

# The study of Inclined Magnetic Field and Thermal Radiation on MHD Oscillatory Convective Flow of Heat Absorbing Visco - Elastic Dusty Fluid Confined In Horizontal Channel

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## Abstract:

The focus of the present study is made on the geometry of free convective unsteady visco elastic conducting dusty fluid flow with the effect of inclined magnetic field and radiation confined by horizontal channel. The influence of oscillatory pressure gradient and the movement of the top plate are incorporated. The heat generated is established in both motions of fluid and dust particles and is high enough to radiate heat. The solutions of velocity distribution, temperature distribution of the fluid, velocity and energy of the dust particle along with skin friction are achieved by employing perturbation scheme. The physical parameters involved in both profiles for fluid and dust particles are examined with aid of graph.

**Keywords:** Magnetohydrodynamic flow, Parallel channel, thermal radiation, inclined magnetic field, visco elastic dusty conducting fluid.

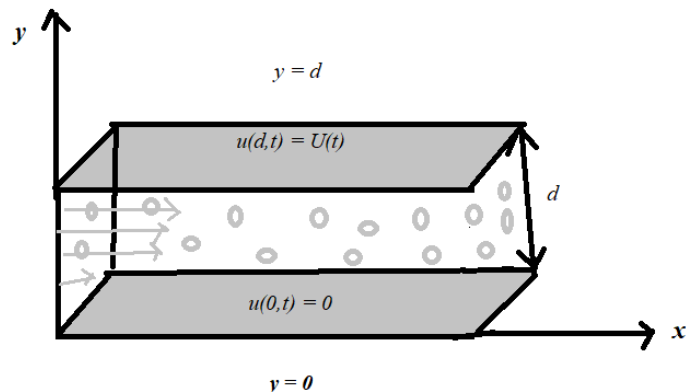
## Introduction

Ahmed, et al, studied about the thermal diffusion effect on a three-dimensional MHD free convection with mass transfer flow from a porous vertical plate. Venkatesh and Kumara, discussed about the exact solution of an unsteady conducting dusty fluid flow between non-tortional oscillating plate and a long wavy wall. Closed form solutions for unsteady free convection flow of a second grade fluid over an oscillating vertical plate is explained by Ali, et al. Dey learnt about dusty hydromagnetic oldryod fluid flow in a horizontal channel with volume fraction and energy dissipation. Sheikh, et al, investigated by comparison and analysis of the Atangana – Baleanu and Caputo – Fabrizio fractional derivatives for generalized Casson fluid model with heat generation and chemical reaction. Dielectrophoretic choking phenomenon of a deformable particle in a converging-diverging microchannel is analyzed by Zhou, T. et al. Dey, explained viscoelastic fluid flow through an annulus with relaxation, retardation effects and external heat source/sink. Bilal, et al, discussed about two-phase fluctuating flow of dusty viscoelastic fluid between non conducting rigid plates with heat transfer. Zhou, et al. learnt AC dielectrophoretic deformable particle-particle interactions and their relative motions. Khan, et al. discussed about effects of relative magnetic field, chemical reaction, heat generation and Newtonian heating on convection flow of casson fluid over a moving vertical plate embedded in a porous medium. Hemamalini discussed about the impact of radiation on MHD oscillatory convective flow of heat absorbing Visco - Elastic dusty fluid confined in horizontal channel. Rajakumar, et al, explained about the influence of dufour and thermal radiation on unsteady mhd walter's liquid model-b flow past an impulsively started infinite vertical plate embedded in a porous medium with chemical reaction, hall and ion slip current.

The flow of dusty fluid has made many investigators or researchers to study on its applications in the field of petroleum industry, polymer technology, high efficient solid rocket propellant, fluid droplet sprays, blood flow, purification process of crude oil and in many fields. In the view of en number of physical problems the research of porous media in magnetic field and radiation plays central role. Extensive research has been carried out and it has paid the way to learn the magnetohydrodynamic flow in channel combined with the inclined magnetic field and radiation .

## Physical Description of the problem

The unsteady oscillating free convective flow of heat absorbing viscoelastic dusty fluid in horizontal plates has been considered inclusive of transverse magnetic field and radiation effect. The movement of the top plate with free stream velocity  $U(t)$  induced by the oscillating pressure gradient.



**Figure 1 Physical Configuration**

The flow is considered along the X- axis. The plate at  $y = d$  fluctuating with freestream velocity  $U(t)$  while the plate at  $y = 0$  is at rest. The fluid and dust particles velocities are given by  $u$  and  $v$ . The temperature between the two plates are high enough to radiate heat. The lower plate is marked with ambient temperature  $T_\infty$  and other plate temperature is sustained with  $T_w$  whereas  $T_p$  denotes the temperature of the particle

$$\frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \left( \nu + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial y^2} - \frac{K_0 N_0}{\rho} (u - v) - \frac{\sigma B_0^2 \sin^2 u}{\rho} + g \beta_T (T - T_\infty) \quad (1)$$

$$m \frac{\partial v}{\partial t} = (u - v) K_0 \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\rho_p C_s}{\rho C_p \gamma_T} (T_p - T) - \frac{Q_0 T}{\rho C_p} - \frac{1}{\rho} \frac{\partial q_r}{\partial y} \quad (3)$$

$$\frac{\partial T_p}{\partial t} = \frac{1}{\gamma_T} (T - T_p) \quad (4)$$

The relevant boundary conditions are

$$\begin{aligned} \text{For } t \leq 0, T(y, 0) = T_\infty, u(y, 0) = 0 \\ t > 0, u(0, t) = 0, T(0, t) = T_w \quad \text{at } y = 0 \\ t > 0, u(d, t) = U(t), T(d, t) = T_\infty \quad \text{at } y = d \end{aligned} \quad (5)$$

Consider  $v(y, t) = \frac{v_0(y)}{e^{-i\omega t}}$ , in order to calculate the velocity of the dust particle and by substituting the value of  $v$  in equation (2),

$$v(y, t) = \left( \frac{K_0}{mi\omega + K_0} \right) u(y, t) \quad (6)$$

Incorporating equation (6) in (1) and at free stream area equation (1) will take the form

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \left( \nu + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial y^2} - \frac{K_0 N_0 (U - u)}{\rho} \left( \frac{K_0}{mi\omega + K_0} - 1 \right) - \frac{\sigma B_0^2}{\rho} (u - U) + g \beta_T (T - T_\infty) \quad (7)$$

For non-dimensionalizing the above equations, the following are considered

$$\begin{aligned}
u^* &= \frac{U}{U_0}, y^* = (yd^{-1}), t^* = \frac{U_0}{d}t, \theta = -\frac{[T_\infty - T]}{T_w - T_\infty}, \theta_p = \frac{-T_p + T_\infty}{-T_w + T_\infty} \tau^* = \frac{1}{\mu\nu} \tau d^2 \\
\text{Re} &= \frac{u_0 d}{\nu}, K_1 = \frac{K_0 N_0 d^2}{\rho\nu}, \alpha = \frac{\alpha_1 u_0}{\rho\nu d}, K_2 = \frac{K_0^2 N_0^2}{\rho\nu(mi\omega + K_0)}, M = \frac{\sigma B_0^2 d^2}{\rho\nu}, \\
Gr &= \frac{g\beta_T(T_w - T_\infty)}{\nu u_0}, \phi = \frac{dQ_0}{\rho C_p}
\end{aligned} \tag{8}$$

The equation of momentum, energy of the fluid and dust particles in dimensionless form is given as:

$$R_e \frac{\partial}{\partial t} u = R_e \frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial}{\partial t} \left( \frac{\partial^2 u}{\partial y^2} \right) + (K_2 - K_1)(U - u) - M(u - U) + G_r \theta \tag{9}$$

$$\frac{\partial \theta}{\partial t} = (Pe)^{-1} \frac{\partial^2 \theta}{\partial y^2} + R \frac{1}{Pe} (\theta_p - \theta) - (\phi + N)\theta \tag{10}$$

$$\frac{\partial \theta_p}{\partial t} = \gamma(\theta - \theta_p) \tag{11}$$

Let assume that  $\theta_{(p)} = \left[ \frac{\gamma}{i\omega + \gamma} \right] \theta$ , then energy equation becomes

$$\frac{\partial \theta}{\partial t} = (pe)^{-1} \frac{\partial^2 \theta}{\partial y^2} + R \frac{1}{Pe} \left[ \frac{1}{i\omega + \gamma} \gamma - \theta \right] - (\phi + N)\theta \tag{12}$$

The corresponding boundary conditions are

$$\begin{aligned}
u(1, t) &= U(t), \theta(0, t) = 1, u(0, t) = 0, \theta(1, t) = 0, \\
U(t) &= 1 + \frac{\epsilon}{2} (e^{i\omega t} + e^{-i\omega t})
\end{aligned} \tag{13}$$

Considering the periodic solution in order to solve the equations

$$\begin{aligned}
u(y, t) &= u_0(y) + \epsilon \frac{1}{2} (u_1(y) + u_2(y)) e^{i\omega t} \\
\theta(y, t) &= \theta_0(y) + \epsilon \theta_1(y) e^{i\omega t}
\end{aligned} \tag{14}$$

Adopting the assumptions and boundary conditions in equation and momentum, the harmonic and non harmonic parts are given by

$$\begin{aligned}
\theta_1(y) &= 0 \\
\theta_0(y) &= \frac{\sinh[\sqrt{m_1} - \sqrt{m_1}y]}{\sinh(\sqrt{m_0})}
\end{aligned} \tag{15}$$

$$\frac{\partial u}{\partial t} \text{Re} = \frac{dU}{dt} \text{Re} + \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^3 u}{\partial t \partial y^2} - (K_1 - K_2) - M(u - U) + Gr \left[ \frac{\sinh[\sqrt{m_1} - \sqrt{m_1}y]}{\sinh(\sqrt{m_0})} \right] \tag{16}$$

Using equation (13),(14) in (16)

$$u_1(y) = \frac{\sinh(\sqrt{m_2}y - \sqrt{m_2})}{\sinh(\sqrt{m_2})} + 1 \tag{17}$$

$$u_2(y) = \frac{\sinh(\sqrt{m_3}y - \sqrt{m_3})}{\sinh(\sqrt{m_3})} + 1 \tag{18}$$

$$u_0(y) = -B \frac{\sinh \sqrt{D} - \sqrt{D}y}{\sinh(\sqrt{D})} + 1 + (B-1) \frac{\sinh \sqrt{m_1} - \sqrt{m_1}y}{\sinh(\sqrt{m_1})} \quad (19)$$

$$u(y,t) = -B \frac{\sinh \sqrt{D} - \sqrt{D}y}{\sinh(\sqrt{D})} + 1 + (B-1) \frac{\sinh \sqrt{m_1} - \sqrt{m_1}y}{\sinh(\sqrt{m_1})} + \frac{\epsilon}{2} \left[ \left( \frac{\sinh \sqrt{m_2}y - \sqrt{m_2}}{\sinh(\sqrt{m_2})} + 1 \right) e^{i\omega t} + \left( \frac{\sinh \sqrt{m_3}y - \sqrt{m_3}}{\sinh(\sqrt{m_3})} + 1 \right) \right] \quad (20)$$

where

$$m_1 = \frac{N(\gamma + i\omega) + R_1(\gamma + i\omega) + Pe\gamma\phi - (\gamma + i\omega)\phi}{\gamma + i\omega}, \quad m_1 = \frac{Re\ i\omega - D}{1 + \alpha}, \quad m_2 = \frac{Re\ i\omega + D}{1 - \alpha}$$

$$D = -(K_2 - K_1) - M, \quad B = 1 + Gr \left( \frac{1}{D - m_1} \right)$$

### Nusselt number

The rate of heat transfer is

$$Nu = \left. \frac{\partial \theta}{\partial y} \right|_{y=0}$$

$$= \sqrt{m_0} \frac{\cosh \left[ \frac{\sqrt{m_0}}{2} \right]}{\sinh \left[ \frac{\sqrt{m_0}}{2} \right]}$$

### Skin Friction

The dimensionless form of skin friction for viscoelastic dusty fluid is given by

$$\tau = Re \frac{\partial u}{\partial y} + \alpha \frac{\partial^2 u}{\partial t \partial y}$$

### Findings:

The predominant physical parameters involved in the flow are discussed using graph.

Fig 2,5,6 and 7 illustrates about the behaviour of velocity graph of fluid for different values of  $\phi$ ,  $M$ ,  $Pe$  and  $R$ . For the increase in these parameters, the fluid velocity profile degrades. whereas fig 3 and 4 depicts about the progress of velocity graph of fluid for incrementing values of  $Gr$  and  $K$ .

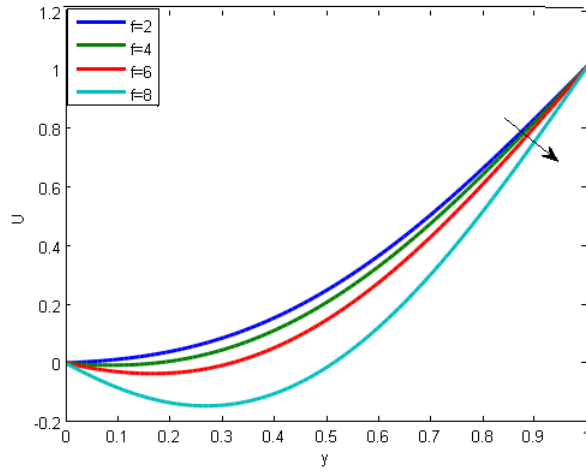


Fig 2: Velocity profile  $U$  for  $\phi$ , [ $Gr=2, M=2, Pe=2, Re=1, K=1$ ]

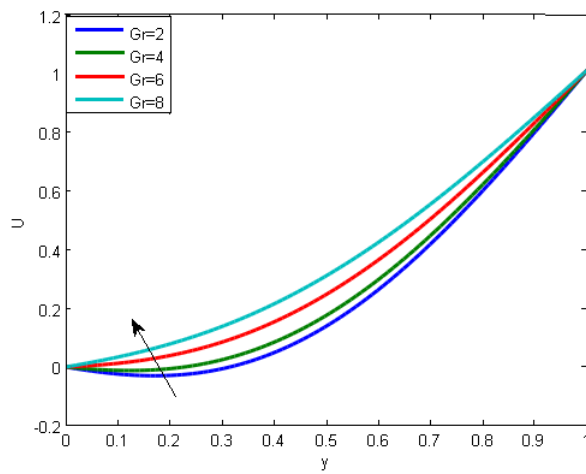


Fig 3: Velocity profile  $U$  for  $Gr$ , [ $\phi=1, M=2, Pe=2, Re=1, K=1$ ]

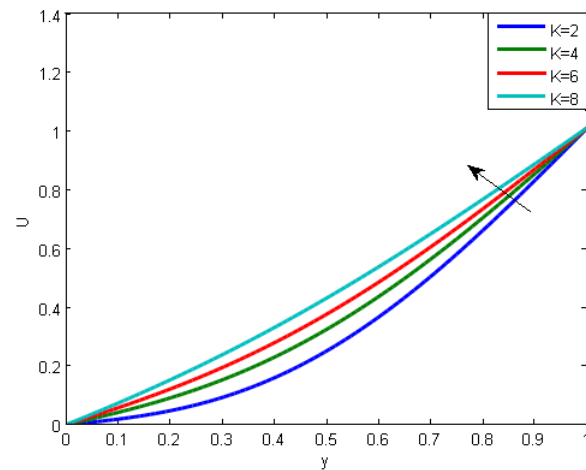


Fig 4: Velocity profile  $U$  for  $K$ , [ $\phi=1, M=2, Pe=2, Re=1, Gr=2$ ]

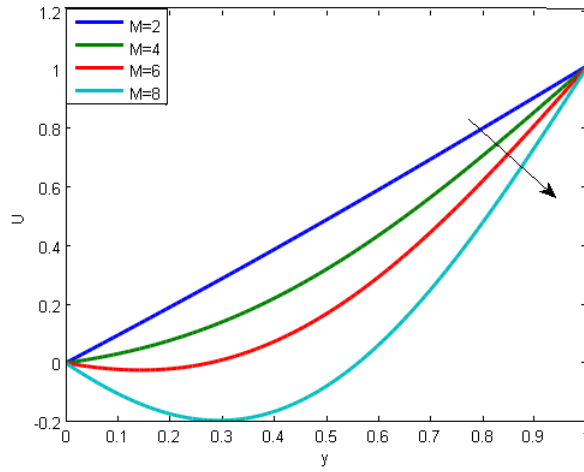


Fig 5: Velocity profile U for M, [ $\phi = 1, Gr = 2, Pe=2, Re=1, K=1$ ]

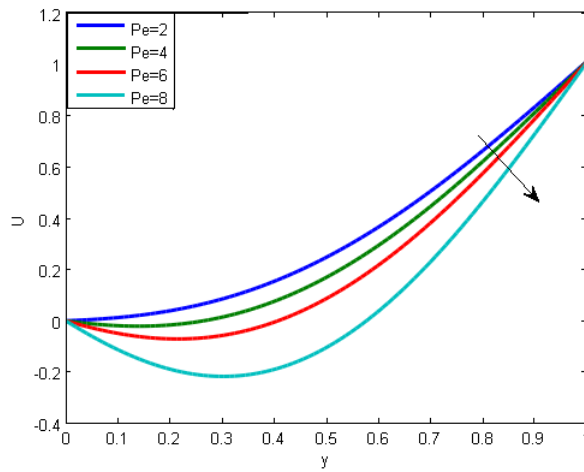


Fig 6: Velocity profile U for Pe, [ $\phi = 1, M=2, Gr=2, Re=1, K=1$ ]

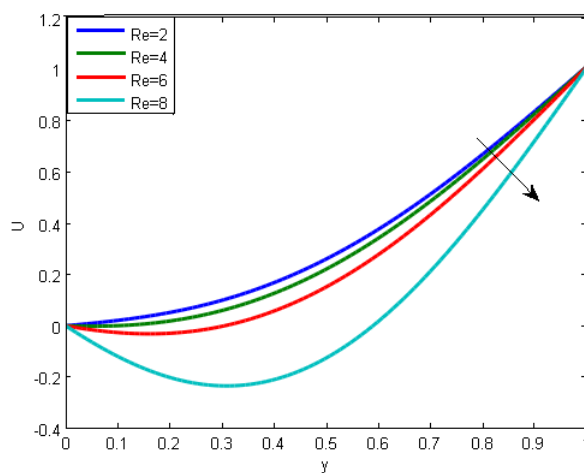


Fig 7: Velocity profile U for Re, [ $\phi = 1, Gr = 2, M=2, Pe=1, K=1$ ]

Figure 8 - 13 represents the influence of velocity of the dusty particle for various values of  $R, Pe, M, \phi, Gr$  and  $m$ . The velocity profile degrades with increasing these physical factors except for the parameter  $m$ . On incrementing the values of  $m$ , it leads to the causes the upgrade in the velocity of dust particles.

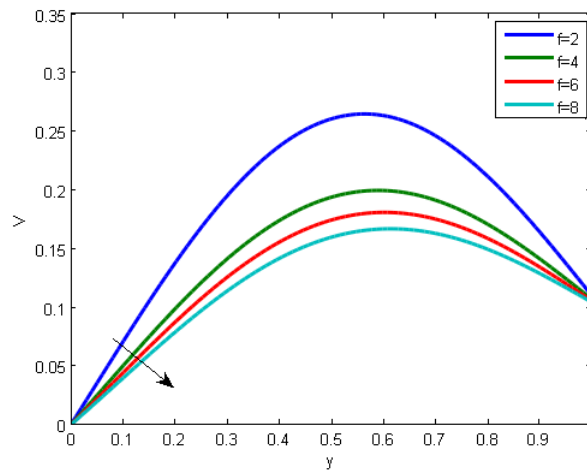


Fig 8: Particle velocity profile  $V$  for  $\phi$ , [ $M=2, Gr = 2, Pe=2, Re=1, K=1, m=1$ ]

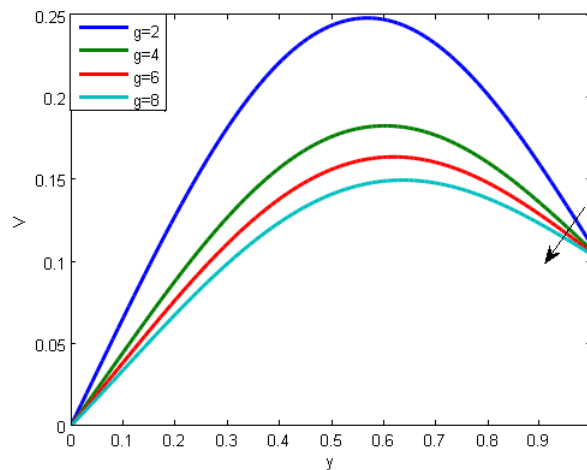


Fig 9: Graph of particle velocity  $V$  for  $Gr$ , [ $\phi = 1, M=2, Pe=2, Re=1, K=1, m=1$ ]

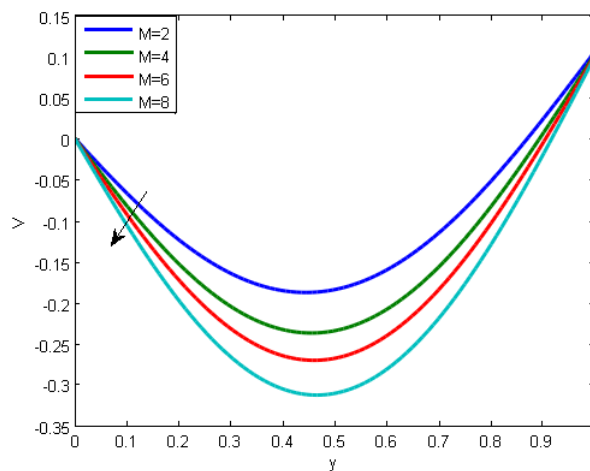


Fig 10: Particle velocity profile  $V$  for  $M$ , [ $\phi = 1, Gr = 2, Pe=2, Re=1, K=1, m=1$ ]

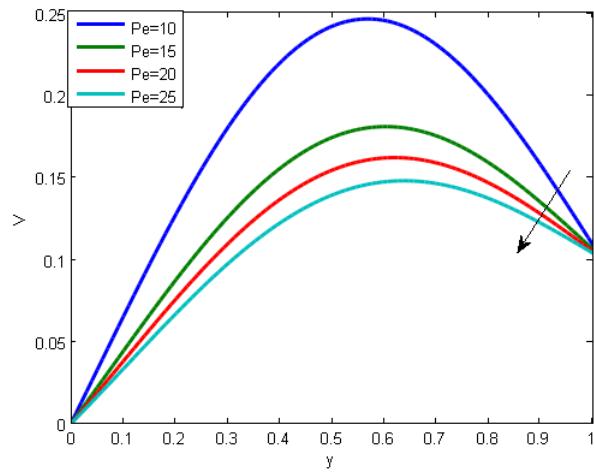


Fig 11: Sketch of particle velocity  $V$  for  $Pe$ , [ $\phi = 1, Gr = 2, M = 2, Re = 1, K = 1, m = 1$ ]

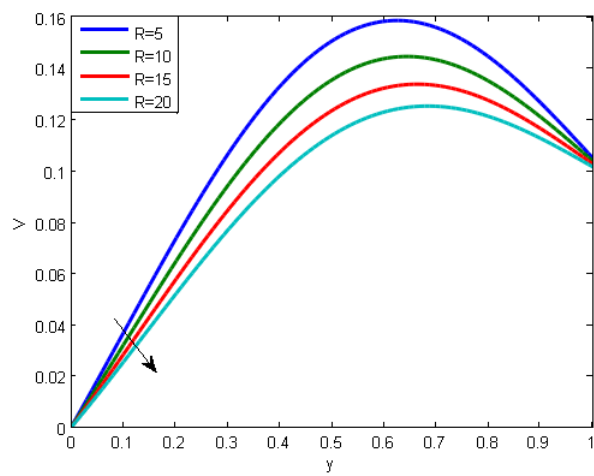


Fig 12: Profile of particle velocity  $V$  for  $R$ , [ $\phi = 1, Gr = 2, Pe = 2, M = 2, K = 1, m = 1$ ]

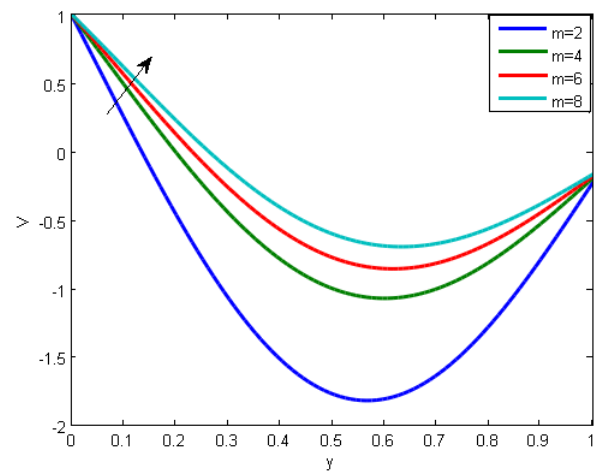


Fig 13: Particle velocity profile  $V$  for  $m$ , [ $\phi = 1, Gr = 2, Pe = 2, M = 2, K = 1, m = 1$ ]



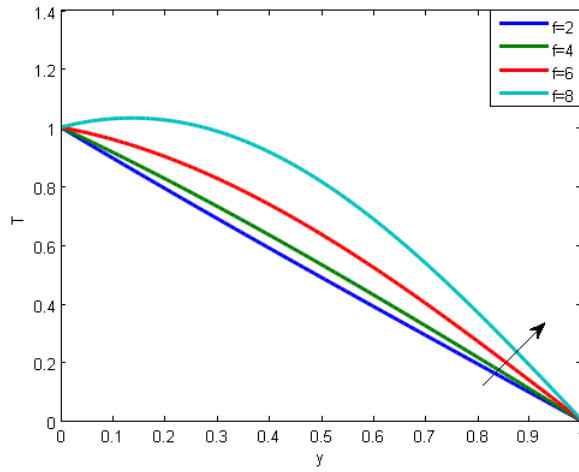


Fig 14: Temperature profile T for  $\phi$ , [R=1,Gr = 2, Pe=2,M=2,K=1,m=1]

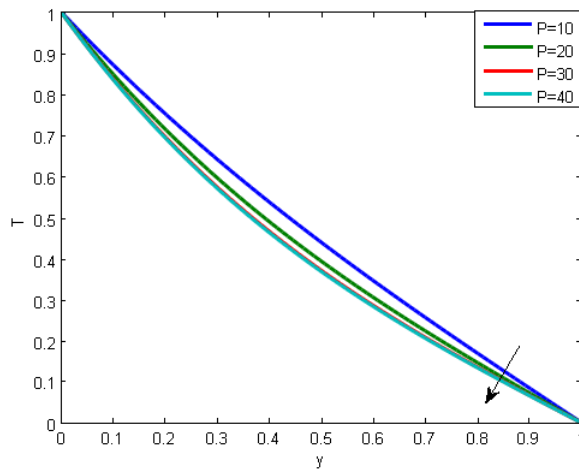


Fig 15: Temperature profile T for Pe, [R=1,Gr = 2,  $\phi$  =1, M=2,K=1,m=1]

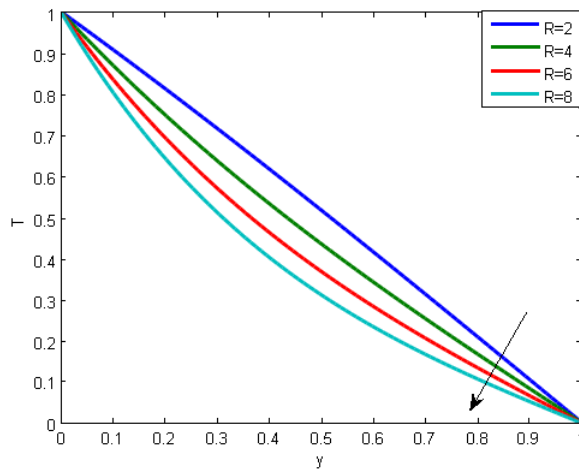


Fig 16: Temperature profile T for R, [Pe=2,Gr = 2,  $\phi$  =1, M=2,K=1,m=1]

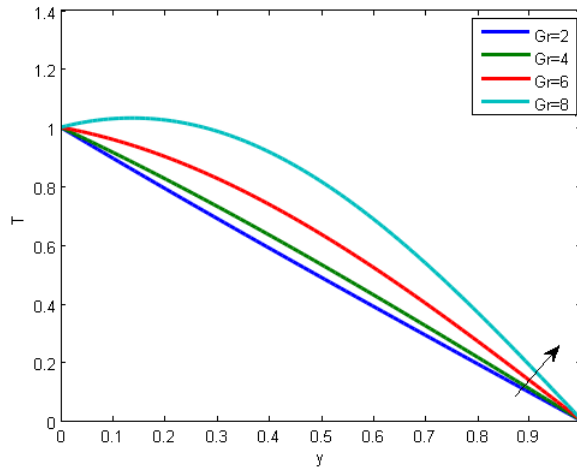


Fig 17: Temperature profile T for Gr, [Pe=2,R = 1,  $\phi = 1$ , M=2,K=1,m=1]

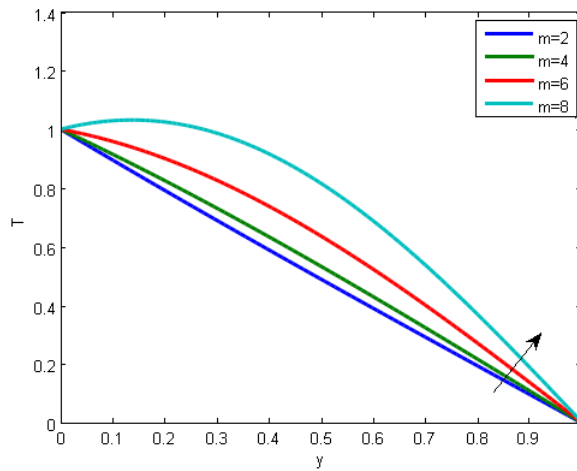


Fig 18: Temperature profile T for m, [Pe=2,R = 1,  $\phi = 1$ , M=2,K=1,Gr=2]

The transformation of temperature profile with different values of Pe,Re Gr and m have been pictured in the figures:15-18 respectively. With the upsurging values of Re,m and Pe temperature profile increases. In the meanwhile with the pileup values of Gr temperature profile decreases. The relation between magnetic parameter, Peclet number and Reynolds number shows retardation in the fluid flow velocity, so this retardation is due to the result of progress in internal resistive forces.

### Conclusion

The present work illustrates about the unsteady magnetohydrodynamic fluctuating flow of viscoelastic fluid bounded by horizontal plates. It is considered that the free stream oscillatory, electrically conducting incompressible with heat radiation has been considered. It is interesting to conclude that on upgrading Gr,  $\Phi$  and K parameters, the velocity profile increases. Considering the velocity profile for dusty particles, the profile degrades by increasing R, Pe, M,  $\Phi$  and  $\Upsilon$  parameters. Whereas the parameters Gr, m involved in the flow shows an increasing temperature profile.

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