2 + 2\beta x^2)e^{-2x}x^{2-1} \times \left[1 + \lambda - 2\lambda\left\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x)\right\}\right]; x > 0, |\lambda| \le 1, \beta > 0 \qquad (1)$$

and CDF of TEtXGD-2 is

$$F(x) = \left(\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x)\right)\times \left[1 + \lambda - \lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x)\}\right] \quad (2)$$

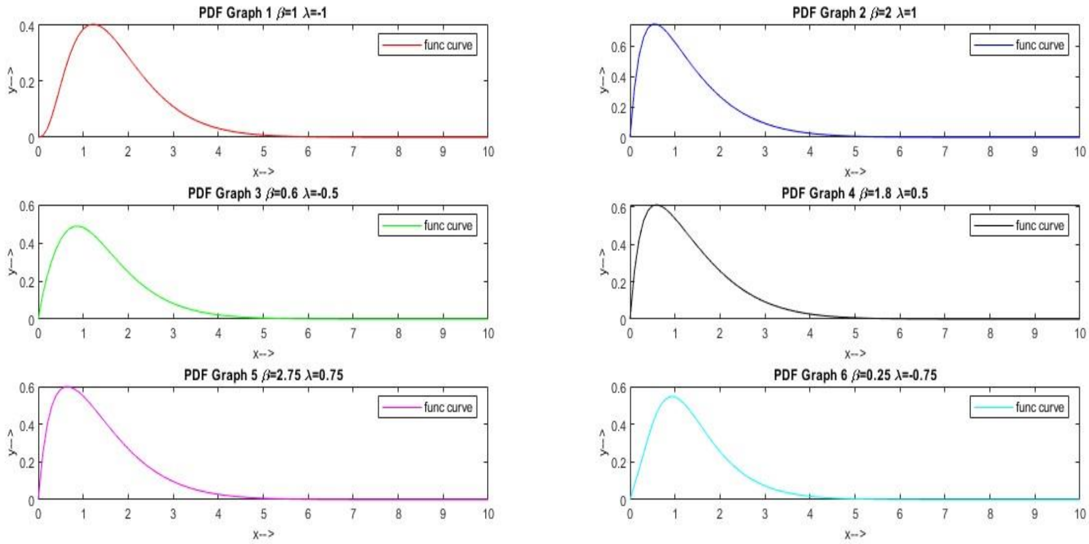

Figure 1: PDF graph

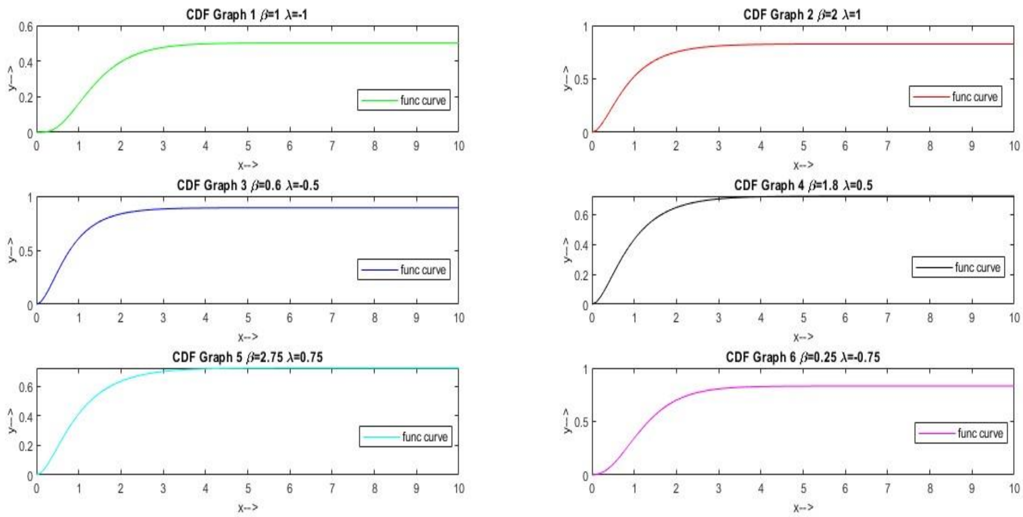

Figure 2: CDF graph

Figure 1 and 2 represents the graph of PDF and CDF for different setups of λ and β.

3 Reliability characteristics

Here we discuss the SF and HRF of the TEtXGD-2. Expressions of SF and HRF of proposed distribution are:

$$S(x) = 1 - \left(\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x)\right) \times \left[1 + \lambda - \lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x)\}\right] \quad (3)$$

and

$$H(x) == \frac{\frac{2^3}{(2+\beta)\Gamma(4)}(2^2+2+2\beta x^2)e^{-2x}x^{2-1}\left[1+\lambda-2\lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x)+\frac{\beta}{4\Gamma(4)}\gamma(4,2x)\}\right]}{1-\left(\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x)+\frac{\beta}{4\Gamma(4)}\gamma(4,2x)\right)\left[1+\lambda-\lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x)+\frac{\beta}{4\Gamma(4)}\gamma(4,2x)\}\right]} \quad (4)$$

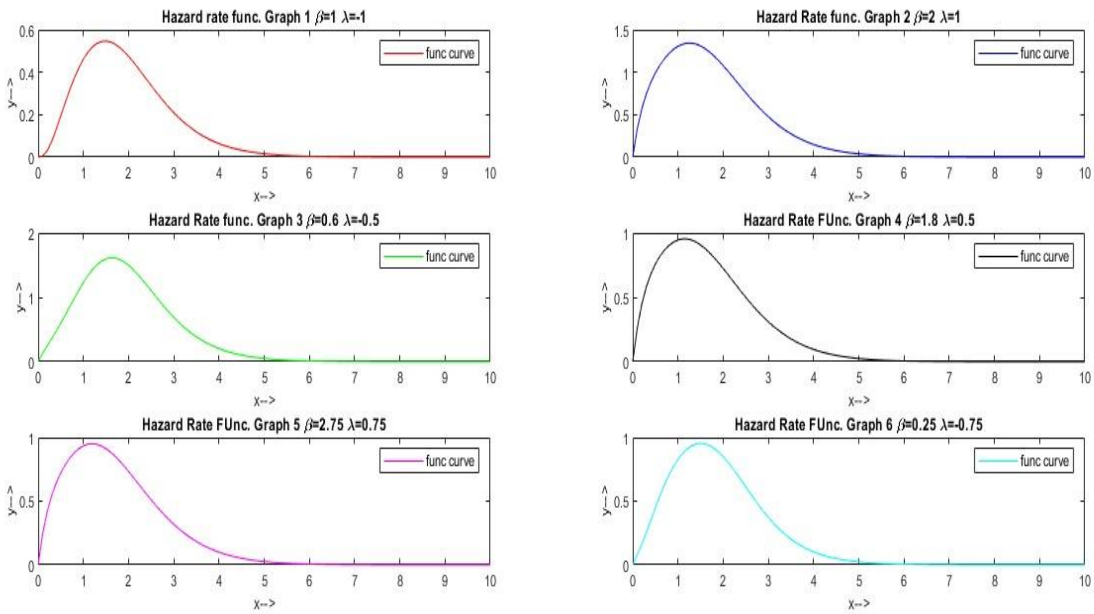

Figure 3: HRF graph

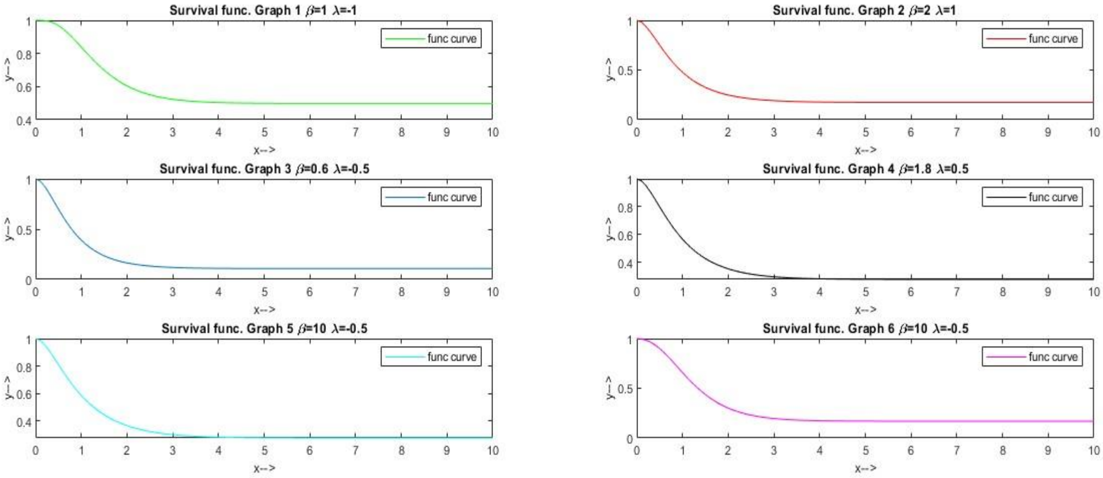

Figure 4: SF graph

Figure 3 and 4 are graphical representation of HRF and SF for variety values of λ and β.

4 Statistical Properties

In this section, we derived the expressions of statistical properties like moment, MGF and quantile function along with their necessary proofs.

4.1 Moments

Moments are useful to decide the shape and peakedness of probability distribution. Let X have a TEtXGD-2. Then the r-th moment of TEtXGD-2 is:

$$E(X^r) = \int_0^\infty x^r f(x) d(x)$$

$$\begin{aligned}
E(X^r) = {} & \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\left\{\frac{6\Gamma(r+2)}{2^{r+2}} + \frac{2\beta\Gamma(r+4)}{2^{r+4}}\right\} \\
& + \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\frac{8\lambda}{(2+\beta)\Gamma(3)}\left\{-\frac{12\Gamma(r+3)}{4^{r+3}} - \frac{6\Gamma(r+2)}{4^{r+2}} + \frac{6\Gamma(r+2)}{2^{r+2}} - \frac{4\beta\Gamma(r+5)}{4^{r+5}} - \frac{2\beta\Gamma(r+4)}{4^{r+4}} + \frac{2\beta\Gamma(r+4)}{2^{r+4}}\right\} \\
& + \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\frac{2\lambda\beta}{4\Gamma(4)}\left\{-\frac{48\Gamma(r+5)}{4^{r+5}} - \frac{72\Gamma(r+4)}{4^{r+4}} + \frac{72\Gamma(r+3)}{4^{r+3}} + \frac{36\Gamma(r+3)}{4^{r+3}} - \frac{36\Gamma(r+3)}{2^{r+3}} - \frac{16\beta\Gamma(r+7)}{4^{r+7}}\right\} \\
& + \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\frac{2\lambda\beta}{4\Gamma(4)}\left\{-\frac{24\beta\Gamma(r+6)}{4^{r+6}} + \frac{24\beta\Gamma(r+5)}{4^{r+5}} + \frac{12\beta\Gamma(r+4)}{4^{r+4}} - \frac{12\beta\Gamma(r+4)}{2^{r+4}}\right\} \qquad (5)
\end{aligned}$$

First four raw moments can be obtain by replacing $r = 1,2,3,4$ in equation (5).

4.2 MGF

Let X have a TEtXGD-2. Then the MGF of X [see, equation (6)]

$$E(e^{tx}) = \int_0^\infty e^{tx} f(x) dx$$

$$E(e^{tx}) = \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\left\{\frac{6\Gamma 2}{(2-t)^2} + \frac{2\beta\Gamma 4}{(2-t)^4}\right\}$$
$$+ \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\frac{8\lambda}{(2+\beta)\Gamma 3}\left\{-\frac{12\Gamma 3}{(4-t)^3} - \frac{6\Gamma 2}{(4-t)^2} + \frac{6\Gamma 2}{(2-t)^2} - \frac{4\beta\Gamma 5}{(4-t)^5} - \frac{2\beta\Gamma 4}{(4-t)^4} + \frac{2\beta\Gamma 4}{(2-t)^4}\right\}$$
$$+ \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\frac{2\lambda\beta}{4\Gamma 4}\left\{-\frac{48\Gamma 5}{(4-t)^5} - \frac{72\Gamma 4}{(4-t)^4} + \frac{72\Gamma 3}{(4-t)^3} + \frac{36\Gamma 3}{(4-t)^3} - \frac{36\Gamma 3}{(2-t)^3} - \frac{16\beta\Gamma 7}{(4-t)^7}\right\}$$
$$+ \frac{8(1+\lambda)}{(2+\beta)\Gamma(4)}\frac{2\lambda\beta}{4\Gamma 4}\left\{-\frac{24\beta\Gamma 6}{(4-t)^6} + \frac{24\beta\Gamma 5}{(4-t)^5} + \frac{12\beta\Gamma 4}{(4-t)^4} - \frac{12\beta\Gamma 4}{(2-t)^5}\right\} \quad (6)$$

4.3 Quantile function

Quantile function (Q(p)) of TEtXGD-2 is,

$$p = F(Q(p))$$

$$p = \frac{2^3}{(2+\beta)\Gamma(4)}(2^2 + 2 + 2\beta Q(p)^2)e^{-2Q(p)}Q(p)^{2-1} \times$$
$$\left[1 + \lambda - 2\lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2Q(p)) + \frac{\beta}{4\Gamma(4)}\gamma(4,2Q(p))\}\right] \quad (7)$$

5 Estimation procedure

In this section maximum likelihood estimation (MLE) for the parameter estimation is discussed. Principal of MLE based on the maximization of likelihood of TEtXGD-2 for parameters. Let $X_1, X_2 \ldots . X_n$ be random sample of size n from equation (2.1) and likelihood function is:

$$L = \prod_{i=1}^{n} \frac{2^3}{(2+\beta)\Gamma(4)}(2^2 + 2 + 2\beta x_i^2)e^{-2x_i}x_i^{2-1} \times$$
$$\left[1 + \lambda - 2\lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x_i) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x_i)\}\right] \quad (8)$$

$$\log L = n * \log\frac{2^3}{(2+\beta)\Gamma(4)} + \sum_{i=1}^{n}\log((2^2 + 2 + 2\beta x_i^2)) + \sum_{i=1}^{n}\log(e^{-2x_i}) + \sum_{i=1}^{n}\log(x_i)$$
$$+ \sum_{i=0}^{n}\log\left\{\left[1 + \lambda - 2\lambda\{\frac{4}{(2+\beta)\Gamma(3)}\gamma(2,2x_i) + \frac{\beta}{4\Gamma(4)}\gamma(4,2x_i)\}\right]\right\} \quad (9)$$

Do the derivative of equation (5.9) with respect to λ and β, and equate them equal to zero to get the estimate of λ and β.

$$\frac{\partial \log L}{\partial \lambda} = 0, \frac{\partial \log L}{\partial \beta} = 0$$

It is not possible to get the tractable form of estimate of parameters λ and β. For the estimation of parameter, we use R programming.

6 Conclusions

In this paper, we propose a two parameter TEtXGD-2. We have obtained the expression of PDF, CDF, SF and HRF respectively. Some of the statistical properties like moment, MGF and quantile function. We discussed the estimation procedure for the estimation of parameters of TEtXGD-2, named as MLE.

7 References

1. Shaw W and Buckley, I. (2007).The alchemy of probability distributions: beyond Gram- Charlier expansions and a skew-kurtotic- normal distribution from a rank transmutation map. Research Report, 2007.
2. Aryal, G. R., and Tsokos, C. P. (2009). On the transmuted extreme value distribution with application. Nonlinear Analysis: Theory, Methods and Applications, 71, 1401-1407.
3. Aryal, G. R., and Tsokos, C. P. (2011). Transmuted Weibull distribution: A generalization of the Weibull probability distribution. European Journal of Pure and Applied Mathematics, 4, 89-102.
4. Gradshteyn, I. S., and Ryzhik, I. M. (2000). Table of Integrals, Series, and Products. San Diego: Academic Press.
5. Johnson, N. L., Kotz, S., and Balakrishnan, N. (1994). Continuous Univariate Distributions. New York: Wiley.
6. Khan, M. S., and King, R. (2013). Transmuted modified Weibull distribution: A generalization of the modified Weibull probability distribution. European Journal of Pure and Applied Mathematics, 6, 66-88.
7. Balaswamy, S. (2018). Transmuted Half Normal Distribution. International Journal of Scientific Research in Mathematical and Statistical Sciences, 5:163–170.
8. Bhatti, F. A., Hamedani, G. G., Sheng, W., and Ahmad, M. (2019). Cubic Rank Transmuted Modified Burr III Pareto Distribution: Development, Properties, Characterizations and Applications. International Journal of Statistics and Probability, 8:94–112, doi:10.5539/ijsp.v8n1p94.
9. Yadav, A. S., Saha, M., Tripathi, H., & Kumar, S. (2021). The Exponentiated XGamma Distribution: a New Monotone Failure Rate Model and Its Applications to Lifetime Data. Statistica, 81(3), 303-334.
10. Kundu, D., and Raqab, M. Z. (2005). Generalized Rayleigh distribution: different methods of estimation. Computational Statistics and Data Analysis, 49, 187-200.
11. Merovci, F. (2013). Transmuted Lindley distribution. International Journal of Open Problems in Computer Science and Mathematics, 6, (to appear).
12. Saha, M., Yadav, AS., Pandey, A., Shukla, S., Maiti, SS & Tripathi, H. (2022). The extended xgamma distribution: characteristics and applications,https://www.researchgate.net/publication/359132685.