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Effect of Singular and Non-Singular Kernel Fractional order derivative on the fractional order PID controller with Volterra based reference tracking

Sachin Gade[1,1a], Sanjay Pardeshi[2], Mahesh Kumbhar[3]

[1] Research Scholar, DOT, Shivaji University Kolhapur Maharashtra India

[1a]Fabtech Technical Campus, College of Engineering and Research, Sangola.

[2] Government Polytechnic, Tasgoan Maharashtra India

[3]Rajarambapu Institute of Technology, Islampur Maharashtra India

Abstract: -*The effect of singular and non-singular kernel fractional order derivative on the performance of fractional order PID controller is discussed in this literature along-with the numerical approximation of Caputo singular and non-singular kernel definition. Volterra based control loop design with the tracking of reference signal confirms the stability conditions for the smooth control action. Three type of proportional integral derivative controllers are implemented for controlling the three different system with the optimum settings of controllers. Integer order and fractional order real time controller is successfully implemented in the embedded environment. Performance comparisons of three controllers have been discussed in this literature.*

***Keywords**: -Singular and non-singular kernel, fractional derivative, singularity points, fractional control system,Fractional Numerical Methods.*

1. Introduction

Concept of changing weighted kernel first appears as Volterra Integral Equations (VIE)of first and second kind [1]. A linear VIE can be a convolution integral as shown in equation (1).

$$x(t) = f(t) + \int_c^t K(t-\tau)\, x(\tau)\, d\tau \quad (1)$$

Continuous closed loop control system using equation (1) is a simple closed loop where f(t) is reference signal, x(t) is a compensator or controller and K(t) treated as the system under control.VIE is producing unstable control loop due to the positive and continuous ringing feedback and hence not suitable to implement as control loop. A negative feedback stable loop is having the form as equation (2)in terms of error signal e(x) and reference signal r(x) is quite popular in control system. Controller or compensator c(t) and g(t) is system to be control.

$$r(x) = e(x) + \int_c^x e(x-t)\left[\int_c^t c(\tau) g(t-\tau) d\tau\right] dt \quad (2)$$

Solving non-local system problem is emerged as the new challenges in the control system. Widespread use of fractional calculusis able to solve the non-local system modelling and control. Caputo fractional derivative equation with singular type kernel satisfies the most of engineering initial value conditions as like real time physical values [2,3]. The Caputo Definition given as,

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_c^t \frac{f'(\tau)\, d\tau}{(t-\tau)^\alpha} \quad (3)$$

Singular type fractional derivative Caputo Kernel shows the changing weighted property as like VIE. This is useful to solve the non-local systems dynamic problems but the initial set of conditions should match with exact physical initial states of controller and system under control. This is found matching with uniform and homogeneous systems where the initial set of conditions not differ much more and having the limited range.

When the domain of lower limit is $c = [-\infty, 0]$constituting maximum range then initial values that does not suitable for the set of initial values of physical control system. Control system defines the lower limit always set to zero or non-zero but impractical to handle large value (-∞)and the singularity present at $t = \tau$. Caputo and Fabrizio suggested non-singular kernel to avoid singularity problem and new non-singular kernel type fractional derivative equation is as given [3],

$$D_t^{\alpha} f(t) = \frac{M(\alpha)}{(1-\alpha)} \int_a^t e^{\left[-\frac{\alpha(t-\tau)}{1-\alpha}\right]} f'(\tau)\, d\tau \quad (4)$$

2. Fractional Numerical Methods

Fractional numerical approximation of Caputo definition proposed in [4] given in equation (5).

$$y^{\alpha}(f(x)) \approx \sum_{n=0}^{N} \left\{ \left[\frac{1}{h^{\alpha}\Gamma(2-\alpha)}\right] \left[\left(y(x_n) G_{n,n}^{\alpha} - y(x_0) G_{n,1}^{\alpha} \right) + \left(\sum_{k=1}^{n-1} y(x_k) M_{n,k}^{\alpha} \right) \right] \right\} \quad (5)$$

Where, $M_{n,k}^{\alpha} = G_{n,k}^{\alpha} - G_{n,k+1}^{\alpha}$; $G_{n,k}^{\alpha} = (n-k+1)^{1-\alpha} - (n-k)^{1-\alpha}$

Lemma 1: - Numerical Approximation of Non-Singular kernel Caputo-Fabrizio definition.

$$D^{\alpha} f(x) \approx \left[\frac{M(\alpha)}{\alpha h}\right] \sum_{n=0}^{N} \left\{ \left[y(x_n) MG_{n,n}^{\alpha,h} - y(x_0) MG_{n,1}^{\alpha,h} \right] + \left[\sum_{k=1}^{n-1} y(x_k) MGG_{n,k}^{\alpha,h} \right] \right\} \quad (6)$$

$MGG_{n,k}^{\alpha,h} = MG_{n,k}^{\alpha,h} - MG_{n,k+1}^{\alpha,h}$ and $MG_{n,k}^{\alpha,h} = e^{\left[\frac{\alpha h}{(1-\alpha)}(k-n)\right]} - e^{\left[\frac{\alpha h}{(1-\alpha)}(k-1-n)\right]}$

Proof :- From equation (4) and $x_n = nh$; $y_n = y_n(x_n) = y_n(nh)$; $x_k = kh$; $y'_k = y'_k(x_k) = y'_k(kh)$; a=0

$$\frac{(1-\alpha)}{M(\alpha)} D^{\alpha} f(x_n) \approx \int_0^t e^{\left[\frac{-\alpha(x_n - x)}{(1-\alpha)}\right]} f'(x)\, dx$$

$$\frac{(1-\alpha)}{M(\alpha)} D^{\alpha} f(x_n) \approx \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} e^{\left[\frac{-\alpha(nh - x)}{(1-\alpha)}\right]} f'(x_k)\, dx$$

Using Taylor Series expansion and neglecting higher order terms, (For Unconditional stable algorithm)

$$f'(x_k) \cong \frac{y(x_k) - y(x_{k-1})}{h}$$

$$\frac{(1-\alpha)}{M(\alpha)} D^{\alpha} f(x_n) \approx \sum_{k=1}^{n} \left[\frac{y(x_k) - y(x_{k-1})}{h}\right] \int_{x_{k-1}}^{x_k} e^{\left[\frac{-\alpha(nh - x)}{(1-\alpha)}\right]} dx \quad \text{; after evaluation}$$

$$D^{\alpha} f(x_n) \approx \left[\frac{M(\alpha)}{\alpha h}\right] \sum_{k=1}^{n} [y(x_k) - y(x_{k-1})] \left[e^{\left[\frac{\alpha h}{(1-\alpha)}(k-n)\right]} - e^{\left[\frac{\alpha h}{(1-\alpha)}(k-1-n)\right]} \right]$$

Let, $MG_{n,k}^{\alpha,h} = e^{\left[\frac{\alpha h}{(1-\alpha)}(k-n)\right]} - e^{\left[\frac{\alpha h}{(1-\alpha)}(k-1-n)\right]}$ and after simplification,(7)

$$D^{\alpha} f(x_n) \approx \left[\frac{M(\alpha)}{\alpha h}\right] \left\{ \left[y(x_n) MG_{n,n}^{\alpha,h} - y(x_0) MG_{n,1}^{\alpha,h} \right] + \left[\sum_{k=1}^{n-1} y(x_k) \left(MG_{n,k}^{\alpha,h} - MG_{n,k+1}^{\alpha,h} \right) \right] \right\}$$

Let, $MGG_{n,k}^{\alpha,h} = MG_{n,k}^{\alpha,h} - MG_{n,k+1}^{\alpha,h}$ (8)

$$D^{\alpha} f(x) \approx \left[\frac{M(\alpha)}{\alpha h}\right] \sum_{n=0}^{N} \left\{ \left[y(x_n) MG_{n,n}^{\alpha,h} - y(x_0) MG_{n,1}^{\alpha,h} \right] + \left[\sum_{k=1}^{n-1} y(x_k) MGG_{n,k}^{\alpha,h} \right] \right\}$$

Corollary 1:- $MG_{n,k}^{0,h} = MG_{n,k}^{\alpha,0} = 0$; $MG_{n,n}^{\alpha,h} = MG_{k,k}^{\alpha,h} = 1 - e^{\left[-\frac{\alpha h}{(1-\alpha)}\right]}$; $MG_{n,n}^{1,h} = MG_{k,k}^{1,h} = 1$

$MG_{n,k}^{\alpha,1} = e^{\left[\frac{\alpha}{(1-\alpha)}\right]} - 1$ at $k - n = 1$; Series is convergent when,

$\int \left| y(x) MGG_{n,k}^{\alpha,h} \right| dx < \infty$ and $\int \left| y(x) MG_{n,n}^{\alpha,h} - y(x_0) MG_{n,1}^{\alpha,h} \right| < \infty$

Relative error between singular and non-singular kernel is given as,

$$\varepsilon = \frac{y(x_n)\left\{ \left[\frac{1}{h^{\alpha}\Gamma(2-\alpha)}\right] - \left[\frac{M(\alpha)}{\alpha h}\left(1 - e^{\left[\frac{-\alpha h}{(1-\alpha)}\right]}\right)\right] \right\} + y(x_0)\left\{ MG_{n,1}^{\alpha,h}\left[\frac{M(\alpha)}{\alpha h}\right] - G_{n,1}^{\alpha}\left[\frac{h^{\alpha}}{\Gamma(2-\alpha)}\right] \right\}}{\left[\frac{1}{h^{\alpha}\Gamma(2-\alpha)}\right]\left[\left(y(x_n) - y(x_0) G_{n,1}^{\alpha} \right) \right]} \quad (9)$$

Where, $MG_{n,n}^{\alpha,h} = 1 - e^{\left[\frac{-\alpha h}{(1-\alpha)}\right]}$(10)

Definition 1: -[6] The fractional Non-singular Caputo-Fabriziodefinition is corrected by Jorge and Juan for the $0 < \alpha < 1$

$$D_t^{\alpha} f(t) = \frac{1}{(1-\alpha)} \int_a^t e^{\left[-\frac{\alpha(t-x)}{1-\alpha}\right]} f'(x)\ dx \quad (11)$$

3. Control Loop Design

Stable Control loop is the special case of Volterra Integral Equation and necessary parameters of control loop should be optimum in order to have better and smother closed loop control. Simple Closed loopcontrol strategy given as,

$$e(t) = r(t) - \int_0^t g(t-\tau) \int_0^t c(t-\tau) e(\tau) d\tau d\tau \ (12)$$

Where, Error signal $(t) = r(t) - y_{PV}(t)$, $r(t)$ is reference signal or Set Point (SP), $y_{PV}(t)$ is output of system to be control.g(t) is the impulse response model of system, $c(t)$ is the impulse response model of Controller. Function of control loop is to provide the desired output of system. The reference signal is the desired output of system. This is achieved with the help of controller. The output of controller is the control signal or manipulating signal given as,

$$y_c(t) = \int_0^t c(t-\tau) e(\tau) d\tau \ (13)$$

Output of system known as Process Value (PV)which is under control and changes with the manipulating signal $y_c(t)$ given as,

$$y_{PV}(t) = \int_0^t g(t-\tau) y_c(\tau) d\tau \ (14)$$

Reference signal is constant throughout closed loop control action.Reference constant function is given as,

$$\emptyset(t) = e(t) + \int_0^t g(t-\tau) \int_0^t c(t-\tau) e(\tau) d\tau d\tau; \ \ \emptyset(t) = r(t) \ (15)$$

Using the value of equation (13) putting in the equation (14),

$$y_{PV}(t) = \int_0^t g(t-\tau) \int_0^t c(t-\tau) e(\tau) d\tau\, d\tau \ (16)$$

Using the definition of error signal and after modification,

$$y_{PV}(t) = \int_0^t g(t-\tau) \int_0^t c(t-\tau)\, r(\tau)\ d\tau d\tau \ - \int_0^t g(t-\tau) \int_0^t c(t-\tau) y_{PV}(\tau)\ d\tau d\tau \ (17)$$

Tracking of output is possible with reference constant function (equation-25),

$$\int_0^t g(t-\tau) \int_0^t c(t-\tau)\ \emptyset(\tau)\ d\tau d\tau \ = y_{PV}(t) + \int_0^t g(t-\tau) \int_0^t c(t-\tau) y_{PV}(\tau)\ d\tau d\tau \ (18)$$

Control loop design reduces to simple form as,

$$Z(t) = y_{PV}(t) + Y(t) \ (19)$$

Where, $Z(t) = \int_0^t g(t-\tau) \int_0^t c(t-\tau)\ \emptyset(\tau)\ d\tau d\tau$ (20)

$$Y(t) = \int_0^t g(t-\tau) \int_0^t c(t-\tau) y_{PV}(\tau)\ d\tau d\tau \ (21)$$

Objective of the control strategy is to make $\emptyset(t)$ constant in the shorter time interval. The tracking of output is easily achieved by rearranging equation (19) as,

$$y_{PV}(t) = Y(t) - Z(t) = \int_0^t g(t-\tau) \int_0^t c(t-\tau) y_{PV}(\tau)\ d\tau d\tau \ - \int_0^t g(t-\tau) \int_0^t c(t-\tau)\ \emptyset(\tau)\ d\tau d\tau$$

It can be easily shown that the solution of above is given using the recursive and iterative method.

Remark 1: - Numerical Fractional derivative (equation-5) is always bounded with $0 \le n \le N$and $1 \le k \le n-1$

Remark 2: - The existence of function $f_1(x)$ is within the bound of $2 \le n \le N$and $1 \le k \le n-1$. $f_1(x) = \left(\sum_{k=1}^{n-1} y(x_k) M_{n,k}^{\alpha}\right)$.

Definition 2: - The Singular kernel Matrix $|M|_{r,c}$ started from $n = 2\ \&\ k = 1$given as,

$$|M|_{r,c} = \begin{bmatrix} s_1 & x & x \\ x & s_2 & x \\ x & x & s_3 \end{bmatrix}$$; where, 's'= singular point and 'x'= non-singular point

The Singular Matrix is square matrix of dimension of $(N-1), (N-1)$

Stability of Control System

The VIE (equation-1) can be written as,

$x_m(t) = x_m(0)e^{\lambda_m t} + \int_0^t e^{\lambda_m(t-\tau)}[r(\tau) - y_{PV}(\tau)]\,d\tau$ (22)

Where, λ_m = Eigenvalues of Singular matrix $|M|_{r,c}$ and $e(t) = r(t) - y_{PV}(t)$

When $\lim_{t\to\infty}|x_m(t)| < \infty$; This condition is satisfying only if $\lim_{t\to\infty}|e^{\lambda_m t}| < \infty$ and $\lim_{t\to\infty}|e^{\lambda_m(t-\tau)}| < \infty$. It is clear that the control system is stable with the eigenvalues such that $\lambda_m < 0$.

The characteristic polynomial of $n \times n$ matrix is,

$f_A(\lambda) = \det[M - \lambda I] = (-\lambda)^n + Tr(M)(-\lambda)^{n-1} + \ldots + \det(M)$(23)

Polynomial $f_A(\lambda)$ has n-roots. det= determinant of matrix, Tr= Trace of matrix

When λ_m is real or complex such that $\lambda_m < 0$ then the control system is always stable and does not have continuous oscillations.

Corollary 2: - The trace of singular matrix is,

$Tr(M) = M^{N-1}(\alpha)[\sum_{i=2}^{N} \quad e(x_i) - (N-1)e(x_0)]$(24)

Where, $\xi_n = M(\alpha)[y(x_n) - y(x_0)]$ and $\xi_2 \le \xi_n \le \xi_N$(25)

4. Fractional Order Controller and illustrative example

Fractional Order Proportional Integral Derivative (FOPID) Controller given as,

$y_c(t) = K_p\, e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t)$(26)

Real time Fractional derivative and integration are calculated using numerical approximated Caputo definition given in equation (5) with the suggestions recommended in definition 2 and property 1 called as the proposed method. The Non-singular type Caputo-Fabrizio fractional derivative and integration are calculated using the numerical approximated definition given in Lemma 1. Integer Order Proportional Integration Derivative (IOPID) results are used to compare with the fractional order controller. IOPID is implemented as per the method mentioned in [7].

Consider a general fractional differential equation as,

$\mathrm{u(t)b} = \mathrm{K_p\, e(t)} + \mathrm{K_i D^{-\lambda} e(t)} + \mathrm{K_d D^{\mu} e(t)}$(27)

Laplace transform of general fractional order differential equation is,

$\mathrm{E(s)} = \frac{\mathrm{s}}{[\mathrm{K_p + K_i(s)^{-\lambda} + K_d(s)^{\mu}}]}\mathrm{b} + \mathrm{e(0)}\left[\frac{[\mathrm{K_i}]}{[\mathrm{K_p(s)^{\lambda+1} + K_i + K_d(s)^{\mu+\lambda}}]} + \frac{[\mathrm{K_d}]}{[\mathrm{K_p(s)^{1-\mu} + K_i(s)^{-\lambda-\mu} + K_d}]}\right]$(28)

b and $\mathrm{e(0)}$ is the coefficient of transfer function. Objective of controller is to minimize the $\mathrm{E(s)}$.

Proposed non-singular kernel and singular kernel fractional definition holds the equality according to the convolution property and the fractional numerical method is used to evaluate the fractional integration as,

$\mathrm{D}^{-\lambda}\mathrm{e(t)} \approx \sum_{n=0}^{N}\left\{\left[\frac{h^{\lambda}}{\Gamma(2+\lambda)}\right]\left[\left((e_n)G_{n,n}^{-\lambda} - y(e_0)G_{n,1}^{-\lambda}\right) + \left(\sum_{k=1}^{n-1}(e_k)M_{n,k}^{-\lambda}\right)\right]\right\}$(29)

The fractional numerical method is used for evaluation of fractional derivative as,

$\mathrm{D}^{\mu}\mathrm{e(t)} \approx \sum_{n=0}^{N}\left\{\left[\frac{1}{h^{\mu}\Gamma(2-\mu)}\right]\left[\left((e_n)G_{n,n}^{\mu} - (e_0)G_{n,1}^{\mu}\right) + \left(\sum_{k=1}^{n-1}(e_k)M_{n,k}^{\mu}\right)\right]\right\}$(30)

Caputo-Fabrizio Non-Singular exponential type kernel fractional order definition is also used to implement the FOPID controller. The fractional numerical method for exponential type non-singular kernel is used to evaluate the fractional integration as,

$\mathrm{D}^{-\lambda}\mathrm{e(t)} \approx \left[\frac{\mathrm{M(-\lambda)}}{(-\lambda)\mathrm{h}}\right]\sum_{\mathrm{n=0}}^{\mathrm{N}}\{[(\mathrm{e_n})\mathrm{MG}_{\mathrm{n,n}}^{-\lambda,\mathrm{h}} - (\mathrm{e_0})\mathrm{MG}_{\mathrm{n,1}}^{-\lambda,\mathrm{h}}] + [\sum_{\mathrm{k=1}}^{\mathrm{n-1}}(\mathrm{e_k})\mathrm{MGG}_{\mathrm{n,k}}^{-\lambda,\mathrm{h}}]\}$(31)

The fractional numerical method for exponential type non-singular kernel is implemented for the evaluation of fractional derivative as,

$\mathrm{D}^{\mu}\mathrm{e(t)} \approx \left[\frac{\mathrm{M(\mu)}}{\mu\mathrm{h}}\right]\sum_{\mathrm{n=0}}^{\mathrm{N}}\{[(\mathrm{e_n})\mathrm{MG}_{\mathrm{n,n}}^{\mu,\mathrm{h}} - (\mathrm{e_0})\mathrm{MG}_{\mathrm{n,1}}^{\mu,\mathrm{h}}] + [\sum_{\mathrm{k=1}}^{\mathrm{n-1}}(\mathrm{e_k})\mathrm{MGG}_{\mathrm{n,k}}^{\mu,\mathrm{h}}]\}$(32)

Integer Order PID (IOPID) controller is also used to control the system and comparing the results with FOPID controller. Integer Order derivative and integration is calculated as follows,

$\frac{de}{dt} = \left[\sum_{n=1}^{N}(-1)^{n+1}\frac{1}{n}\left(\sum_{i=0}^{n}(-1)^{n+1}\frac{n!}{(n-i)!\,i!}e_{(n-i)}\right)\right]$(33)

$\int_0^n e\,dt = \frac{1}{3}\left[(e_0 + e_n) + 2\sum_{j=1}^{\frac{n}{2}-1}e_{2j} + 4\sum_{j=1}^{\frac{n}{2}}e_{(2j-1)}\right]$(34)

Conceptual Block diagram of Present work

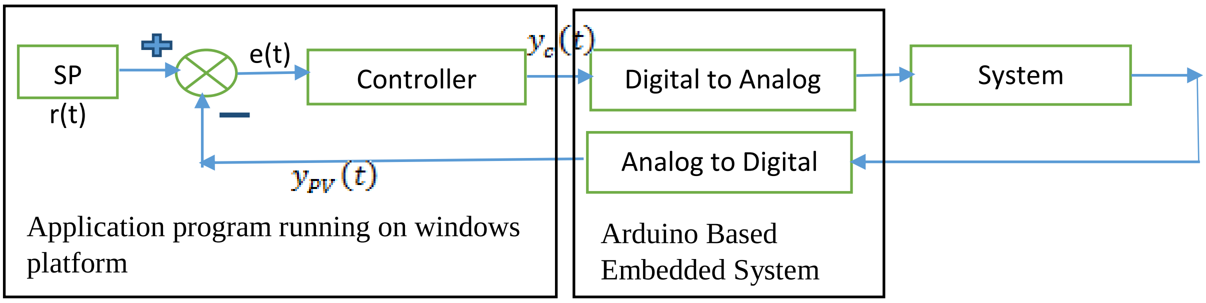


Figure 1:- Conceptual Block Diagram

Present research work is broadly classified into software method and hardware method. Software method composed of application software which is developed using Visual Basic 2008 toolset and intended to run on the windows 7 operating system. Front-end of application program is shown in the figure 2. Universal Serial Bus (USB) 2.0 communication protocol is used for exchanging the data between application program and Arduino based embedded system. System which is under control is connected with embedded system as shown in the figure 1. Hardware method consists of Arduino open access hardware which is responsible for the necessary data exchange as shown in figure 1.Averaging filter is used to remove out the noise signal present in the system output and implemented in the embedded system.Application program takes the data from embedded hardware through USB port and error signal is calculated using the user defined set point. The controller parameters are also user defined and user can input the parameters.Application program is able to export the practical data in excel format for the further analysis. User can able to select the IOPID or FOPID controller from the options. The singular or non-singular kernel is also selected using the check box provided. If check box is clicked, then it is non-singular kernel otherwise singular kernel is used for the FOPID controller.Calculated controller output is transmitted to the embedded system through the USB communication.

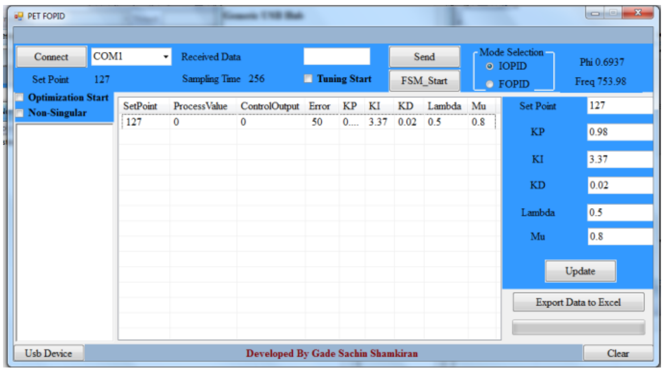


Figure 2:- Front-end of application program

Three different controllers are used to check the performance of the control action as per the control loop design (see equation26 to 34). Dynamics of control system is kept under the stable boundary region as per the stability criteria mentioned above.

Controllers used in this present work are,

1. IOPID with the optimum system parameters
2. FOPID with singular kernel and used with optimum parameters
3. FOPID with exponential non-singular kernel with optimum parameters.

Sensitivity = 1.6479, phase of controller = 0.6937 and Gain Margin = 18.6704 dB.$\lambda = 0.8$, $\mu = 0.5$. Set point = 127. These parameters are also used for the stable control actionthroughout the experimentation.

Three systems are used for comparison of results and validation of fractional numerical method for control problems as,

1. First Order System (FOS) with following transfer function,

$$G1(s) = \frac{10}{s+10} \quad (35)$$

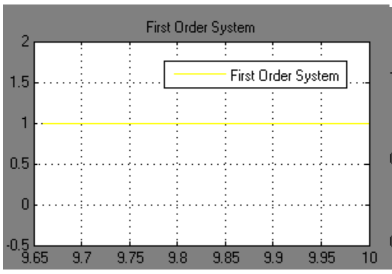
2. Second Order System (SOS) with following transfer function

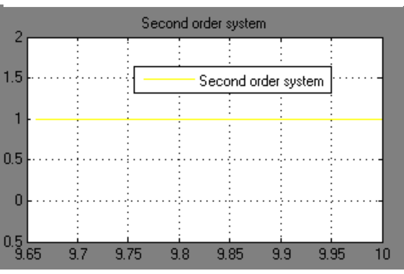
$$G2(s) = \frac{10000}{s^2 + 300s + 10000} \quad (36)$$

3. Third Order System (TOS) with following transfer function.

$$G3(s) = \frac{10^9}{s^3 + 5\times10^3 s^2 + 6\times10^6 s + 10^9} \quad (37)$$

Step response of $1(s)$, $G1(s)$ and $G1(s)$ is shown in the figure 3,4 and 5 respectively.





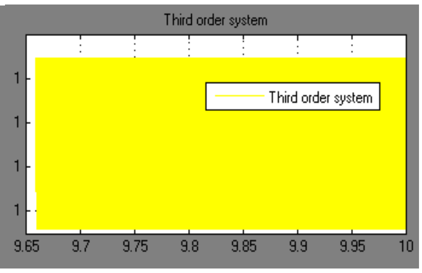


Figure 3:- Step Response of FOSFigure 4:- Step Response of SOSFigure 5:- Step Response of TOS

First Order System Performance of Three Controllers

Table 1 shows the controller parameters used during the experimentation work for the controlling of the FOS.FOS is an open loop stable system having the stable step response (Figure 3). Singular kernel type controller shows the minimum root mean square (RMS) value of 22.75872571 and maximum value 23.29065361 shown by non-singular kernel (see table 2). The IOPID shows the moderate performance of 22.82697323. Manipulating signal effort is minimum in case of singular type controller which is18.98151514 and the non-singular type controller has maximum manipulating signal effort (19.66289255). No peak overshoot observed in three controllers. However, the IOPID settles down quickly (3.328 sec.) for the FOS as compare with the FOPID. Singular controller settles earlier than the non-singular controller for the first order system.

Table 1:- Parameters used for FOS

Controller parameters used for First Order System					
	KP	KI	KD	Lambda	Mu
IOPID	1.489048	0.00742	0.274031	NA	NA
FOPID_Singular	2.485801	0.00862	0.002998	0.5	0.8
FOPID_Non_Singular	2.485801	0.09882	0.011697	0.5	0.8

Table 2:- Performance comparison for FOS

First Order System Performance			
Performance Parameter	IOPID	FOPID	
		FOPID_Singular	FOPID_Non-Singular
RMS	22.82697323	22.75872571	23.29065361
Manipulating Signal	19.07872653	18.98151514	19.66289255
Peak Overshoot	0	0	0
Settling Time	3.328	3.84	4.096

Second order System Performance of Three Controllers

Non-singular kernel controller shows the best performance as compare with the IOPID and singular FOPID (Table 4).Second order system is open loop stable system. Non-singular type controller shows the minimum

RMS value of 18.77145359 as compared with the maximum RMS value of 27.4062997 shown by singular FOPID. Non-singular FOPID has very low manipulating signal effort of 16.140762 as compared with the other two controllers. Whereas, IOPID shows the moderate performance.Non-singular FOPID settling time is 3.584 sec. which is minimum of that of IOPID and singular FOPID.Even though it is minimum than the FOS settling time.No peak overshoot is observed in case of both singular and non-singular FOPID but found the nominal overshoot of 0.787402% in case of IOPID.Table 3 shows the controller parameters used for second order system.

Table 3:- Parameters used for SOS

Controller parameters used for Second Order System					
	KP	KI	KD	Lambda	Mu
IOPID	0.835533	0.00563	0.166745	NA	NA
FOPID_Singular	2.485801	0.0384	0.170244	0.5	0.8
FOPID_Non_Singular	2.485801	0.14821	0.011892	0.5	0.8

Table 4:- Performance comparison for SOS

Second Order System Performance			
	IOPID	FOPID_Singular	FOPID_Non_Singular
RMS	22.94243	27.4062997	18.77145359
Manipulating Signal	20.40981	25.2667482	16.140762
% Peak Overshoot	0.787402	0	0
Setlling time	5.12	9.984	3.584

Third Order System Performance of Three Controllers

Third order system is behaving oscillatory for the input unit step signal but does not provide the unbounded output and hence the TOS is an open loop stable system. Non-singular FOPID controller settles the output at 3.84 sec which is much more set quickly as compare with the other two controllers and other two systems. RMS of non-singular controller is minimum 21.24799456 shows better performance index than other two controllers.Manipulating signal effort is 18.27177685 for non-singular controller. IOPID shows least performance and singular FOPID shows the moderate performance index.

Table 5: - Parameters used for TOS

Controller parameters used for Third Order System					
	KP	KI	KD	Lambda	Mu
IOPID	1.576329	0.06637	0.005	NA	NA
FOPID_Singular	2.485801	0.05072	0.005879	0.5	0.8
FOPID_Non_Singular	2.485801	0.14247	0.016146	0.5	0.8

Table 6:- Performance comparison for TOS

Third Order System Performance			
	IOPID	FOPID_Singular	FOPID_Non_Singular
RMS	32.95523232	31.28640537	21.24799456
Manipulating Signal	31.45726472	29.50853401	18.27177685
% Peak Overshoot	0	0	0
Setlling time	17.408	12.288	3.84

Output response for the FOS, SOS and TOS using the three different controllers are shown in the figure 6, 7 and 8respectively. Among the three controllers the non-singular kernel FOPID is shown the best performance for the higher order system.

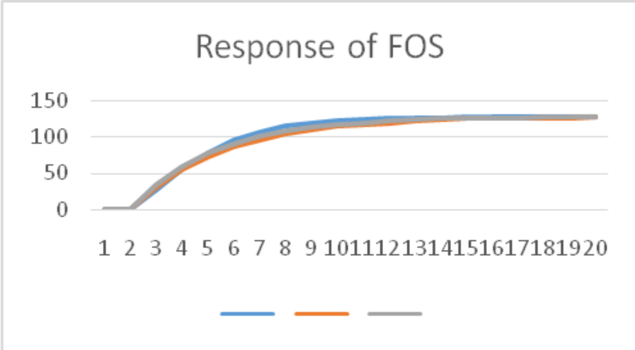


Figure 61:- Response of FOS

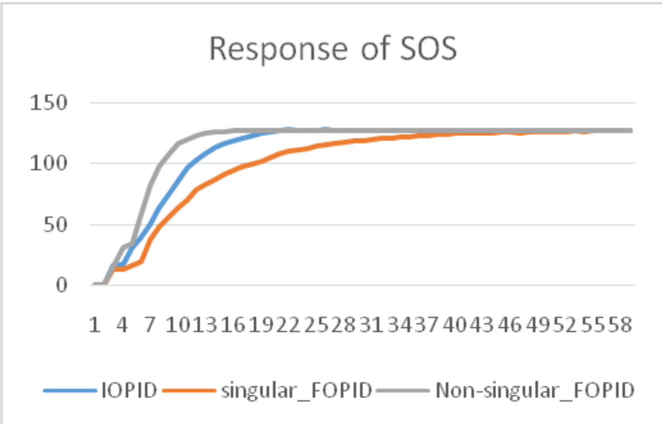


Figure 72:- Performance of SOS

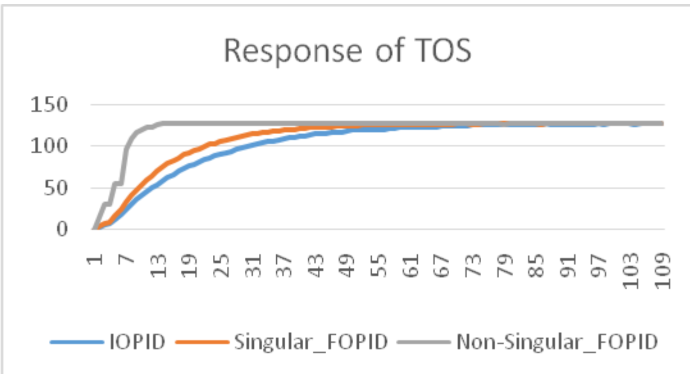


Figure8:- Response of TOS

5. Conclusion

Non-singular kernel Caputo-Fabrizio definition is useful in the controlling of the system which shows the optimum best performance for the higher order system.Caputo-Fabrizio non-singular kernel settled down quickly for the higher order system and hence useful in the controlling of the quick and faster control action.The effect of singular and non-singular kernel fractional order derivative on the performance of fractional order PID controller have been studied in this work and it was found that the performance of non-singular definition was better as compared with the other definition. Volterra based control loop design with the tracking of reference signal showed the smooth control action. In the real time embedded system the IOPID and FOPID implemented successfully.

Declaration of interests

"The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper".

Statement of Ethics

"The authors have no ethical conflicts to disclose"

Disclosure Statement

"The authors have no conflict of interest to declare"

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