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A Note on Properties of Mittag - Leffler Function Under Generalized Fractional Integral Operators

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Abstract

The purpose of this paper is to study various properties of Mittag-Leffler function and introduce some new theorems which give the images of this Mittag - Leffler function under the generalized fractional integral operators in terms of Beta, Mellin, Laplace, Whittaker and K-transforms. On account of general nature of results in the form of Mittag - Leffler function, several known and new results involving simpler functions can be obtained by taking suitable values of parameters involved.

Keywords: Generalized fractional calculus operators; Mittag-Leffler function; I-function; Beta transform; Mellin transform; Laplace transform; Whittaker transform; K-transform.

1. Introduction

In the literature, it is evident that many researchers have worked on special functions involving with fractional calculus operators and integral transforms and studied their properties with applications in science and technology [1,2,3,4,7,8,10,12,14,15,16,18]. Following the sequence, in this paper our objective is to study some properties of Mittag-Leffler function [11] and identify some new image formulas under the generalized fractional integral operators with the use of integral transformations like Laplace transform [19], Mellin Transform [5], Beta transform [19], Whittaker transform [20] and K-transform [6]. The generalized fractional operators those we used here, involves well known I-function given by Saxena [17] as kernel, are the extension of the operators given by Saxena and Kumbhat [16]. In study of the properties of Mittag-Leffler function and the related image formulas, we first find the images of the function under the generalized fractional integral operators and then we obtain the integral transforms of these images.

The I-function, given by Saxena [17] is defined as:

$$I[z] = I^{m,n}_{p_i,q_i:l}\left\{z\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\} = \frac{1}{2\pi\omega}\int_L \phi(\xi)z^{\xi}\,d\xi \quad \dots (1.1)$$

where $\omega = \sqrt{-1}$ and

$$\phi(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{l}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\ \prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right\}} \quad \dots (1.2)$$

For more details, one can refer [17].

The generalized Mittag-Leffler Function introduced by Prabhakar [13] and defined as

$$E^{\gamma}_{\lambda,\beta}(z) = \sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)n!}z^n \qquad ;\ (\lambda,\beta,\gamma\in C); Re(\lambda)>0, Re(\beta)>0, Re(\gamma)>0 \quad \dots (1.3)$$

The Generalized Fractional Integral Operators involving I-function as kernel defined and used here, are the extension of fractional integral operators given by Saxena and Kumbhat [16] and can be verified smoothly for I-function along with the conditions mentioned here. These operators are defined as

$$A^{\mu,\alpha}_{x,r}[f(x)] = rx^{-\mu-r\alpha-1}\int_0^x t^{\mu}\,(x^r-t^r)^{\alpha}\ I^{m,n}_{p_i,q_i:r}\left\{kX\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}f(t)\,dt \quad \dots (1.4)$$

$$K_{x,r}^{\varepsilon,\alpha}[f(x)] = rx^{\varepsilon}\int_0^x t^{-\varepsilon-r\alpha-1}\,(t^r - x^r)^{\alpha}\ I_{p_i,q_i:r}^{m,n}\left\{kY\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}f(t)\,dt$$

… (1.5)

Where

$$X = \left(\frac{t^r}{x^r}\right)^{\tau}\left(1-\frac{t^r}{x^r}\right)^{\upsilon}\ and\ Y = \left(\frac{x^r}{t^r}\right)^{\tau}\left(1-\frac{x^r}{t^r}\right)^{\upsilon}\quad ;\tau,\upsilon > 0$$

… (1.6)

The sufficient conditions of these operators are

$(i) 1 \le p,q < \infty, p^{-1}+q^{-1} = 1;$

$(ii) Re\left(\mu + r\tau\left(\frac{b_j}{\beta_j}\right)\right) > -q^{-1};\ Re\left(\alpha + r\upsilon\left(\frac{b_j}{\beta_j}\right)\right) > -q^{-1};$

$$Re\left(\varepsilon+\alpha + r\tau\left(\frac{b_j}{\beta_j}\right)\right) > -p^{-1}; j = 1,\dots,m$$

$(iii)\ f(x)\epsilon L_p(0,\infty)$

$(iv)\ |argk| \le \frac{\pi\lambda}{2}, \lambda > 0$

where

$$\lambda = \sum_{j=1}^{m}(\alpha_i) + \sum_{j=1}^{n}(\beta_i) - \max_{1\le i\le r}\left[\sum_{j=n+1}^{p_i}(\alpha_{ji}) + \sum_{j=m+1}^{q_i}(\alpha_{ji})\right]$$

The Beta Transform [19] of a function $f(t)$ is defined as

$$B\{f(t);a,b\} = \int_0^1 t^{a-1}(1-t)^{b-1}\,f(t)\,dt\quad ;a,b\in\mathbb{C}, Re(a) > 0, Re(b) > 0$$

… (1.7)

The Mellin Transform [5] of a function $f(t)$ is defined as

$$M\{f(t);s\} = \int_0^{\infty} t^{s-1}\,f(t)dt\quad ;Re(s) > 0$$

… (1.8)

The well-known Laplace Transform [19] of a function $f(t)$, is defined as

$$L\{f(t);s\} = \int_0^{\infty} e^{-st}\,f(t)dt\quad ;Re(s) > 0$$

… (1.9)

The Whittaker transform given by Whittaker and Watson [20] holds the following result

$$\int_0^{\infty} e^{-\frac{t}{2}}\,t^{\zeta-1}W_{\chi,\omega}(t)dt = \frac{\Gamma\left(\frac{1}{2}+\omega+\zeta\right)\Gamma\left(\frac{1}{2}-\omega+\zeta\right)}{\Gamma(1-\chi+\omega)}\quad ;Re(\omega\pm\zeta) > -\frac{1}{2}$$

… (1.10)

where $W_{\chi,\omega}(t)$ is the Whittaker confluent hypergeometric function and defined as

$$W_{\chi,w}(t) = \frac{\Gamma(-2w)}{\Gamma\left(\frac{1}{2}-\chi-w\right)}M_{\chi,w}(t) + \frac{\Gamma(2w)}{\Gamma\left(\frac{1}{2}+\chi+w\right)}M_{\chi,-w}(t)$$

where

$$M_{\chi,w}(t) = t^{\left(\frac{1}{2}+w\right)}e^{-\frac{t}{2}}\,{}_1F_1\left(\frac{1}{2}+w-\chi;2w+1;t\right)$$

… (1.11)

The K-Transform [6] is defined as

$$\Re_v[f(x);p] = g[p;v] = \int_0^\infty (px)^{\frac{1}{2}} K_v\,(px) f(x)dx$$

… (1.12)

where $Re(p) > 0;\ K_v(x)$ is the Bessel function of the second kind [13], defined by

$$K_v(x) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} W_{0,v}(2x)$$

… (1.13)

here $W_{0,v}(.)$ is the Whittaker function [20].

The following result given by Mathai [9] will be used here in evaluating the integrals

$$\int_0^\infty t^{\rho-1} K_v(ax)dx = 2^{\rho-2} a^{-p}\, \Gamma\left(\frac{\rho \pm v}{2}\right);\ Re(a) > 0;\ Re(\rho \pm v) > 0$$

… (1.14)

2. Main Results

In this section we study the properties and obtain the images of Mittag-Leffler function under the generalized fractional integral operators defined in (1.4) and (1.5).

Theorem 2.1 Let $a, \lambda, \beta, \vartheta, \gamma \in \mathbb{C}, \lambda > 0, x > 0, Re(\lambda) > 0, Re(\vartheta) > 0, Re(\beta) > 0, Re(\gamma) > 0, 1 \le p \le 2$ then

$$A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1} E_{\lambda,\beta}^{\gamma}(at^v)\right)(x) = x^{\vartheta-1} \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\lambda n+\beta)} \frac{(ax^v)^n}{n!}$$

$$\times\ I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k \left| \begin{matrix} (a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}, \left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right), (-\alpha, v) \\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha, \tau+v\right), (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right.\right\}$$

… (2.1)

Provided the conditions, mentioned with operators (1.4) are satisfied.

Proof. To establish the result (2.1), using (1.6) and definition of I-function on left-hand side of (2.1), we have

$$A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1} E_{\lambda,\beta}^{\gamma}(at^v)\right)(x) = \Delta_1(say)$$

$$= rx^{-\mu-r\alpha-1} \int_0^x t^{\mu+\vartheta-1}\,(x^r - t^r)^\alpha \left\{\frac{1}{2\pi\omega}\int_L \phi(\xi)(kX)^\xi d\xi\right\} \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\lambda n+\beta)n!}(at^v)^n\ dt$$

Now, changing the order of the integration which is valid under the given conditions, we get

$$\Delta_1 = rx^{-\mu-r\alpha-1} \sum_{n=0}^{\infty} \frac{(\gamma)_n a^n}{\Gamma(\lambda n+\beta)n!} \times \frac{1}{2\pi\omega}\int_L \phi(\xi).\,k^\xi x^{r\alpha-r\tau\xi}\left\{\int_0^x t^{\mu+\vartheta+vn+r\tau\xi-1}\left(1-\frac{t^r}{x^r}\right)^{\alpha+v\xi} dt\right\} d\xi$$

Let $t^r/x^r = y \Rightarrow\ t = xy^{1/r}$, we get

$$\Delta_1 = x^{\vartheta-1} \sum_{n=0}^{\infty} \frac{(\gamma)_n a^n}{\Gamma(\lambda n+\beta)\,n!} \frac{x^{vn}}{2\pi\omega} \int_L \phi(\xi)\ k^\xi x^{v\xi} \times \left\{\int_0^1 y^{\left(\frac{\mu+\vartheta+vn}{r}\right)+\tau\xi}(1-y)^{\alpha+v\xi}\ dy\right\} d\xi$$

Using the definition of Beta function in the inner integral, we have

$$\Delta_1 = x^{\vartheta-1} \sum_{n=0}^{\infty} \frac{(\gamma)_n (ax^v)^n}{\Gamma(\lambda n+\beta)\,n!} \frac{1}{2\pi\omega} \int_L \phi(\xi)\ k^\xi \times \frac{\Gamma\big(((\mu+\vartheta+vn)/r)+\tau\xi\big)\Gamma(\alpha+1+v\xi)}{\Gamma\big(((\mu+\vartheta+vn)/r)+\alpha+1+(\tau+v)\xi\big)} d\xi$$

Further by using the definition of I-function under (1.1) and (1.2), we reached at the desired result.

Theorem 2.2 Let $a, \lambda, \beta, \vartheta, \gamma \in \mathbb{C}, x > 0, Re(\lambda) > 0, Re(\vartheta) < 1,\ 1 \le p \le 2,$ then

$$K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta} E_{\lambda,\beta}^{\gamma}(at^{-v})\right)(x) = x^{-\vartheta} \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\lambda n+\beta)n!}\ (ax^{-v})^n\ \times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\epsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

… (2.2)

Provided the conditions, mentioned with operator (1.5) are satisfied.

Proof. Using (1.6) and definition of I-function on left-hand side of (2.2), we have

$K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)(x)=\Delta_2(say)$

$$=rx^{\varepsilon}\int_x^{\infty}t^{-\varepsilon-\vartheta-r\alpha-1}\,(t^r-x^r)^{\alpha}\left\{\frac{1}{2\pi\omega}\int_L\phi(\xi)(kY)^{\xi}d\xi\right\}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}(at^{-v})^n dt$$

Now, changing the order of the integration which is valid under the given conditions, we get

$$\Delta_2=rx^{\varepsilon}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}\frac{1}{2\pi\omega}\int_L\phi(\xi)\;k^{\xi}x^{r\tau\xi}\left\{\int_x^{\infty}t^{-\varepsilon-\vartheta-vn-r\tau\xi-1}\left(1-\frac{x^r}{t^r}\right)^{\alpha+v\xi}dt\right\}d\xi$$

Let $x^r/t^r=y\ then\ t=x/y^{(1/r)}$ in the above term and using beta function, we get

$$\Delta_2=x^{-\vartheta-1}\sum_{n=0}^{\infty}\frac{(\gamma)_n a^n}{\Gamma(\lambda n+\beta)\,n!}\frac{x^{-vn}}{2\pi\omega}\int_L\phi(\xi)\;k^{\xi}\left\{\int_0^1 y^{\left(\frac{\varepsilon+\vartheta+vn}{r}\right)+\tau\xi-1}(1-y)^{\alpha+v\xi}\,dy\right\}d\xi$$

Using the definition of Beta function in the inner integral, we have

$$\Delta_2=x^{-\vartheta}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}(ax^{-v})^n\frac{1}{2\pi\omega}\int_L\phi(\xi)k^{\xi}\times\frac{\Gamma\left(\left((\varepsilon+\vartheta+vn)/r\right)+\tau\xi\right)\Gamma(\alpha+1+v\xi)}{\Gamma\left(\left((\varepsilon+\vartheta+vn)/r\right)+\alpha+1+(\tau+v)\xi\right)}d\xi$$

Further by using the definition of I-function under (1.1) and (1.2), we reached at the desired result.

Theorem 2.3 The operator defined in (2.1) and $Re(\psi+\vartheta)>0$, then

$$x^{\psi}A_{x,r}^{\mu,\alpha}\left[t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right](x)=A_{x,r}^{\mu-\psi,\alpha}\left[t^{\psi+\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right](x)$$

…(2.3)

Provided the conditions, mentioned with operator (1.4) are satisfied.

Proof. Let us use (2.1) in left hand side of (2.3), we get

$$x^{\psi}R_{x,r}^{\mu,\alpha}\left[t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right](x)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}a^n\,x^{\vartheta+\psi+vn-1}$$

$$\times I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

…(2.4)

By using (2.1) in right hand side of (2.3), we have

$A_{x,r}^{\mu-\psi,\alpha}\left[t^{\psi+\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right](x)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}a^n\,x^{\vartheta+\psi+vn-1}\times$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

…(2.5)

By observing (2.4) and (2.5), the result (2.3) is true.

Theorem 2.4 The operator defined in (2.2) and $Re(\psi+\vartheta)>0$, then

$$x^{-\psi}K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)(x)=K_{x,r}^{\varepsilon-\psi,\alpha}\left(t^{-\vartheta-\psi}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)(x) \quad \dots(2.6)$$

Proof. Using (2.2) in left side of (2.4), we get

$$x^{-\psi}K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)(x)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}a^n\,x^{-\vartheta-\psi-vn}\times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,-v)\\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\} \quad \dots(2.7)$$

using (2.2) in right side of (2.4), we have

$$K_{x,r}^{\varepsilon-\psi,\alpha}\left(t^{-\vartheta-\psi}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)(x)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}(a)^n\,x^{-\vartheta-\psi-vn}\times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\} \quad \dots(2.8)$$

By observing (2.7) and (2.8), the result (2.6) is true.

Theorem 2.5 The Beta – transform of the operator defined in Theorem 2.1, gives the following result

$$B\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right);c,d\right\}=\sum_{n=0}^{\infty}\frac{(\gamma)_n a^n}{\Gamma(\lambda n+\beta)n!}\frac{\Gamma(c+\vartheta-1+vn)\Gamma(d)}{\Gamma(c+d+\vartheta-1+vn)}$$

$$\times I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\} \quad \dots(2.9)$$

Provided that the conditions mentioned with the operator and Beta-transform are satisfied.

Proof. Applying Beta – transform defined in (1.7), on the operator (2.1), we get

$$B\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right);c,d\right\}=\sum_{n=0}^{\infty}\frac{(\gamma)_n a^n}{\Gamma(\lambda n+\beta)n!}\times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

$$\times\int_0^1 t^{c+\vartheta+vn-1-1}\,(1-t)^{d-1}\,dt$$

Further by using the definition of Beta-function, we readily arrive at the desired result.

Theorem 2.6 The Beta – transform of the operator defined in Theorem 2.2, gives the following result

$$B\left\{K_{x,r}^{\mu,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right);c,d\right\}=\sum_{n=0}^{\infty}\frac{(\gamma)_n(a^n)}{\Gamma(\lambda n+\beta)n!}\frac{\Gamma(c-\vartheta+vn)\Gamma(d)}{\Gamma(c+d-\vartheta+vn)}\times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},-\tau\right)\right),(-\alpha,-v)\\\left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,-(\tau+v)\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

… (2.10)

Provided that the conditions mentioned with the operator and Beta-transform are satisfied.

Proof: Applying Beta – transform defined in (1.7), on the operator (2.2), we get

$$B\left\{K_{x,r}^{\mu,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right);c,d\right\}=\sum_{n=0}^{\infty}\frac{(\gamma)_n a^n}{\Gamma(\lambda n+\beta)n!}\times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},-\tau\right)\right),(-\alpha,v)\\\left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}\times$$

$$\int_0^1 t^{c-\vartheta-vn-1}\ (1-t)^{d-1}\,dt$$

Further by using the definition of Beta-function, we readily arrive at the desired result.

Theorem 2.7. The Mellin – transform of the operator defined in Theorem 2.2, gives the following result

$$M\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right)\right\}(s)$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{a^n}{n!}\frac{1}{(s+\vartheta+vn-1)}$$

$$\times I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\\left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

… (2.11)

Provided that the conditions mentioned with the operator and Mellin-transform are satisfied and $Re(s)>Re(v)$.

Proof: Applying Mellin – transform defined in (1.8), on the operator (2.1), we get

$$M\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right)\right\}(s)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{a^n}{n!}\times$$

$$I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\\left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

$$\times\int_0^{\infty}t^{s-1}t^{-\vartheta-vn}\,dt$$

Solving the integral and with simple simplifications, we readily arrive at the desired result.

Theorem 2.8. The Mellin – transform of the operator defined in Theorem 2.2, gives the following result

$$M\left\{K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)\right\}(s)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{a^n}{n!}\frac{1}{(s-\vartheta-vn)}$$

$$\times I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},-\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

… (2.12)

Provided that the conditions mentioned with the operator and Mellin-transform are satisfied and $Re(1-\vartheta)<1,\ Re(s)>Re(v)$.

Proof: Applying Mellin – transform defined in (1.8), on the operator (2.2), we get

$$M\left\{K^{\varepsilon,\alpha}_{x,r}\left(t^{-\vartheta}E^{\gamma}_{\lambda,\beta}(at^{-v})\right)\right\}(s)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{a^n}{n!}\times$$

$$I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},-\tau\right)\right),(-\alpha,-v)\\ \left(-\frac{(\mu+\vartheta+1+vn)}{r}-\alpha,-(\tau+v)\right),(b_j,\beta_j)_{1,m};(a_{ji},\alpha_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

$$\times\int_0^{\infty}t^{s-1}\,t^{-\vartheta-vn}dt$$

Solving the integral and with simple simplifications, we readily arrive at the desired result.

Theorem 2.9 The Laplace – transform of the operator defined in Theorem 2.1, gives the following result

$$L\left\{R^{\mu,\alpha}_{x,r}(t^{\vartheta-1})E^{\gamma}_{\lambda,\beta}(at^{v});s\right\}=s^{-\vartheta}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{(as^{-v})^n}{n!}\Gamma(\vartheta+vn)\times$$

$$I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

... (2.13)

Provided that the conditions mentioned with the operator and Laplace-transform are satisfied and $Re(\vartheta+vn)>0$.

Proof: Applying Laplace – transform defined in (1.9), on the operator (2.1), we get

$$L\left\{A^{\mu,\alpha}_{x,r}\left[t^{\vartheta-1}E^{\gamma}_{\lambda,\beta}(at^{v})\right];s\right\}$$

$$=L\left\{t^{\vartheta-1}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{(at^{v})^n}{n!}I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}\right\}$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{(a)^n}{n!}I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

$$\times\int_0^{\infty}e^{-st}t^{\vartheta+vn-1}\,dt$$

Using the definition of Laplace transform and with simple simplifications, we readily arrive at the desired result.

Theorem-2.10: The Laplace – transform of the operator defined in Theorem 2.2, gives the following result

$$L\left\{K^{\varepsilon,\alpha}_{x,r}\left(t^{-\vartheta}E^{\gamma}_{\lambda,\beta}(at^{-v})\right)\right\}(s)=s^{\vartheta-1}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{(as^{v})^n}{n!}\,\Gamma(1-\vartheta-vn)\times$$

$$I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},-\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}.$$

… (2.14)

Provided that the conditions mentioned with the operator and Laplace-transform are satisfied and $Re(1-\vartheta-vn)>0$.

Proof: Applying Laplace – transform defined in (1.9), on the operator (2.2), we get

$$L\left\{K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)\right\}(s)$$

$$=L\left\{t^{-\vartheta}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{(at^{-v})^n}{n!}I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}\right\}$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{a^n}{n!}I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}\times\int_0^{\infty}e^{-st}\,t^{-(\vartheta+vn)}\,dt$$

Using the definition of Laplace transform and with simple simplifications, we readily arrive at the desired result.

Theorem 2.11. The Whittakar – transform of the operator defined in Theorem 2.1, gives the following result

$$\int_0^{\infty}e^{-\varphi t/2}t^{\zeta-1}W_{\chi,\omega}(\varphi t)\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right)\right\}dt$$

$$=\varphi^{1-\vartheta-\zeta}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\,n!}(a\varphi^{-v})\;\frac{\Gamma(w+\vartheta+\zeta+vn-1/2)\Gamma(\vartheta-w+\zeta+vn-1/2)}{\Gamma(\vartheta-\chi+\zeta+vn)}$$

$$\times I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(a_{ji},\alpha_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

… (2.15)

Provided that the conditions mentioned with the operator and Whittakar-transform are satisfied and $Re[w\pm(\vartheta+\zeta+vn-1)]>\frac{1}{2}$.

Proof: Applying Whittakar – transform defined in (1.10), on the operator (2.1), we get

$$\int_0^{\infty}e^{-\varphi t/2}t^{\zeta-1}W_{\chi,\omega}(\varphi t)\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right)\right\}dt$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n\,a^n}{\Gamma(\lambda n+\beta)\,n!}I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

$$\times\int_0^{\infty}e^{-\varphi t/2}\,t^{(\vartheta+\zeta+vn-1)-1}W_{\chi,w}(\varphi t)dt$$

Let us take $\varphi t=z,$ then we have

$$\int_0^{\infty}e^{-\varphi t/2}t^{\zeta-1}W_{\chi,\omega}(\varphi t)\left\{A_{x,r}^{\mu,\alpha}\left(t^{\vartheta-1}E_{\lambda,\beta}^{\gamma}(at^{v})\right)\right\}dt$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n\,a^n}{\Gamma(\lambda n+\beta)\,n!}I_{p_i+2,q_i+1;l}^{m,n+2}\left\{k\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(a_{ji},\alpha_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

$$\times\varphi^{1-\vartheta-\zeta-vn}\int_0^{\infty}e^{-z/2}\,z^{(\vartheta+\zeta+vn-1)-1}W_{\chi,\omega}(z)dz$$

Now by using the result (1.10), we reached at the desired result.

Theorem 2.12. The Whittakar – transform of the operator defined in Theorem 2.2, gives the following result ,

$$\int_0^{\infty}e^{-\varphi t/2}\,t^{\zeta-1}W_{\chi,\omega}(\varphi t)\left\{K_{x,r}^{\varepsilon,\alpha}\left(t^{-\vartheta}E_{\lambda,\beta}^{\gamma}(at^{-v})\right)\right\}dt$$

$$= \varphi^{\vartheta-\zeta} \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\, n!} (a\varphi^{v}) \times \frac{\Gamma(\omega-\vartheta+\zeta-vn+1/2)\Gamma(-\vartheta-\omega+\zeta-vn+1/2)}{\Gamma(1-\vartheta-\chi+\zeta-vn)}$$

$$\times I^{m,n+2}_{p_i+2,q_i+1;l}\left\{ k \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i}, \left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right), (-\alpha, v) \\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha, \tau+v\right), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right\} \quad \ldots (2.16)$$

Provided that the conditions mentioned with the operator and Whittakar-transform are satisfied and $Re[\omega \pm (-\vartheta+\zeta-vn-1)] > 1/2$.

Proof: Applying Whittakar – transform defined in (1.10), on the operator (2.2), we get

$$\int_0^\infty e^{-\varphi t/2}\, t^{\zeta-1} W_{\chi,\omega}(\varphi t) \left\{ K^{\varepsilon,\alpha}_{x,r}\left(t^{-\vartheta} E^{\gamma}_{\lambda,\beta}(a t^{-v})\right)\right\} dt$$

$$= \sum_{n=0}^{\infty} \frac{(\gamma)_n\, a^n}{\Gamma(\lambda n+\beta)\, n!} I^{m,n+2}_{p_i+2,q_i+1;l}\left\{ k \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i}, \left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right), (-\alpha, v) \\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha, \tau+v\right), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right\}$$

$$\times \int_0^\infty e^{-\varphi t/2}\, t^{(-\vartheta+\zeta-vn)-1} W_{\chi,\omega}(\varphi t) dt$$

Let us take $\varphi t = z$, and proceeding on the similar lines that we have taken in the proof of Theorem 2.12, we get the desired result.

Theorem 2.13. The K – transform of the operator defined in Theorem 2.1, gives the following result

$$\int_0^\infty t^{\rho-1} K_\ell \left(wt \left\{ A^{\mu,\alpha}_{x,r}\left(t^{\vartheta-1} E^{\gamma}_{\lambda,\beta}(a t^{v})\right)\right\}\right) dt$$

$$= 2^{\rho+\vartheta-3} (w)^{(1-\rho-\vartheta)} \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\, n!} \left(a \left(\frac{2}{w}\right)^{v}\right) . \Gamma\left(\frac{(\rho+\vartheta+vn-1)\pm \ell}{2}\right)$$

$$\times I^{m,n+2}_{p_i+2,q_i+1;l}\left\{ k \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i}, \left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right), (-\alpha, v) \\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha, \tau+v\right), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right\} \quad \ldots (2.17)$$

Provided that the conditions mentioned with the operator and K-transform are satisfied and $Re(w) > 0, Re\big((\rho+\vartheta+vn-1) \pm \ell\big) > 0$.

Proof. Applying K – transform defined in (1.12), on the operator (2.1), we get

$$\int_0^\infty t^{\rho-1} K_\ell \left(wt \left\{ A^{\mu,\alpha}_{x,r}\left(t^{\vartheta-1} E^{\gamma}_{\lambda,\beta}(a t^{v})\right)\right\}\right) dt$$

$$= \sum_{n=0}^{\infty} \frac{(\gamma)_n\, a^n}{\Gamma(\lambda n+\beta)\, n!} I^{m,n+2}_{p_i+2,q_i+1;l}\left\{ k \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i}, \left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right), (-\alpha, v) \\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha, \tau+v\right), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right\}$$

$$\times \int_0^\infty (t)^{(\rho+\vartheta+vn-1)-1} K_\ell(\omega t) dt,$$

In view of (1.14), we readily arrive at the desired result.

Theorem 2.14. The K – transform of the operator defined in Theorem 2.2, gives the following result

$$\int_0^\infty t^{\rho-1} K_\ell \left(wt \left\{ K^{\varepsilon,\alpha}_{x,r}\left(t^{\vartheta-1} E^{\gamma}_{\lambda,\beta}(a t^{-v})\right)\right\}\right) dt$$

$$= 2^{\rho-\vartheta-2}(\omega)^{(\vartheta-\rho)} \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\lambda n+\beta)\, n!}\left(a\left(\frac{w}{2}\right)^{v}\right).\Gamma\left(\frac{(\rho-\vartheta-vn)\pm\ell}{2}\right)$$

$$\times I^{m,n+2}_{p_i+2,q_i+1;l}\left\{k \left| \begin{matrix} (a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v) \\ \left(-\frac{(\varepsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right.\right\}$$

… (2.18)

Provided that the conditions mentioned with the operator and K-transform are satisfied and $Re(w) > 0, Re\big((\rho-\vartheta-vn)\pm\ell\big) > 0$.

Proof: By taking K-transform of the operator (2.2) and proceeding like the proof of Theorem 2.13, we get the desired result.

3. Special cases

If we take $l = 1$ in I-function, then it reduces to well known H-function [9,17] and the results (2.1) and (2.2) gives following corollaries-

Corollary 1

$$A^{\mu,\alpha}_{x,r}\left(t^{\vartheta-1}E^{\gamma}_{\lambda,\beta}(at^{v})\right)(x) = x^{\vartheta-1}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)}\frac{(ax^{v})^n}{n!}H^{m,n+2}_{p+2,q+1}\left\{k\left|\begin{matrix}(a_p,\alpha_p),\left(1-\left(\frac{(\mu+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\mu+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_q,\beta_q)\end{matrix}\right.\right\}$$

… (3.1)

Provided the conditions, mentioned with (2.1) are satisfied.

Corollary 2

$$K^{\varepsilon,\alpha}_{x,r}\left(t^{-\vartheta}E^{\gamma}_{\lambda,\beta}(at^{-v})\right)(x) = x^{-\vartheta}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\lambda n+\beta)n!}\,(ax^{-v})^n H^{m,n+2}_{p+2,q+1}\left\{k\left|\begin{matrix}(a_p,\alpha_p),\left(1-\left(\frac{(\varepsilon+\vartheta+vn)}{r},\tau\right)\right),(-\alpha,v)\\ \left(-\frac{(\epsilon+\vartheta+vn)}{r}-\alpha,\tau+v\right),(b_q,\beta_q)\end{matrix}\right.\right\}$$

… (3.2)

Provided the conditions, mentioned with (2.2) are satisfied.

Conflict of Interest – None

Declaration of Interest – None

Conclusion

In the present paper, we have studied the properties of Mittag-Leffler function under the extension of generalized fractional integral operators given by Saxena and Kumbhat [15] and developed some new images. The results obtained here involves special functions like Mittag Leffler function and I -function, due to their general nature and usefulness in the theory of integral operators and relevant part of computational mathematics they may have an important place in the literature. Also, the special functions involved here can be reduced in simpler functions, those have variety of applications in different domains of science and technology and can be observed as special cases, those we have not mentioned here explicitly.

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