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DEVELOPMENT AND OPTIMIZATION OF EXPERIMENTAL DATA BASED MODELS FOR TANDUMDRIVE HUMAN POWERED FLYWHEEL MOTOR(HPFM)

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Abstract
This paper presents the development of the experimental data based mathematical models for tandum drive human powered flywheel motor. There were various dependent and independent variables involved in tandum drive human powered flywheel motor. Therefore, apart from formulation and development of the model, the optimization was done to find the best sets of the independent variable to achieve the responses as an output. The tandum drive HPFM was designed, fabricated and based on the theory of experimentation, the experiment was performed and total 40 sets of readings were recorded. In this work, the responses of single response variables flywheel angular velocity were experimentally studied and the experimental data based models for this response variables are optimized to getthe best set of independent variables human energy, Effectiveness of mechanism and mass moment of inertia of fly wheel involved in the tandum drive HPFM.
Keywords HPFM, Optimization, Experimentation, Pi Terms, Dependant Variables, Independent Variables, Human Power, Pedal Power, Flywheel Motor, Mathematical Modelling, Buckingham's π theorem,.

1. Introduction

The Tandum human powered flywheel motor (HPFM)is bicycle driven energy unit comprising of speed raising gear pair and a flywheel whereasmechanicalpower transmission system consists of a clutch and torque amplification gear pair. The schematic arrangement of the machine is shown in figure 1 whereas figure 2 shows the actually designed and fabricated experimental setup tandum drive HPFM. In order to meet the research objectives clearly and quickly with the relevant sort of data and adequate sample size, proper experiment planning is essential. The performance of tandum drive HPFM is influenced by a number of factors. The goal of this study is to present the adopted experimental design in detail and to generate design data in the form of evolving experimental data based models for various dependent/ responsive variables of the Human Powered Flywheel Motor.

1.1. Experimental Setup

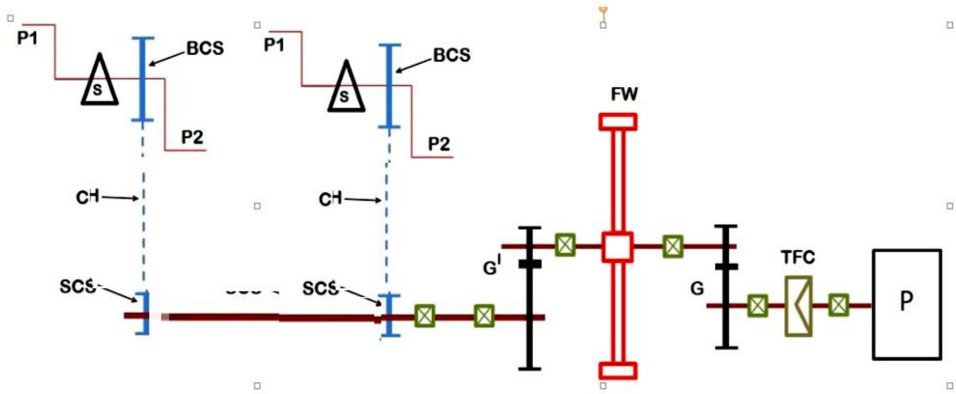

Figure 1: HPFM Tandum Drive

S = Seat,P1, P2 = Pedals, BCS = Big Chain Sprocket, SCS = Small Chain Sprocket,CH = Bicycle Chain Drive, GI = Speed increasing gear pair, FW = Flywheel, TFC/SJC = Torsionally Flexible Clutch / Spiral Jaw Clutch,P = Process Unit, G = Torque Amplification Gear pair
This novel machine system comprises of three subsystem viz
(1) Energy unit

(2) Transmission
(3) Process machine.
The Schematics of the system is shown in Fig 1.

Energy unit: This comprises a bicycle like pedaling system, speed increasing gear pair G' and flywheel FW. Transmission comprises TFC (the torsionally flexible Clutch) and the torque amplification gear pair G and (3) the process unit.

Working:

Step 1: A young boy (20-25 years), slim stature, middle height peddles the bicycle like system for 1 minute and speeds up the flywheel FW to the speed of 500-1000 rpm. The FW is 0.8m Rim diameter X 10cm rim width X 2cm rim thickness, The energy stored in the flywheel is 3500-4000kgf-m.

Step 2: Upon FW reaching the rated speed TFC is engaged and the energy stored in the FW is made available to the process unit P. The energy stored in the FW is exhausted in 5 to 15 seconds depending on the process unit.

Operation Principle of Setup

Figure 2: Experimental Setup-HPFM Tandum Drive

In the entire work done so far only one peddler is tried. As the system is introducing a new employment guarantee scheme with energy application or new process unit, it is worthwhile trying more than one peddler either two (i.e. tandum drive)schematically as shown in Fig.1 or three or even four. This innovation of the Energy unit may store more than 3000-4000 kgf-m energy in the flywheel[1,2,3,4,6,12,16 to 25,26 to 40]. This enhancement of stored energy in the flywheel during the same peddling time i.e.60 seconds or 1 minute may prove to be worth for energizing other process units manufacturing other rural/village/interior based products/processes which are otherwise being energized by other conventional prime movers either electric Motor /Engine/Air motors etc. in the h.p.range 8 to 24 hp.

- To enhance the power range of so far developed HPFM energized process Machines [1-40]*
- To establish an expression for ωT.
- To establish an expression for maximum stored energy in the flywheel.
- To establish an expression for maximum efficiency of the system

2. Methodology of Theory of Experimentation

The method of Design of Experiment (DoE) is applied in this work. Design of Experiment (DoE) has its advantages such as it can increase the result achieved, reduce variability, and make the result closer to the target, shorten the amount of time and reduce the cost.. The approach of Design of Experimentation [10] is applied for formulating generalized experimental data based model which includes identification of variables, reduction of independent variables by dimensional analysis, test planning (comprising of determination of test envelope, test points, test sequence and experimental plan), physical design of experimental set up, execution of experimentation, purification of experimental data, model formulation, reliability of models, model optimization. The various dependent and independent variables involved tandum drive HPFM are shown in table 1

Mathematical Model

2.1 Dimensional Analysis

Dimensional analysis is a useful mathematics tool for reducing variables by creating non-dimensional sets of variables known as pi (π) terms. The number of independent terms in an experiment is reduced when the dimensional equation for a phenomenon is deduced. The intended model is the exact mathematical form of this dimensional problem. As a result, this dimensional analysis method allows for more systematic experimental planning and the presenting of results in a more informative and simple manner.

2.2 Variables Identification

The term "variables" is used in a broad sense to refer to any physical quantity that change. It is an independent variable if physical quantities may be modified independently of other physical quantities. The term "dependent" or "responsive variable" refers to a physical quantity those changes in reaction to the modification of one or more independent factors. An extraneous variable is a physical quantity that impacts our test and changes in a random and uncontrolled manner. The different dependent or response variables, independent variables, and extraneous variables affecting the phenomena are determined based on the functioning condition of the tandum drive HPFM. In addition, data purification is carried out to reduce the effect of extraneous variables on the phenomena in order to avoid their unfavourable impact.

Table 1 shows Tandum Drive HPFM Various Dependent and Independent Variables

Variables	Unit	MLT
ω Angular velocity of the flywheel in rad/sec reached after time interval T seconds	rad /s	T^{-1}
G = Gear Ratio	---	$M^0L^0T^0$
I Mass moment of inertia of the flywheel	Kg-m2	ML^2
R Input energy by the rider,	Kgf-m	ML^2T^{-2}
T peddling time, in seconds	Seconds	T
EM effectiveness of the mechanism, M	--	$M^0L^0T^0$
Acceleration due to gravity ,g	m/s2	LT^{-2}

2.3 Buckingham's Π - Theorem is used to reduce Pi terms.

The pi (Π) terms for all dependent/response and independent factors impacting the phenomenon of tandem drive human powered flywheel motor are formed using Buckingham's Π- Theorem approach.

The dimensional equation would be obtained as under.

$\omega = f\,[I, R, T, g, G, EM]$ Eq.(1)

R=R1+R2,

R1: Energy Input by One Rider.

R2: Energy Input by Other Rider.

Hence Eq.1 can be rewrite as

$$\omega T = f\left[\left(\frac{I}{R^2T}\right),(EM),(G)\right] \quad \text{Eq.(2)}$$

In general it can be stated as under

$$\pi(0) = f\left[(\pi 1),(\pi 2),(\pi 3)\right] \quad \text{Eq.(3)}$$

In equation (3),

$$\pi 1 = \left[\frac{I}{R^2T}\right], \pi 2 = [EM], \pi 3 = [G] \text{ and } \pi 0 = [\omega T]$$

where ω Angular velocity of the flywheel in rad/sec reached after time interval T seconds

The exponential form of dimensional from eq 1.

$$\omega T = k\left[\left(\frac{I}{R^2T}\right)^a,(EM)^b,(G)^c\right] \quad \text{Eq.(4)}$$

Where K stands for a curve-fitting constant, and a, b, c stand for constant exponent.. The effects for these factors are found by means of multiple regression analysis and a suitable computer platform

3. Experimentation

The classical plan of experimentation is used to carry out the experimentation. In the classical plan of experimentation all the independent pi terms except one are maintained constant at their planned fixed level values and the said independent pi terms under consideration is to be varied over its widest possible range as decided by test envelope. The experimentation is carried out to cover the entire test envelope and all the test points within the test envelope. During experimentation, the gear ratio varies from1.14, 1.5, 2.0,4.0 and Effective ness of mechanism varies from 1 to 1.13. Thus the different types of gear ratio and effectiveness of mechanism are used during experimentation for monitoring the actualfeasibility of the machine. During experimentation speed of flywheel is measured using Non-Contact of tachometer.

The total 36 sets of the readings were taken.Table 3 and Table 4 shows the experimental plan and responses recorded.Table 2 shows the Human energy calculation for each peddler. Table 4 shows the experimental observation for tandum drive human powered flywheel motor. Table 5 shows the values of independent and dependent variablevalues in term of pi term.

S.N.	Name of peddler	AGE in Yrs	Wt.in Kg(w1)	Pulse P1 before climbing	Pulse P2 after climbing	Rise in pulse P2-P1=P	Time t1(sec)g	h1= Height Climbed(m)	Work Done for time t1 and pulse rise rate P WD=W1Xh1
1	Bhavesh Dhole	21	50	81	115	35	15	4	200
2	Roshan Hatwar	21	61	77	120	43	13	4	244
3	Ritik Lende	21	87	88	110	22	13.4	3.6	313.2
4	Ravikant Chopkar	21	65	80	120	40	16	4	260
5	Lokesh	21	64	83	120	37	14	4	256
6	Jagdish	21	50	78	106	28	18	4.26	213

Table 2 : Human energy calculation

Table 3 : Experimental results of tandum drive HPFM

S.N.	Peddler 1						Peddler 2						EM		Gear ratio	I = flywheel's mas moment of inertia in Kg-m2	Speed of Flywheel N
	Name	Pulse P1 before Pedalling	Pulse P2 after Pedalling	Rise in pulse P2-P1=P_{OP}	Time t_{op} Sec)	R = Input energy by the rider, Kgf-m =WDX (t_1/t_{op}) X(P_{op}/P)	Name	Pulse P1 before Pedalling	Pulse P2 after Pedalling	Rise in pulse P2-P1=P_{OP}	Time t_{op} Sec)	R = Input energy by the rider, Kgf-m =WDX(t_1/t_{op})X(P_{op}/P)	Peddler 1	Peddler2			
1	Jagdish	72	167	95	300	33.725	Bhavesh	73	161	88	300	25.14286	1	1	1.14	10.41238037	399
2	Lokesh	81	136	55	300	17.75855856	Roshan	71	116	45	300	11.06512	1	1	1.14	10.41238037	405
3	Ritik Lende	80	129	49	300	18.51493694	Ravikant Chopkar	75	136	61	300	21.14667	1	1	1.14	10.41238037	401
4	Jagdish	73	166	93	300	33.015	Bhavesh	75	151	76	300	21.71429	1	1	1.5	10.41238037	455
5	Lokesh	84	133	49	300	15.82126126	Roshan	71	104	33	300	8.114419	1	1	1.5	10.41238037	445
6	Ritik Lende	86	130	44	300	16.62565766	Ravikant Chopkar	80	121	41	300	14.21333	1	1	1.5	10.41238037	465
7	Jagdish	71	165	94	300	33.37	Bhavesh	74	160	86	300	24.57143	1	1	2	10.41238037	480
8	Lokesh	82	131	49	300	15.82126126	Roshan	72	113	41	300	10.08155	1	1	2	10.41238037	495
9	Ritik Lende	83	124	41	300	15.49209009	Ravikant Chopkar	78	131	53	300	18.37333	1	1	2	10.41238037	505
10	Jagdis	75	161	86	300	30.53	Bhavesh	77	157	80	300	22.85714	1	1	4	10.412380	590

	h															37	
11	Lokesh	86	141	55	300	17.75855856	Roshan	70	109	39	300	9.589767	1	1	4	10.41238037	523
12	Ritik Lende	88	130	42	300	15.86994595	Ravikant Chopkar	81	125	44	300	15.25333	1	1	4	10.41238037	515
13	Jagdish	73	133	60	300	21.3	Bhavesh	71	143	72	300	20.57143	1	1.3	1.14	10.41238037	399
14	Lokesh	84	139	55	300	17.75855856	Roshan	72	106	34	300	8.36031	1	1.3	1.14	10.41238037	415
15	Ritik Lende	80	125	45	300	17.00351351	Ravikant Chopkar	74	118	44	300	15.25333	1	1.3	1.14	10.41238037	411
16	Jagdish	74	143	69	300	24.495	Bhavesh	75	155	80	300	22.85714	1	1.3	1.5	10.41238037	455
17	Lokesh	85	148	63	300	20.34162162	Roshan	71	113	42	300	10.32744	1	1.3	1.5	10.41238037	465
18	Ritik Lende	88	130	42	300	15.86994595	Ravikant Chopkar	79	128	49	300	16.98667	1	1.3	1.5	10.41238037	471
19	Jagdish	72	128	56	300	19.88	Bhavesh	72	141	69	300	19.71429	1	1.3	2	10.41238037	488
20	Lokesh	82	132	50	300	16.14414414	Roshan	71	102	31	300	7.622636	1	1.3	2	10.41238037	495
21	Ritik Lende	81	120	39	300	14.73637838	Ravikant Chopkar	75	115	40	300	13.86667	1	1.3	2	10.41238037	475
22	Jagdish	75	141	66	300	23.43	Bhavesh	77	157	80	300	22.85714	1	1.3	4	10.41238037	514
23	Lokesh	86	146	60	300	19.37297297	Roshan	70	109	39	300	9.589767	1	1.3	4	10.41238037	530
24	Ritik Lende	88	130	42	300	15.86994595	Ravikant Chopkar	81	125	44	300	15.25333	1	1.3	4	10.41238037	520
25	Jagdis	71	133	62	300	22.01	Bhavesh	73	136	63	300	18	1.3	1.3	1.14	10.412380	411

	h															37	
26	Lokesh	80	137	57	300	18.40432432	Roshan	72	107	35	300	8.606202	1.3	1.3	1.14	10.41238037	416
27	Ritik Lende	82	117	35	300	13.22495495	Ravikant Chopkar	76	123	47	300	16.29333	1.3	1.3	1.14	10.41238037	420
28	Jagdish	73	138	65	300	23.075	Bhavesh	74	147	73	300	20.85714	1.3	1.3	1.5	10.41238037	465
29	Lokesh	82	143	61	300	19.69585586	Roshan	71	105	34	300	8.36031	1.3	1.3	1.5	10.41238037	485
30	Ritik Lende	89	125	36	300	13.60281081	Ravikant Chopkar	78	119	41	300	14.21333	1.3	1.3	1.5	10.41238037	490
31	Jagdish	72	131	59	300	20.945	Bhavesh	74	135	61	300	17.42857	1.3	1.3	2	10.41238037	480
32	Lokesh	81	135	54	300	17.43567568	Roshan	73	105	32	300	7.868527	1.3	1.3	2	10.41238037	491
33	Ritik Lende	83	115	32	300	12.09138739	Ravikant Chopkar	78	121	43	300	14.90667	1.3	1.3	2	10.41238037	476
34	Jagdish	74	135	61	300	21.655	Bhavesh	75	145	70	300	20	1.3	1.3	4	10.41238037	555
35	Lokesh	84	141	57	300	18.40432432	Roshan	72	101	29	300	7.130853	1.3	1.3	4	10.41238037	560
36	Ritik Lende	89	125	36	300	13.60281081	Ravikant Chopkar	79	118	39	300	13.52	1.3	1.3	4	10.41238037	563

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Table 4 : Experimental results of dependent and independent pi terms

S.N	R	T	EM	G	ω=Angular Velocity	Π1=I/R2T	Π2=EM	Π3=G	Π01=ωT
1	58.86786	300	1	1.14	41.762	1E-05	1	1.14	20881
2	28.82367	300	1	1.14	42.39	4.18E-05	1	1.14	21195
3	39.6616	300	1	1.14	41.97133	2.21E-05	1	1.14	20985.67
4	54.72929	300	1	1.5	47.62333	1.16E-05	1	1.5	23811.67
5	23.93568	300	1	1.5	46.57667	6.06E-05	1	1.5	23288.33
6	30.83899	300	1	1.5	48.67	3.65E-05	1	1.5	24335
7	57.94143	300	1	2	50.24	1.03E-05	1	2	25120
8	25.90281	300	1	2	51.81	5.17E-05	1	2	25905
9	33.86542	300	1	2	52.85667	3.03E-05	1	2	26428.33
10	53.38714	300	1	4	61.75333	1.22E-05	1	4	30876.67
11	27.34833	300	1	4	54.74067	4.64E-05	1	4	27370.33
12	31.12328	300	1	4	53.90333	3.58E-05	1	4	26951.67
13	41.87143	300	1.15	1.14	41.762	1.98E-05	1.15	1.14	20881
14	26.11887	300	1.15	1.14	43.43667	5.09E-05	1.15	1.14	21718.33
15	32.25685	300	1.15	1.14	43.018	3.34E-05	1.15	1.14	21509
16	47.35214	300	1.15	1.5	47.62333	1.55E-05	1.15	1.5	23811.67
17	30.66906	300	1.15	1.5	48.67	3.69E-05	1.15	1.5	24335
18	32.85661	300	1.15	1.5	49.298	3.22E-05	1.15	1.5	24649
19	39.59429	300	1.15	2	51.07733	2.21E-05	1.15	2	25538.67
20	23.76678	300	1.15	2	51.81	6.14E-05	1.15	2	25905
21	28.60305	300	1.15	2	49.71667	4.24E-05	1.15	2	24858.33
22	46.28714	300	1.15	4	53.79867	1.62E-05	1.15	4	26899.33
23	28.96274	300	1.15	4	55.47333	4.14E-05	1.15	4	27736.67
24	31.12328	300	1.15	4	54.42667	3.58E-05	1.15	4	27213.33
25	40.01	300	1.3	1.14	43.018	2.17E-05	1.3	1.14	21509
26	27.01053	300	1.3	1.14	43.54133	4.76E-05	1.3	1.14	21770.67
27	29.51829	300	1.3	1.14	43.96	3.98E-05	1.3	1.14	21980
28	43.93214	300	1.3	1.5	48.67	1.8E-05	1.3	1.5	24335
29	28.05617	300	1.3	1.5	50.76333	4.41E-05	1.3	1.5	25381.67
30	27.81614	300	1.3	1.5	51.28667	4.49E-05	1.3	1.5	25643.33
31	38.37357	300	1.3	2	50.24	2.36E-05	1.3	2	25120
32	25.3042	300	1.3	2	51.39133	5.42E-05	1.3	2	25695.67
33	26.99805	300	1.3	2	49.82133	4.76E-05	1.3	2	24910.67
34	41.655	300	1.3	4	58.09	2E-05	1.3	4	29045
35	25.53518	300	1.3	4	58.61333	5.32E-05	1.3	4	29306.67
36	27.12281	300	1.3	4	58.92733	4.72E-05	1.3	4	29463.67

Table 5: Test Envelop

$\pi 1$ **=I/R²T**	$\pi 2$ **=EM**	$\pi 3$ **=G**
1.00E-05	1	1.14

to	to	to
6.14E-05	1.3	4

Table 6: Test Range

$\pi 1$=**I/R^{2}T**	$\pi 2$ =**EM**	$\pi 3$ =**G**
1.00E-05	1	1.14
1.03E-05	1.15	1.5
1.16E-05	1.3	2
1.22E-05		4
1.55E-05		
1.62E-05		
1.80E-05		
1.98E-05		
2.00E-05		
2.17E-05		
2.21E-05		
2.36E-05		
3.03E-05		
3.22E-05		
3.34E-05		
3.58E-05		
3.65E-05		
3.69E-05		
3.98E-05		
4.14E-05		
4.18E-05		
4.24E-05		
4.41E-05		
4.49E-05		
4.64E-05		
4.72E-05		
4.76E-05		
4.76E-05		
5.09E-05		
5.17E-05		
5.32E-05		
5.42E-05		
6.06E-05		
6.14E-05		

4. Development of the Model

During experimentation, the data generated belongs to the dependent variables. The data of various independent variables are gathered during experimentation. In case of tandum drive HPFM, there were seven independent variables form and one dependent variable. All these variables are represented by pi terms corresponding to each variable. The mathematical model is nothing but formulating correlation between these independent pi terms and a dependent pi term. The mathematical model is called as generalized experimental data based model as it is formulated on the data generated through experimentation. Equation 3 is the developed mathematical modelsangular velocity of fly wheel in rad/sec for pedalling time T sec.
The Exponential form of mathematical model is

Model of dependent pi term π_{01} as under-

$$\omega T = K_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1}$$

$$\pi 01 = K_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1}$$

Taking log on the both sides of equation for $\pi_{01,}$ getting eight unknown terms in the equations,

$$\therefore \text{Log}\pi_{01} = log K_1 \times log(\pi_1)^{a1} \times log(\pi_2)^{b1} \times log(\pi_3)^{c1} \qquad 4$$

Let, Z1= log π01, K1 = log k1, A = log π1, B = log π2, C = log π3

Putting the values in equations 4, the same can be written as

$$Z_1 = K_1 + a_1 A + b_1 B \quad (5)$$

Equation 5 is a regression equation of Z on A, B, and C in an n dimensional co-ordinate system. This signifies a regression hyper plane [Spiegel 1980]. To determine the regression hyper plane, determines a_1, b_1 andc_1 in equation 5 so that

$$\Sigma Z_1 = nK_1 + a_1 \Sigma A + b_1 \Sigma B + c_1 \Sigma C$$

$$\Sigma Z_1 \times A = K_1 \Sigma A + a_1 \Sigma A \times A + b_1 \Sigma B \times A + c_1 \Sigma C \times A$$

$$\Sigma Z_1 \times B = K_1 \Sigma B + a_1 \Sigma A \times B + b_1 \Sigma B \times B + c_1 \Sigma C \times B$$

$$\Sigma Z_1 \times C = K_1 \Sigma C + a_1 \Sigma A \times C + b_1 \Sigma B \times C + c_1 \Sigma C \times C$$

In the above set of equations, the values of the multipliers K1, a1, b1 and c1 are replaced to calculate the values of the unknowns (viz. K1, a1, b1 and c1). The values of the terms on L H S and the multipliers of K1, a1, b1 and c1 in the set of equations are calculated and tabulated in the Table 3 to 6. After replacing the values in the equations 5 one will obtain a set of 8 equations, which can be solved simultaneously to get the values of K1, a1, b1 and c1. The above equations can be verified in the matrix form and further values of K1, a1, b1 and c1 can be obtained by using matrix analysis.

$$X_1 = \mathrm{inv}(W \quad (6)$$

The matrix method of solving these equations using 'MATLAB' is given below.
W = 3 x 3 matrix of the multipliers of K_1, a_1, b_1 andc_1
P_1 = 3 x 1 matrix of the terms on L H S and
X_1 = 3 x 1 matrix of solutions of values of K_1, a_1, b_1 andc_1
Then, The matrix obtained is given by,

Matrix

$$Z_1 x \begin{bmatrix} 1 \\ A \\ B \\ C \end{bmatrix} = \begin{bmatrix} n & A & B & C \\ A & A^2 & BA & CA \\ B & AB & B^2 & CB \\ C & AC & BC & C^2 \end{bmatrix} x \begin{bmatrix} K_1 \\ a_1 \\ b_1 \\ c_1 \end{bmatrix} \ldots \quad (7)$$

$$P_1 = W_1 \times X_1$$

$$\begin{vmatrix} 158.1765 \\ -714.08 \\ 9.215054 \\ 45.22983 \end{vmatrix} \begin{vmatrix} 36 & -162.529 & 2.095694 & 10.22477 \\ -162.52869 & 735.6103 & -9.36069 & -46.0735 \\ 2.09569431 & -9.36069 & 0.200008 & 0.595222 \\ 10.2247749 & -46.0735 & 0.595222 & 4.386074 \end{vmatrix} \begin{vmatrix} K \\ a1 \\ b1 \\ c1 \end{vmatrix}$$

$$[P_1] = [W_1][X_1]$$

Using Matlab, , after solving X_1 matrix with K_1 and indices K_1, a_1, b_1, c_1, d_1, e_1, f_1, g_1 and $h_1$are as follows

K	4.3581
a1	0.0061
b1	0.082
c1	0.205

But k1 is the log value so convert into normal value so take antilog of k_1.

Antilog (4.3581) = 22808.67201
Hence the model for dependent term pi01 is

$$(\pi_{01}) = K_1\{(\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1}\}$$

$$\pi_{01} = 22808.67201.(\pi_1)^{0.0061} (\pi_2)^{0.082} (\pi_3)^{0.205}$$

Where, π_1 = I/R2T, π_2 =EM, π_3 =G, K=22808.67201

$$\omega T = 22808.67201\left[\left(\frac{I}{R^2T}\right)^{0.0061}, (EM)^{0.082}, (G)^{0.205}\right] \text{------------------------}(8)$$

Graphical representation of Comparison between Model and Experimental data of all six dependents Pi terms for general model.

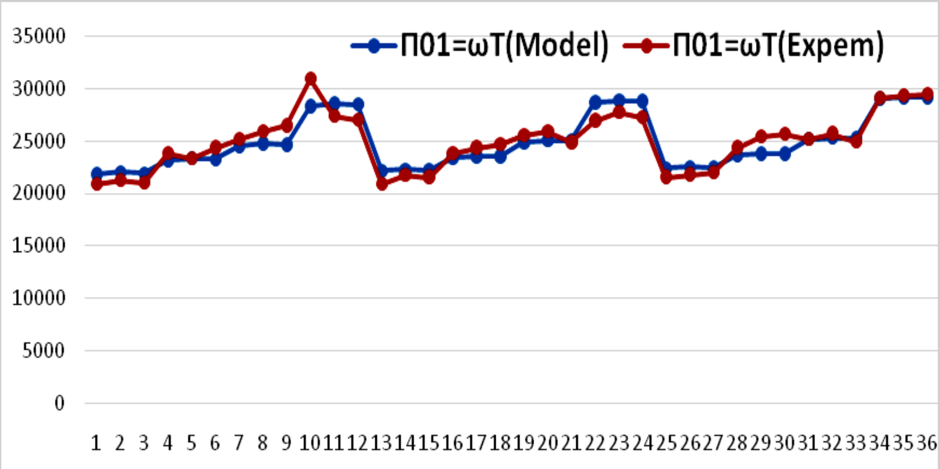


Figure 3: Comparison between Field Data and Mathematical Model For angular speed

4.1 Reliability of model:

Table 7:Reliability

ERROR	FREQ.	%Error (fi)	Frequency (xi)	fi*xi
0	5	0	5	0
1	4	1	4	4
2	6	2	6	12
3	6	3	6	18
4	7	4	7	28
5	2	5	2	10
6	4	6	4	24
7	1	7	1	7
8	1	8	1	8
		36	**36**	**111**
				3.08333333
		Mean Error	**Σfi*xi/xi**	**3.08333333**
		Reliability	**(100-Mean Error)**	**96.91666667**

Reliability of model is established using relation Reliability =100-% mean error and Mean error = Σ(xi. fi)/Σ(xi),where xi is % error and fi is frequency of occurrence [8]. Therefore the reliability of General model ΠD1 is equal to 96.91 %.

4.2 R- Coefficient:

Table 7: *R- Coefficient*

fi	Yi	(Yi-fi)	(Yi-fi)^2	(Yi-Y)	(Yi-Y)^2
PMC(Model)	**PMC(Expem)**				
21840.85	20881	-959.8529901	921317.8	-4018.04	16144645
22031.96	21195	-836.9617005	700504.9	-3704.04	13719912
21946.33	20985.67	-960.6677534	922882.5	-3913.37	15314491
23125.39	23811.67	686.2779117	470977.4	-1087.37	1182381
23359.9	23288.33	-71.56591034	5121.68	-1610.71	2594376
23287.79	24335	1047.208633	1096646	-564.04	318141.1
24513.16	25120	606.8401993	368255	220.96	48823.32
24755.11	25905	1149.885904	1322238	1005.96	1011956

24674.29	26428.33	1754.039331	3076654	1529.293	2338738
28284.22	30876.67	2592.447994	6720787	5977.627	35732021
28515.98	27370.33	-1145.650419	1312515	2471.293	6107291
28471.04	26951.67	-1519.369463	2308484	2052.627	4213276
22184.62	20881	-1303.617034	1699417	-4018.04	16144645
22312.72	21718.33	-594.3853396	353293.9	-3180.71	10116895
22255.34	21509	-746.3354963	557016.7	-3390.04	11492371
23433.3	23811.67	378.3693423	143163.4	-1087.37	1182381
23557.8	24335	777.1962123	604034	-564.04	318141.1
23538.01	24649	1110.989703	1234298	-250.04	62520
24911.16	25538.67	627.5112712	393770.4	639.6267	409122.3
25066.76	25905	838.2430928	702651.5	1005.96	1011956
25010.18	24858.33	-151.8432363	23056.37	-40.7067	1657.033
28660.09	26899.33	-1760.7547	3100257	2000.293	4001173
28824.49	27736.67	-1087.827378	1183368	2837.627	8052125
28799.2	27213.33	-1585.871408	2514988	2314.293	5355954
22421.21	21509	-912.2080499	832123.5	-3390.04	11492371
22528.94	21770.67	-758.2735115	574978.7	-3128.37	9786720
22504.55	21980	-524.5510057	275153.8	-2919.04	8520795
23691.73	24335	643.2722499	413799.2	-564.04	318141.1
23821.7	25381.67	1559.967657	2433499	482.6267	232928.5
23824.2	25643.33	1819.137189	3309260	744.2933	553972.6
25172.48	25120	-52.47547482	2753.675	220.96	48823.32
25300.68	25695.67	394.988106	156015.6	796.6267	634614
25280.69	24910.67	-370.0198546	136914.7	11.62667	135.1794
28986.94	29045	58.06280715	3371.29	4145.96	17188984
29160.51	29306.67	146.1526991	21360.61	4407.627	19427173
29139.06	29463.67	324.6034713	105367.4	4564.627	20835817
			□(Yi-fi)^2		**□(Yi-Y)^2**
895192.4	896365.3	1172.963048	**40000295**	-0.10667	**2.46E+08**
	Y				
24866.45	24899.04	32.58230689	1111119	-0.00296	6830985

x=□(Yi-fi)^2/□(Yi-Y)^2		**0.162659**		
	R^2=1-x			
	R^2	**0.837341**	**R**	**0.915064**

yi= Observed value of dependant variable for ith Experimental sets (Experimental data)
fi=Observed value of dependant variable for ith pridicted value sets (Model data)
Y= Meanof Yi Y=24899.03704
R^2 = C0-efficient of Determination

4.3 Optimization of Model:

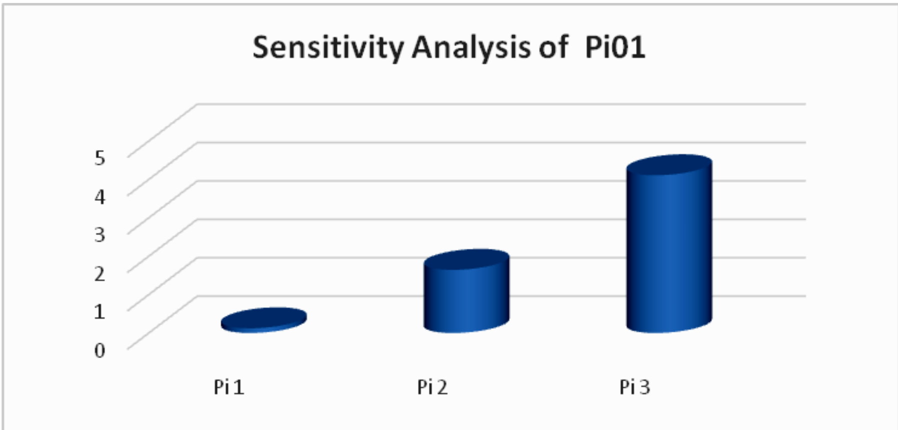
K1	log k	22808.67201	**4.3581**
	Solution		**Max**
Z	4.46517543		29186.05728
X1	-4.211511915		6.14452E-05
X2	0.113943352		1.3
X3	0.602059991		4
	Constrain	**C**	**Log C**
X1<=C1MAX	-4.211511915	6.14452E-05	-4.21151
X1>=C1MIN	-4.211511915	1.00155E-05	-4.99933
X2<=C2 MAX	0.113943352	1.3	0.113943
X2>=C2MIN	0.113943352	1	0
X3<=C3 MAX	0.602059991	4	0.60206
X3>=C3 MIN	0.602059991	1.14	0.056905

Maxima Minima:

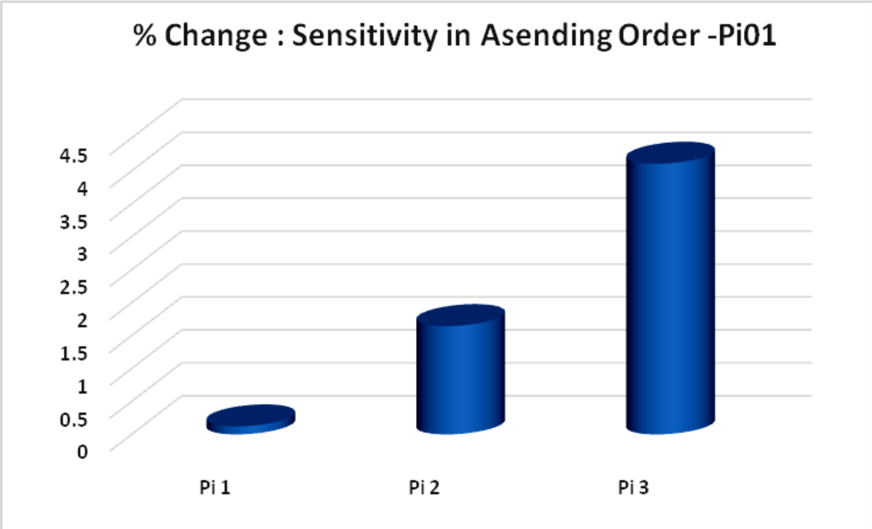
	$\pi 1$ **=I/R²T**	$\pi 2$ **=EM**	$\pi 3$ **=G**	Π01=ωT
MAXIMA	6.14452E-05	1.3	4	**29186.0573**
MINIMA	1.00155E-05	1	1.14	**21840.853**

4.4 ***Sensitivity Analysis:***

	Sensitivity of Pi			
	Pi 1	**Pi 2**	**Pi 3**	**Pi01**
Avg.	**3.44E-05**	1.15	2.16	**25376**
10%	**3.79E-05**	1.15	2.16	**25390**
-10%	**3.1E-05**	1.15	2.16	**25359**
	% Change			**0.1224**
Avg.	3.44E-05	**1.15**	2.16	**25376**
10%	1.96E+12	**1.265**	2.16	**25575**
-10%	1.96E+12	**1.035**	2.16	**25157**
	% Change			**1.6448**
Avg.	3.44E-05	1.15	**2.16**	**25376**
10%	3.44E-05	1.15	**2.376**	**25876**
-10%	3.44E-05	1.15	**1.944**	**24833**
	% Change			**4.1098**



Pi Terms	% Change
Pi 1	0.122405
Pi 2	1.64484
Pi 3	4.109803



5. Conclusions

The angular velocity with respect to pedalling time of tandum drive is established through theory of experimentation. The data in the present work are collected by performingactual experimentation due to which the finding of the present study truly represents the degree of interaction of variousindependent variables in tandum drive human powered flywheel motor. The trends for the behaviour of the modelis demonstrated by graphical analysis and it has been noted that the mathematical models can be successfully used for thecomputation of dependent terms for a given set of independent terms for tandum drive flywheel motor. The model was formulatedfor angular velocity of flywheel fortotal36 sets of observations andis optimized. The optimized values of dependent pi terms of angular velocity are $\pi1$ =6.14452E-05seconds, $\pi2$ =1.3and $\pi3$ =4

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