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A new class of local function on Nano Ideal Topological Spaces

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Abstract

The intention of this paper is to introduce a new class of local function namely, $ns_{\alpha}g-$ local function in the $n\mathcal{J}-$ topological spaces. We investigate certain properties and characteristics of $ns_{\alpha}g-$ open set and $ns_{\alpha}g-$ local function in $n-$ topological space and $n\mathcal{J}-$topological space respectively. Furthermore, we construct a new class of topology and discuss certain characteristics.

Keywords: nano ideal topological space, $ns_{\alpha}g-$ closure, $ns_{\alpha}g-$ interior, $ns_{\alpha}g-$ local function, $\mathcal{N}^{s_{\alpha}g^{\star}}-$ topology

1.INTRODUCTION

Hamlett and Jankovic[9] have considered the local function in ideal topological space and they have obtained a new topology. Parimala et.al[5] introduced a notion of nano ideal topological space by introducing a similar type of local function, namely nano local function. Pasunkilipandian et.al [10] introduced a new class of nano generalized closed set in $n-$ topological space namely, $ns_{\alpha}g-$ closed set. In this paper, we introduce the notion of $ns_{\alpha}g-$ local function using $ns_{\alpha}g-$ open sets and study its properties in nano ideal topological space. Further, we construct a topology $\mathcal{N}^{s_{\alpha}g^{\star}}$ for Γ using $ns_{\alpha}g-$ open set and an ideal $\mathcal{J}$ on Γ.

2.PRELIMINARIES

We recall the following definitions, which will be used in sequel.

Definition 2.1[2] Let Γ be a nonempty finite set of objects called the universe and $\mathcal{R}$ be an equivalence relation on Γ named as indiscernibility relation. Then Γ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\Gamma,\mathcal{R})$ is said to be an approximation space. Let $\mathcal{X}\subseteq\Gamma$. Then,

(i)The lower approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be for certain classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and is denoted by $L_{\mathcal{R}}(\mathcal{X})$. That is, $L_{\mathcal{R}}(\mathcal{X})=\cup_{x\in\Gamma}\{\mathcal{R}(\mathcal{X})\subseteq X: x\in\Gamma\}$ where $\mathcal{R}(\mathcal{X})$ denotes the equivalence class determined by $x\in\Gamma$.

(ii)The upper approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be possibly classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and is denoted by $U_{\mathcal{R}}(\mathcal{X})$. That is, $U_{\mathcal{R}}(\mathcal{X}) = \cup_{x\in\Gamma} \{\mathcal{R}(\mathcal{X}): \mathcal{R}(\mathcal{X}) \cap X \neq \emptyset, x \in \Gamma\}$ where $\mathcal{R}(\mathcal{X})$ denotes the equivalence class determined by $x \in \Gamma$.

(iii) The boundary region of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be classified neither as $\mathcal{X}$ nor as not $-\mathcal{X}$ with respect to $\mathcal{R}$ and is denoted by $B_{\mathcal{R}}(\mathcal{X})$. That is, $B_{\mathcal{R}}(\mathcal{X}) = U_{\mathcal{R}}(\mathcal{X}) - L_{\mathcal{R}}(\mathcal{X})$.

Definition 2.2 [2] Let Γ be a universe, $\mathcal{R}$ be an equivalence relation on Γ and $\mathcal{N}_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U}, \emptyset, L_{\mathcal{R}}(\mathcal{X}), U_{\mathcal{R}}(\mathcal{X}), B_{\mathcal{R}}(\mathcal{X})\}$, where $X \subseteq \Gamma$, satisfies the following axioms:

(i) $\mathcal{U}, \emptyset \in \mathcal{N}_{\mathcal{R}}(\mathcal{X})$.

(ii) The union of the elements of any sub-collection of $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ is in $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$.

(iii) The intersection of the elements of any finite subcollection of $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ is in $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$.

Therefore, $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ is a topology on Γ called the nano topology on Γ with respect to $\mathcal{X}$. We call $(\Gamma, \mathcal{N}_{\mathcal{R}}(\mathcal{X}))$ as the nano topological space. The elements of $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ are called nano open sets (briefly, n- open sets). The complement of a nano open set is called a nano closed set (briefly, $n -$ closed set).

Definition 2.3 [6] A subset $\mathcal{C}$ of a nano topological space $(\Gamma, \mathcal{N})$ is said to be nano semi $\alpha -$ open set (briefly, $NS_\alpha -$ O.S) if there exists a $n\alpha -$ open set $\mathcal{P}$ in Γ such that $\mathcal{P} \subseteq \mathcal{C} \subseteq n - cl(\mathcal{P})$ or equivalently if $\mathcal{C} \subseteq n - cl(n\alpha - int(\mathcal{P}))$.The family of all $NS_\alpha -$ O.S of $\mathcal{U}$ is denoted by $NS_\alpha O(\mathcal{U}, \mathcal{M})$.

Definition 2.4 [7] A subset $\mathcal{H}$ of a nano ideal topological space $(\Gamma, \mathcal{N}, \mathcal{I})$ is said to be nano ideal semi $\alpha -$ generalized closed set (briefly, $nIs_\alpha g -$ closed set) if $\mathcal{H}_n^* \subseteq \mathcal{K}$ whenever $\mathcal{H} \subseteq \mathcal{K}$ and $\mathcal{K}$ is nano semi $\alpha -$ open.

Definition 2.5 [8] Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be an ideal topological space and $x \in A \subseteq X$. Then $\mathcal{X}$ is said to be an $\mathcal{I}\alpha -$ interior point of A if A contain an $\mathcal{I}\alpha -$ open neighborhood set for $\mathcal{X}$. The set of all $\mathcal{I}\alpha -$ interior points of A is called $\mathcal{I}\alpha -$ interior set and simply is denoted by $\mathcal{I}\alpha - int(A)$.

Definition 2.6 [5]Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a nano ideal topological space with an ideal $\mathcal{I}$ on Γ and $(.)^*$ be a set operator from 2^Γ to 2^Γ (2^Γ is the set of all subsets of Γ). For a subset $\mathcal{A} \subset \Gamma$, $\mathcal{A}_n^*(\mathcal{I}, \mathcal{N}) = \{x \in \Gamma: \mathcal{G}_n \cap \mathcal{A} \notin \mathcal{I}$, for every $\mathcal{G}_n \in \mathcal{G}_n(x)\}$, where $\mathcal{G}_n(x) = \{\mathcal{G}_n: x \in \mathcal{G}_n, \mathcal{G}_n \in \mathcal{N}\}$ is a family of nano open sets which contain x, is called the nano local function (briefly, $n -$ local function) of $\mathcal{A}$ with respect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write $\mathcal{H}_n^*$ for $\mathcal{H}_n^*(\mathcal{I}, \mathcal{N})$.

Definition 2.7 [5]A subset $\mathcal{H}$ of a nano ideal topological space $(\Gamma, \mathcal{N}, \mathcal{I})$ is $n^* -$ dense in itself (resp.$n^* -$ perfect and $n^* -$ closed) if $\mathcal{H} \subseteq \mathcal{H}_n^*$ (resp.$\mathcal{H} = \mathcal{H}_n^*$ and $\mathcal{H}_n^* \subseteq \mathcal{H}$).

Definition 2.8 [5]A basis $\beta(\mathcal{I}, \mathcal{N})$ for $\mathcal{N}^*$ can be described as follows: $\beta(\mathcal{I}, \mathcal{N}) = \{A - B: A \in \mathcal{N}, B \in \mathcal{I}\}$.

Definition 2.9 [5] $\mathcal{N}^*(\mathcal{I}, \mathcal{N}) = \{V \subset \Gamma: n - cl^*(\Gamma - V) = \Gamma - V\}$ is called nano*$-$ topology which is finer than $\mathcal{N}$. The elements of $\mathcal{N}^*(\mathcal{I}, \mathcal{N})$ are called nano$*$ $-$open (briefly, $n * -$ open) and the complement of an $n * -$ open set is called nano$*$ $-$ closed (briefly, $n * -$ closed).

3. $ns_\alpha g -$ closure and $ns_\alpha g -$ interior

Definition 3.1. For every set $\mathcal{H} \subseteq (\Gamma, \mathcal{N})$,

(i) $ns_\alpha g -$ closure of $\mathcal{H}$ is defined as the intersection of all $ns_\alpha g -$ closed sets containing $\mathcal{H} (i.e.,) ns_\alpha g - cl(\mathcal{H}) = \cap \{\mathcal{F}: \mathcal{H} \subseteq \mathcal{F}, \mathcal{F} \in ns_\alpha g -$ closed set$\}$.

(ii) $ns_\alpha g -$ interior of $\mathcal{H}$ is defined as the union of all $ns_\alpha g -$ open sets contained in $\mathcal{H}$ (i.e.,) $ns_\alpha g - int(\mathcal{H}) = \cup \{\mathcal{K}: \mathcal{K} \subseteq \mathcal{H}, \mathcal{K} \in ns_\alpha g -$ open set$\}$.

Lemma 3.2 If $ns_\alpha g - cl(\Gamma, \mathcal{N})$ is closed under finite union, then $ns_\alpha g -$ closure is a Kuratowski operator on $(\Gamma, \mathcal{N}, \mathcal{I})$.

Proof: (i) $ns_\alpha g - cl(\emptyset) = \emptyset$ and $ns_\alpha g - cl(\Gamma) =$. Also, $\mathcal{H} \subseteq ns_\alpha g - cl(\mathcal{H})$.

(ii) Suppose that $\mathcal{H}$ and $\mathcal{K}$ are two subsets of Γ, then $ns_{\alpha}g-cl(\mathcal{H})\subseteq ns_{\alpha}g-cl(\mathcal{H}\cup\mathcal{K})$ and $ns_{\alpha}g-cl(\mathcal{K})\subseteq ns_{\alpha}g-cl(\mathcal{H}\cup\mathcal{K})$. Hence, $ns_{\alpha}g-cl(\mathcal{H})\cup ns_{\alpha}g-cl(\mathcal{K})\subseteq ns_{\alpha}g-cl(\mathcal{H}\cup\mathcal{K})$. If $\gamma\notin ns_{\alpha}g-cl(\mathcal{H})\cup ns_{\alpha}g-cl(\mathcal{K})$, then there exists $\mathcal{A},\mathcal{B}\in ns_{\alpha}g-cl(\Gamma,\mathcal{N},\mathcal{I})$ such that $\mathcal{H}\subseteq\mathcal{A},\gamma\notin\mathcal{A},\mathcal{K}\subseteq\mathcal{B}$ and $\gamma\notin\mathcal{B}$. Hence, $\mathcal{H}\cup\mathcal{K}\subseteq\mathcal{A}\cup\mathcal{B}$ and $\gamma\notin\mathcal{A}\cup\mathcal{B}$. By hypothesis, $\mathcal{A}\cup\mathcal{B}$ is $ns_{\alpha}g-$ closed. Thus, $\gamma\notin ns_{\alpha}g-cl(\mathcal{A}\cup\mathcal{B})$ implies that $\gamma\notin ns_{\alpha}g-cl(\mathcal{H}\cup\mathcal{K})$. Hence, $ns_{\alpha}g-cl(\mathcal{H}\cup\mathcal{K})\subseteq ns_{\alpha}g-cl(\mathcal{H})\cup ns_{\alpha}g-cl(\mathcal{K})$. Thus, $ns_{\alpha}g-cl(\mathcal{H}\cup\mathcal{K})=ns_{\alpha}g-cl(\mathcal{H})\cup ns_{\alpha}g-cl(\mathcal{K})$.

(iii) Let $\mathcal{H}\subseteq\Gamma$ and $\mathcal{F}$ be a $ns_{\alpha}g-$ closed set containing $\mathcal{H}$. Then by Definition 3.1.(i), $ns_{\alpha}g-cl(\mathcal{H})\subseteq\mathcal{F}$ and $ns_{\alpha}g-cl\big(ns_{\alpha}g-cl(\mathcal{H})\big)\subseteq\mathcal{F}$. Since $ns_{\alpha}g-cl\big(ns_{\alpha}g-cl(\mathcal{H})\big)\subseteq\mathcal{F}$, $ns_{\alpha}g-cl\big(ns_{\alpha}g-cl(\mathcal{H})\big)\subseteq\cap\{\mathcal{F}:\mathcal{H}\subseteq\mathcal{F},\mathcal{F}\in ns_{\alpha}g-cl(\Gamma,\mathcal{N},\mathcal{I})\}$. Hence, $\big(ns_{\alpha}g-cl(\mathcal{H})\big)\subset ns_{\alpha}g-cl\big(ns_{\alpha}g-cl(\mathcal{H})\big)$ and by Definition 3.1.(i), $ns_{\alpha}g-cl\big(ns_{\alpha}g-cl(\mathcal{H})\big)\subset\big(ns_{\alpha}g-cl(\mathcal{H})\big)$ implies that $\big(ns_{\alpha}g-cl(\mathcal{H})\big)=ns_{\alpha}g-cl\big(ns_{\alpha}g-cl(\mathcal{H})\big)$. Thus, $ns_{\alpha}g-$ closure is a Kuratowski closure operator on Γ.

Theorem 3.3 Let $\mathcal{H}$ be a subset of the $n\mathcal{I}-$ topological space $(\Gamma,\mathcal{N})$. Then $\gamma\in ns_{\alpha}g-cl(\mathcal{H})$ if and only if $\mathcal{H}\cap\mathcal{G}\neq\emptyset$ for every $ns_{\alpha}g-$ open set $\mathcal{G}$ containing γ.

Proof: Suppose that $\gamma\in ns_{\alpha}g-cl(\mathcal{H})$.Let $\mathcal{G}$ be a $ns_{\alpha}g-$ open set containing γ such that $\mathcal{H}\cap\mathcal{G}=\emptyset$ so that $\mathcal{H}\subset\mathcal{G}^c$. But $\mathcal{G}^c$ is a $ns_{\alpha}g-$ closed set and $ns_{\alpha}g-cl(\mathcal{H})\subset\mathcal{G}^c$. Since $\gamma\notin\mathcal{G}^c$, $\gamma\notin ns_{\alpha}g-cl(\mathcal{H})$ which is contrary to the hypothesis. Hence, $\mathcal{G}\cap\mathcal{H}\neq\emptyset$ for every $ns_{\alpha}g-$ open set $\mathcal{G}$ containing γ. Conversely, suppose that every $ns_{\alpha}g-$ open set of Γ containing γ such that $\mathcal{H}\cap\mathcal{G}\neq\emptyset$. If $\gamma\notin ns_{\alpha}g-cl(\mathcal{H})$, then there exists a $ns_{\alpha}g-$ closed set $\mathcal{F}$ of Γ such that $\mathcal{H}\subset\mathcal{F}$ and $\gamma\notin\mathcal{F}$. Hence, $\gamma\in\mathcal{F}^c$ and $\mathcal{F}^c$ is an $ns_{\alpha}g-$ open set containing γ. Thus, $\mathcal{F}^c\cap\mathcal{H}=\emptyset$ which is a contradiction. Hence, $\gamma\in ns_{\alpha}-cl(\mathcal{H})$.

Theorem 3.4 Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a $n\mathcal{I}-$ topological space and $\mathcal{H}$ be a subset of Γ, then the following axioms are satisfied.

(i) $\big(ns_{\alpha}g-int(\mathcal{H})\big)^c=ns_{\alpha}g-cl(\mathcal{H}^c)$.

(ii) $ns_{\alpha}g-int(\mathcal{H})=\big(ns_{\alpha}g-cl(\mathcal{H}^c)\big)^c$.

(iii) $ns_{\alpha}g-cl(\mathcal{H})=\big(ns_{\alpha}g-int(\mathcal{H}^c)\big)^c$.

Proof: (i) Let $\gamma\in\big(ns_{\alpha}g-int(\mathcal{H})\big)^c$. Then $\gamma\notin ns_{\alpha}g-int(\mathcal{H})$. That is, every $ns_{\alpha}g-$ open set $\mathcal{F}$ containing γ is such that $\mathcal{F}\nsubseteq\mathcal{H}$. That is, every $ns_{\alpha}g-$ open set $\mathcal{F}$ containing γ is such that $\mathcal{F}\cap\mathcal{H}^c\neq\emptyset$. Since $\gamma\in ns_{\alpha}g-cl(\mathcal{H}^c)$, $\big(nIs_{\alpha}g-int(\mathcal{H})\big)^c\subseteq ns_{\alpha}g-cl(\mathcal{H}^c)$. Conversely, let $\gamma\in ns_{\alpha}g-cl(\mathcal{H})^c$. Then, every $ns_{\alpha}g-$ open set $\mathcal{F}$ containing γ is such that $\mathcal{F}\cap\mathcal{H}^c\neq\emptyset$. That is, every $ns_{\alpha}g-$ open set $\mathcal{F}$ containing γ is such that $\mathcal{F}\nsubseteq\mathcal{H}$. By the Definition 3.1. (ii), $\gamma\notin ns_{\alpha}g-int(\mathcal{H})$. That is, $\gamma\in\big(ns_{\alpha}g-int(\mathcal{H})\big)^c$ so that $ns_{\alpha}g-cl(\mathcal{H}^c)\subset\big(ns_{\alpha}g-int(\mathcal{H})\big)^c$. Thus, $\big(ns_{\alpha}g-int(\mathcal{H})\big)^c=ns_{\alpha}g-cl(\mathcal{H}^c)$.

(ii) Follows by taking complements in (i).

(iii) Follows by replacing $\mathcal{H}$ by $\mathcal{H}^c$ in (i).

4. $ns_{\alpha}g-$ local function

Definition 4.1 Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a $n\mathcal{I}-$ topological space and $(.)^{s_{\alpha}g^*}$ be a set operator from 2^{Γ} to 2^{Γ}, where 2^{Γ} is the set of all subsets of Γ. For a subset $\mathcal{H}\subset\Gamma$, $\mathcal{H}^*_{ns_{\alpha}g}(\mathcal{I},\mathcal{N})=\{x\in\Gamma:\mathcal{G}_n\cap\mathcal{H}\notin\mathcal{I}$, for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)\}$ is called the nano semi α generalized local function (briefly, $ns_\alpha g-$ local function) of $\mathcal{H}$ with respect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write $\mathcal{H}^*_{ns_\alpha g}$ instead of $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I},\mathcal{N})$.

Remark 4.2 Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a $n\mathcal{I}-$ topological space. The minimal ideal is $\{\emptyset\}$ for any $n\mathcal{I}-$ topological space $(\Gamma,\mathcal{N},\mathcal{I})$ and the maximal ideal is 2^Γ.

Remark 4.3 For every $\mathcal{H} \subseteq \Gamma$,

(i) If $\mathcal{I}=\{\emptyset\}$, then $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H}) \neq n - cl(\mathcal{H})$.

(ii) If $\mathcal{I}=2^\Gamma$, then $\mathcal{H}^*_{ns_\alpha g} = \emptyset$.

(iii) If $\mathcal{H} \in \mathcal{I}$, then $\mathcal{H}^*_{ns_\alpha g} = \emptyset$.

(iv) Neither $\mathcal{H} \subset \mathcal{H}^*_{ns_\alpha g}$ nor $\mathcal{H}^*_{ns_\alpha g} \subset \mathcal{H}$, in general.

Proposition 4.4 Let $(\Gamma,\mathcal{N})$ be a nano topological space with ideals $\mathcal{I},\mathcal{I}'$ on Γ and $\mathcal{H},\mathcal{K}$ be subsets of Γ. Then,

(i) $(\emptyset)^*_{ns_\alpha g} = \emptyset$.

(ii) $\mathcal{H} \subseteq \mathcal{K} \backslash implies \mathcal{H}^*_{ns_\alpha g} \subseteq \mathcal{K}^*_{ns_\alpha g}$.

(iii) $\mathcal{I} \subseteq \mathcal{I}' \Longrightarrow \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}') \subseteq \mathcal{H}^*_{ns_\alpha g}(\mathcal{I})$.

(iv) $\mathcal{H}^*_{ns_\alpha g} \subseteq ns_\alpha g - cl(\mathcal{H})$.

(v) $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) \subseteq ns_\alpha g - cl(\mathcal{H})$, $(\mathcal{H}^*_{ns_\alpha g} *$ is a \$ns_\alpha g-\$ closed subset of $\mathcal{H}$).

(vi) $\left(\mathcal{H}^*_{ns_\alpha g}\right)^*_{ns_\alpha g} \subseteq \mathcal{H}^*_{ns_\alpha g}$.

(vii) $\mathcal{H}^*_{ns_\alpha g} \cup \mathcal{K}^*_{ns_\alpha g} = (\mathcal{H} \cup \mathcal{K})^*_{ns_\alpha g}$.

(viii) $\mathcal{H}^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} = (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} \subseteq (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g}$.

(ix) $\mathcal{I} \in \mathcal{I} \backslash implies (\mathcal{H} \setminus \mathcal{I})^*_{ns_\alpha g} \subseteq \mathcal{H}^*_{ns_\alpha g} = (\mathcal{H} \cup \mathcal{I})^*_{ns_\alpha g}$.

Proof: (i) The proof is trivial.

(ii) Let $\mathcal{H} \subset \mathcal{K}$ and $\gamma \in \mathcal{H}^*_{ns_\alpha g}$. Assume that $\gamma \notin \mathcal{K}^*_{ns_\alpha g}$. We have $\mathcal{G}_n \cap \mathcal{K} \in \mathcal{I}$ for some $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$. Since $\mathcal{G}_n \cap \mathcal{H} \subseteq \mathcal{G}_n \cap \mathcal{K}$ and $\mathcal{G}_n \cap \mathcal{K} \in \mathcal{I}$, we obtain $\mathcal{G}_n \cap \mathcal{H} \in \mathcal{I}$.Thus, $\gamma \notin \mathcal{H}^*_{ns_\alpha g}$ which is a contradiction. Clearly, $\mathcal{H}^*_{ns_\alpha g} \subseteq \mathcal{K}^*_{ns_\alpha g}$.

(iii) Let $\mathcal{I} \subseteq \mathcal{I}'$ and $\gamma \in \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}')$. Then, $\mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}'$ for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$. By hypothesis, $\mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}$. So $\gamma \in \mathcal{H}^*_{ns_\alpha g}(\mathcal{I})$.Therefore, $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I}') \subseteq \mathcal{H}^*_{ns_\alpha g}(\mathcal{I})$.

(iv) Let $\gamma \in \mathcal{H}^*_{ns_\alpha g}$. Then by Definition 4.1, for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma), \mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}$ which implies $\mathcal{G}_n \cap \mathcal{H} = \emptyset$. Hence, $\gamma \in ns_\alpha g - cl(\mathcal{H})$, therefore, $\mathcal{H}^*_{ns_\alpha g} \subseteq ns_\alpha g - cl(\mathcal{H})$.

(v) $\mathcal{H}^*_{ns_\alpha g} \subseteq ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g})$ hold in general. Let $\gamma \in ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g})$. Then $\mathcal{H}^*_{ns_\alpha g} \cap \mathcal{G}_n \neq \emptyset$ for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$. Therefore, there exists some $\lambda \in \mathcal{H}^*_{ns_\alpha g} \cap \mathcal{G}_n$ and $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$. Since $\lambda \in \mathcal{H}^*_{ns_\alpha g}$, $\mathcal{H} \cap \mathcal{G}_n \notin \mathcal{I}$ and hence $\gamma \in \mathcal{H}^*_{ns_\alpha g}$. Thus, $ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) \subseteq \mathcal{H}^*_{ns_\alpha g}$. Now, $ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) = \mathcal{H}^*_{ns_\alpha g}$. Then $\mathcal{H} \cap \mathcal{G}_n \notin \mathcal{I}$ for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$. This implies that $\mathcal{H} \cap \mathcal{G}_n \neq \emptyset$ for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$ and so $\gamma \in ns_\alpha g - cl(\mathcal{H})$. Consequently, $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) \subseteq ns_\alpha g - cl(\mathcal{H})$ and $\mathcal{H}^*_{ns_\alpha g}$ is a $ns_\alpha g -$ closed.

(vi) From (v), $\left(\mathcal{H}^*_{ns_\alpha g}\right)^*_{ns_\alpha g} \subseteq ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) = \mathcal{H}^*_{ns_\alpha g}$, since $\mathcal{H}^*_{ns_\alpha g}$ is a $ns_\alpha g -$ closed set.

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(vii) $\mathcal{H} \subset \mathcal{H} \cup \mathcal{K}$ and $\mathcal{K} \subset \mathcal{H} \cup \mathcal{K}$ and $\mathcal{H}^*_{ns_\alpha g} \cup \mathcal{K}^*_{ns_\alpha g} \subseteq (\mathcal{H} \cup \mathcal{K})^*_{ns_\alpha g}$ by (ii). Conversely, let $\gamma \in (\mathcal{H} \cup \mathcal{K})^*_{ns_\alpha g}$. Then for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$, $\mathcal{G}_n \cap (\mathcal{H} \cup \mathcal{K}) \notin \mathcal{I}$ implies that $(\mathcal{G}_n \cap \mathcal{H}) \cup (\mathcal{G}_n \cap \mathcal{K}) \notin \mathcal{I}$. Therefore, $\mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}$ and $\mathcal{G}_n \cap \mathcal{K} \notin \mathcal{I}$. This implies that $\gamma \in \mathcal{H}^*_{ns_\alpha g}$ and $\gamma \in \mathcal{K}^*_{ns_\alpha g}$, that is, $\gamma \in \mathcal{H}^*_{ns_\alpha g} \cup \mathcal{K}^*_{ns_\alpha g}$. So we obtain the equality.

(viii) Clearly, $(\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} \subseteq (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g}$. Since $\mathcal{H} \setminus \mathcal{K} \subseteq \mathcal{H}$, by (ii), $(\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \subseteq \mathcal{H}^*_{ns_\alpha g}$ and hence $(\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} \subseteq \mathcal{H}^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g}$. Conversely, $\mathcal{H} \subseteq (\mathcal{H} \setminus \mathcal{K}) \cup \mathcal{K}$, by (vii) $\mathcal{H}^*_{ns_\alpha g} \subseteq (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \cup \mathcal{K}^*_{ns_\alpha g}$ and hence $\mathcal{H}^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} \subseteq \left((\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \cup \mathcal{K}^*_{ns_\alpha g}\right) \setminus \mathcal{K}^*_{ns_\alpha g}$. Therefore, $\mathcal{H}^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} \subseteq (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \setminus \left(\mathcal{K}^*_{ns_\alpha g} \cup \mathcal{K}^*_{ns_\alpha g}\right)$ so that $\mathcal{H}^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} \subset (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g}$. Therefore, $\mathcal{H}^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g} = (\mathcal{H} \setminus \mathcal{K})^*_{ns_\alpha g} \setminus \mathcal{K}^*_{ns_\alpha g}$.

(ix) By (vii) and Remark 3.2 (iii), $(\mathcal{H} \cup \mathcal{I})^*_{ns_\alpha g} = \mathcal{H}^*_{ns_\alpha g} \cup \mathcal{I}^*_{ns_\alpha g} = \mathcal{H}^*_{ns_\alpha g} \cup \emptyset = \mathcal{H}^*_{ns_\alpha g}$. Since $\mathcal{H} \setminus \mathcal{I} \subset \mathcal{H}$ by (ii),$(\mathcal{H} \setminus \mathcal{I})^*_{ns_\alpha g} \subseteq \mathcal{H}^*_{ns_\alpha g}$. Hence, we get the result.

Remark 4.5 In general, the reverse implications of (ii),(iii) and (iv) of Proposition 4.4 does not hold.\\

For instance, consider the $n\mathcal{I}-$ topological spaces $(\Delta, \mathcal{N}, \mathcal{I}')$, $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ and $(\Delta_1, \mathcal{N}_1, \mathcal{I}'_1)$ as follows:

(a) $\Delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta/\mathcal{R} = \{\{\delta_1\}, \{\delta_2, \delta_3\}, \{\delta_4\}\}$; $\mathcal{X} = \{\delta_1, \delta_3\}$; $\mathcal{N} = \{\emptyset, \Delta, \{\delta_1\}, \{\delta_1, \delta_2, \delta_3\}, \{\delta_2, \delta_3\}\}$; $\mathcal{I}' = \{\emptyset, \{\delta_2\}, \{\delta_3\}, \{\delta_4\}, \{\delta_2, \delta_3\}, \{\delta_3, \delta_4\}, \{\delta_2, \delta_4\}, \{\delta_2, \delta_3, \delta_4\}\}$.

(b) $\Delta_1 = \{\delta_1, \delta_2, \delta_3\}$; $\Delta_1/\mathcal{R} = \{\{\delta_1, \delta_2\}, \{\delta_3\}\}$; $\mathcal{X} = \{\delta_2\}$; $\mathcal{N}_1 = \{\emptyset, \Delta_1, \{\delta_1, \delta_2\}\}$; $\mathcal{I}_1 = \emptyset, \{\delta_3\}\}$.

(c) $\Delta_1 = \{\delta_1, \delta_2, \delta_3\}$; $\Delta_1/\mathcal{R} = \{\{\delta_1, \delta_2\}, \{\delta_3\}\}$; $\mathcal{X} = \{\delta_2\}$; $\mathcal{N}_1 = \{\emptyset, \Delta_1, \{\delta_1, \delta_2\}\}$; $\mathcal{I}'_1 = \emptyset, \{\delta_2\}\}$.

(i) Let $\mathcal{H} = \{\delta_2, \delta_3\}$; $\mathcal{K} = \{\delta_2\}$; $\mathcal{H}^*_{ns_\alpha g} = \{\delta_3\} \subset \mathcal{K}^*_{ns_\alpha g} = \{\delta_1, \delta_3\}$, but $\mathcal{H} \not\subseteq \mathcal{K}$.

(ii) For $\mathcal{H} = \{\delta_1, \delta_2\}$, $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I}') = \Delta_1 \subset \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}) = \{\delta_1, \delta_3\}$ but $\mathcal{I} \not\subseteq \mathcal{I}'$.

(iii) For $\mathcal{H} = \{\delta_1, \delta_2\}$, $\mathcal{H}^*_{ns_\alpha g} = \{\delta_1, \delta_4\} = ns_\alpha g - cl\left(\mathcal{H}^*_{ns_\alpha g}\right)$; $ns_\alpha g - cl(\mathcal{H}) = \{\delta_1, \delta_2, \delta_4\}$. Here, $ns_\alpha g - cl(\mathcal{H}) /\subseteq \mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl\left(\mathcal{H}^*_{ns_\alpha g}\right)$.

Theorem 4.6 Let $(\Gamma, \mathcal{N})$ be a nano topological space with ideals $\mathcal{I}, \mathcal{I}'$ on Γ and let $\mathcal{H}$ be a subset of Γ. Then $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I} \cap \mathcal{I}') = \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}) \cup \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}')$.

Proof: Since $\mathcal{I} \cap \mathcal{I}' \subset \mathcal{I}$ and $\mathcal{I} \cap \mathcal{I}' \subseteq \mathcal{I}'$, by Proposition 4.4 (iii), $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I}) \subseteq \mathcal{H}^*_{ns_\alpha g}(\mathcal{I} \cap \mathcal{I}')$ and $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I}') \subseteq \mathcal{H}^*_{ns_\alpha g}(\mathcal{I} \cap \mathcal{I}')$.Hence, $\mathcal{H}^*_{ns_\alpha g}(\mathcal{I}) \cup \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}') \subseteq \mathcal{H}^*_{ns_\alpha g}(\mathcal{I} \cap \mathcal{I}')$. Conversely, let $\gamma \in \mathcal{H}^*_{ns_\alpha g}(\mathcal{I} \cap \mathcal{I}')$. Then for every $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma), \mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I} \cap \mathcal{I}'$ hence, $\mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}$ or $\mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}'$. This shows that $\gamma \in \mathcal{H}^*_{ns_\alpha g}(\mathcal{I})$ or $\gamma \in \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}')$. Therefore, $\gamma \in \mathcal{H}^*_{ns_\alpha g}(\mathcal{I}) \cup \mathcal{H}_{ns_\alpha g^*}(\mathcal{I}')$. Hence, we get the result.

Lemma 4.7 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. If $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$,then $\mathcal{G}_n \cap \mathcal{H}^*_{ns_\alpha g} = \mathcal{G}_n \cap (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g} \subseteq (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g}$ for any subset $\mathcal{H}$ of Γ.

Proof: Suppose that $\mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$ and $\gamma \in \mathcal{G}_n \cap \mathcal{H}^*_{ns_\alpha g}$. Then $\gamma \in \mathcal{G}$ and $\gamma \in \mathcal{H}^*_{ns_\alpha g}$. Let $\mathcal{F}$ be any $ns_\alpha g-$ open set containing γ. Then $\mathcal{F} \cap \mathcal{G}_n \in \mathcal{N}^{s_\alpha g}(\gamma)$ and $\mathcal{F} \cap (\mathcal{G}_n \cap \mathcal{H}) = (\mathcal{F} \cap \mathcal{G}_n) \cap \mathcal{H} \notin \mathcal{I}$, by Definition 4.1. This shows that $\gamma \in (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g}$ and hence we obtain $\mathcal{G}_n \cap \mathcal{H}^*_{ns_\alpha g} \subseteq (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g}$. Moreover, $\mathcal{G}_n \cap \mathcal{H}^*_{ns_\alpha g} \subseteq \mathcal{G}_n \cap (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g}$. Since $\mathcal{G}_n \cap \mathcal{H} \subseteq \mathcal{H}$ by Proposition 4.4 (iii), $(\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g} \subseteq \mathcal{H}^*_{ns_\alpha g}$ and $\mathcal{G}_n \cap (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g} \subseteq \mathcal{G}_n \cap \mathcal{H}^*_{ns_\alpha g}$. Therefore, $\mathcal{G}_n \cap \mathcal{H}^*_{ns_\alpha g} = \mathcal{G}_n \cap (\mathcal{G}_n \cap \mathcal{H})^*_{ns_\alpha g}$.

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Theorem 4.8 If $(\Gamma, \mathcal{N}, \mathcal{I})$ is a $n\mathcal{I}-$ topological space and $\mathcal{H} \subseteq \mathcal{H}^*_{ns_\alpha g}$, then $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) = ns_\alpha g - cl(\mathcal{H})$.

Proof: By Proposition 4.4 (iv), for every $\mathcal{H} \subseteq \Gamma$ we have $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) \subseteq ns_\alpha g - cl(\mathcal{H})$. From the hypothesis, $\mathcal{H} \subseteq \mathcal{H}^*_{ns_\alpha g}$ implies that $ns_\alpha g - cl(\mathcal{H}) \subseteq ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g})$ so that $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H}^*_{ns_\alpha g}) = ns_\alpha g - cl(\mathcal{H})$.

5. The open sets of $\mathcal{N}^{s_\alpha g*}$

Definition 5.1 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. The set operator $ns_\alpha g - cl^*$ is called a nano semi α generalized $*$ closure and is defined as $ns_\alpha g - cl^*(\mathcal{H}) = \mathcal{H} \cup \mathcal{H}^*_{ns_\alpha g}$ for $\mathcal{H} \subseteq \Gamma$.

Remark 5.2 (i) From Remark 4.2(i), if $\mathcal{I} = \{\emptyset\}$ then $\mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl(\mathcal{H})$. In this case, $ns_\alpha g - cl^*(\mathcal{H}) = ns_\alpha g - cl(\mathcal{H})$.

(ii) If $(\Gamma, \mathcal{N}, \mathcal{I})$ is a $n\mathcal{I}-$ topological space with $\mathcal{I} = \{\emptyset\}$, then $\mathcal{N}^{s_\alpha g*} = \mathcal{N}^{s_\alpha g}$.

Definition 5.3 $\mathcal{N}^{s_\alpha g*}(\mathcal{I}) = \{\mathcal{F} \subset \Gamma : ns_\alpha g - cl^*(\mathcal{F}^c) = \mathcal{F}^c\}$. $\mathcal{N}^{s_\alpha g*}(\mathcal{I}, \mathcal{N}^{s_\alpha g})$ is called nano semi α generalized $*-$ topology (briefly, $\mathcal{N}^{s_\alpha g*}-$ topology), which is finer than $\mathcal{N}^{s_\alpha g}$. We simply write $\mathcal{N}^{s_\alpha g*}$ instead of $\mathcal{N}^{s_\alpha g*}(\mathcal{I}, \mathcal{N}^{s_\alpha g})$. The elements of $\mathcal{N}^{s_\alpha g*}(\mathcal{I}, \mathcal{N}^{s_\alpha g})$ are called nano semi α generalized $*-$ open sets (briefly, $ns_\alpha g^*-$ open sets) and the complement of a $ns_\alpha g^*-$ open set is called $ns_\alpha g^*-$ closed set. Here, $ns_\alpha g - cl^*(\mathcal{H})$ and $ns_\alpha g - int^*(\mathcal{H})$ will denote the $ns_\alpha g-$ closure and $ns_\alpha g-$ interior of $\mathcal{H}$ in $(\Gamma, \mathcal{N}^{s_\alpha g*})$.

Remark 5.4 The topology $\mathcal{N}^{s_\alpha g*}$ is finer than $\mathcal{N}^{s_\alpha g}$.

Proposition 5.5 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space and $\mathcal{H} \subseteq \Gamma$. If $\mathcal{H} \subseteq \mathcal{H}^*_{ns_\alpha g}$, then

(i) $ns_\alpha g - cl(\mathcal{H}) = ns_\alpha g - cl^*(\mathcal{H})$.
(ii) $ns_\alpha g - int(\mathcal{H}^c) = ns_\alpha g - int^*(\mathcal{H}^c)$.

Proof: (i) The proof follows from Theorem 4.8.

(ii) If $\mathcal{H} \subseteq \mathcal{H}^*_{ns_\alpha g}$, then $ns_\alpha g - cl(\mathcal{H}) = ns_\alpha g - cl^*(\mathcal{H})$ by (i) so that $(ns_\alpha g - cl^*(\mathcal{H}))^c = (ns_\alpha g - cl^*(\mathcal{H}))^c$.Therefore, $ns_\alpha g - int(\mathcal{H}^c) = ns_\alpha g - int^*(\mathcal{H}^c)$.

Proposition 5.6 The set operator $ns_\alpha g - cl^*$ satisfies the following conditions:

(i) $\mathcal{H} \subseteq ns_\alpha g - cl^*(\mathcal{H})$.
(ii) $ns_\alpha g - cl^*(\emptyset) = \emptyset$ and $ns_\alpha g - cl^*(\Gamma) = \Gamma$.
(iii) If $\mathcal{H} \subseteq \mathcal{K}$, then $ns_\alpha g - cl^*(\mathcal{H}) \subseteq ns_\alpha g - cl^*(\mathcal{K})$.
(iv) $ns_\alpha g - cl^*(\mathcal{H}) \cup ns_\alpha g - cl^*(\mathcal{K}) = ns_\alpha g - cl^*(\mathcal{H} \cup \mathcal{K})$.
(v) $ns_\alpha g - cl^*(ns_\alpha g - cl^*(\mathcal{H})) = ns_\alpha g - cl^*(\mathcal{H})$.
(vi) $ns_\alpha g - cl^*(\mathcal{H} \cap \mathcal{K}) \subseteq ns_\alpha g - cl^*(\mathcal{H}) \cap ns_\alpha g - cl^*(\mathcal{K})$.
(vii) If $\mathcal{G}_n$ is $ns_\alpha g-$ open, then $\mathcal{G}_n \cap (ns_\alpha g - cl^*(\mathcal{H})) \subseteq ns_\alpha g - cl^*(\mathcal{G}_n \cap \mathcal{H})$.

Proof: (i) By Definition 3.1, we obtain $\mathcal{H} \subseteq \mathcal{H} \cup \mathcal{H}^*_{ns_\alpha g} = ns_\alpha g - cl^*(\mathcal{H})$.

(ii) $ns_\alpha g - cl^*(\emptyset) = (\emptyset)^*_{ns_\alpha g} \cup \emptyset = \emptyset$, by Proposition 4.4 and $ns_\alpha g - cl^*(\Gamma) = \Gamma \cup \Gamma^*_{ns_\alpha g} = \Gamma$.

(iii) Since $\mathcal{H} \subseteq \mathcal{K}$ and $\mathcal{H}^*_{ns_\alpha g} \subseteq \mathcal{K}^*_{ns_\alpha g}$ implies that $\mathcal{H} \cup \mathcal{H}^*_{ns_\alpha g} \subset \mathcal{K} \cup \mathcal{K}^*_{ns_\alpha g}$ so that $ns_\alpha g - cl^*(\mathcal{H}) \subseteq ns_\alpha g - cl^*(\mathcal{K})$.

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(iv) $ns_{\alpha}g-cl^*(\mathcal{H}\cup\mathcal{K})=(\mathcal{H}\cup\mathcal{K})\cup(\mathcal{H}\cup\mathcal{K})^*_{ns_{\alpha}g}=(\mathcal{H}\cup\mathcal{K})\cup\left(\mathcal{H}^*_{ns_{\alpha}g}\cup\mathcal{K}^*_{ns_{\alpha}g}\right)=\left(ns_{\alpha}g-cl^*(\mathcal{H})\right)\cup\left(ns_{\alpha}g-cl^*(\mathcal{K})\right)$.

(v) $ns_{\alpha}g-cl^*\left(ns_{\alpha}g-cl^*(\mathcal{H})\right)=ns_{\alpha}g-cl^*\left(\mathcal{H}\cup\mathcal{H}^*_{ns_{\alpha}g}\right)=\left(\mathcal{H}^*_{ns_{\alpha}g}\cup\mathcal{H}\right)\cup\left(\mathcal{H}^*_{ns_{\alpha}g}\cup\mathcal{H}\right)^*_{ns_{\alpha}g}=\left(\mathcal{H}^*_{ns_{\alpha}g}\cup\mathcal{H}\right)\cup\left(\left(\mathcal{H}^*_{ns_{\alpha}g}\right)^*_{ns_{\alpha}g}\cup\mathcal{H}^*_{ns_{\alpha}g}\right)=\mathcal{H}^*_{ns_{\alpha}g}\cup\mathcal{H}=ns_{\alpha}g-cl^*(\mathcal{H})$.

(vi) Let $\gamma\in ns_{\alpha}g-cl^*(\mathcal{H}\cap\mathcal{K})$. But $\mathcal{H}\cap\mathcal{K}\subseteq\mathcal{H}$ and $\mathcal{H}\cap\mathcal{K}\subseteq\mathcal{K}$. Therefore, $(\mathcal{H}\cap\mathcal{K})^*_{ns_{\alpha}g}\subseteq\mathcal{H}^*_{ns_{\alpha}g}$ and $(\mathcal{H}\cap\mathcal{K})^*_{ns_{\alpha}g}\subseteq\mathcal{K}^*_{ns_{\alpha}g}$. Thus, $ns_{\alpha}g-cl^*(\mathcal{H}\cap\mathcal{K})=(\mathcal{H}\cap\mathcal{K})\cup(\mathcal{H}\cap\mathcal{K})^*_{ns_{\alpha}g}\subseteq\mathcal{H}\cup\mathcal{H}^*_{ns_{\alpha}g}=ns_{\alpha}g-cl^*(\mathcal{H})$. Similarly, $ns_{\alpha}g-cl^*(\mathcal{H}\cap\mathcal{K})\subseteq ns_{\alpha}g-cl^*(\mathcal{K})$. Since $\gamma\in ns_{\alpha}g-cl^*(\mathcal{H}\cap\mathcal{K})$, $\gamma\in ns_{\alpha}g-cl^*(\mathcal{H})$ and $\gamma\in ns_{\alpha}g-cl^*(\mathcal{K})$. Thus, $\gamma\in ns_{\alpha}g-cl^*(\mathcal{H})\cap ns_{\alpha}g-cl^*(\mathcal{K})$. Hence, $ns_{\alpha}g-cl^*(\mathcal{H}\cap\mathcal{K})\subseteq ns_{\alpha}g-cl^*(\mathcal{H})\cap ns_{\alpha}g-cl^*(\mathcal{K})$.

(vii) Since $\mathcal{G}$ is $ns_{\alpha}g-$ open, by Lemma 4.7, we have $\mathcal{G}_n\cap\left(\mathcal{H}\cup\mathcal{H}^*_{ns_{\alpha}g}\right)=(\mathcal{G}_n\cap\mathcal{H})\cup\left(\mathcal{G}_n\cap\mathcal{H}^*_{ns_{\alpha}g}\right)\subseteq(\mathcal{G}_n\cap\mathcal{H})\cup(\mathcal{G}_n\cap\mathcal{H})^*_{ns_{\alpha}g}$. Therefore, $\mathcal{G}_n\cap\left(ns_{\alpha}g-cl^*(\mathcal{H})\right)\subseteq ns_{\alpha}g-cl^*(\mathcal{G}_n\cap\mathcal{H})$.

Definition 5.7 A subset $\mathcal{H}$ of a $n\mathcal{I}-$ topological space $(\Gamma,\mathcal{N},\mathcal{I})$ is $\mathcal{N}s_{\alpha}g^*-$ dense in itself (resp. $\mathcal{N}s_{\alpha}g^*-$ perfect and $\mathcal{N}s_{\alpha}g^*-$ closed) if $\mathcal{H}\subseteq\mathcal{H}^*_{ns_{\alpha}g}$ (resp. $\mathcal{H}=\mathcal{H}^*_{ns_{\alpha}g}$ and $\mathcal{H}^*_{ns_{\alpha}g}\subseteq\mathcal{H}$).

Remark 5.8 The relationship related to sets defined in Definition 5.7, we have the following diagram.

$$\mathcal{N}s_{\alpha}g^*-\text{dense in itself}\longleftarrow\mathcal{N}s_{\alpha}g^*-\textit{perfect}\longrightarrow\mathcal{N}s_{\alpha}g^*-\textit{closed}$$

Remark 5.9 The reverse implication of the above figure are not true.

For instance, consider the $n\mathcal{I}-$ topological space $(\Delta_2,\mathcal{N}_2,\mathcal{I}_2)$ as follows: $\Delta_2=\{\delta_1,\delta_2,\delta_3,\delta_4\}$; $\Delta_2/\mathcal{R}=\{\{\delta_1\},\{\delta_2,\delta_4\},\{\delta_3\}\}$; $\mathcal{X}=\{\delta_2,\delta_4\}$; $\mathcal{N}_2=\{\emptyset,\Delta_2,\{\delta_2,\delta_4\}\}$; $\mathcal{I}_2=\{\emptyset,\{\delta_2\}\}$. Here, the sets $\mathcal{H}=\{\delta_1,\delta_2,\delta_3\}$ is $\mathcal{N}s_{\alpha}g^*-$ closed and $\mathcal{H}=\{\delta_1\}$ is $\mathcal{N}s_{\alpha}g^*-$ dense-in-itself. But both are not $\mathcal{N}s_{\alpha}g^*-$ perfect.

Lemma 5.10 Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a $n\mathcal{I}-$ topological space and $\mathcal{H}\subseteq\Gamma$. If $\mathcal{H}$ is $ns_{\alpha}g^*-$ dense in itself, then $\mathcal{H}^*_{ns_{\alpha}g}=ns_{\alpha}g-cl\left(\mathcal{H}^*_{ns_{\alpha}g}\right)=ns_{\alpha}g-cl(\mathcal{H})=ns_{\alpha}g-cl^*(\mathcal{H})$.

Proof: Let $\mathcal{H}$ be a $ns_{\alpha}g^*-$ dense in itself. Then $\mathcal{H}\subseteq\mathcal{H}^*_{ns_{\alpha}g}$. From Proposition 5.6, we obtain $\mathcal{H}^*_{ns_{\alpha}g}=ns_{\alpha}g-cl\left(\mathcal{H}^*_{ns_{\alpha}g}\right)=ns_{\alpha}g-cl(\mathcal{H})=ns_{\alpha}g-cl^*(\mathcal{H})$. The result follows from Theorem 4.8 and Definition 5.1 .

Definition 5.11 A basis $\mathcal{B}(\mathcal{I},\mathcal{N})$ for $\mathcal{N}^{s_{\alpha}g^*}$ can be described as follows : A subset $\mathcal{H}$ of a $n\mathcal{I}-$ topological space $(\Gamma,\mathcal{N},\mathcal{I})$ is said to be $\mathcal{N}^{s_{\alpha}g^*}-$ closed if $\mathcal{H}^*_{ns_{\alpha}g}\subset\mathcal{H}$. Thus, we have $\mathcal{G}\in\mathcal{N}^{s_{\alpha}g^*}$ if and only if $\mathcal{G}^c$ is $\mathcal{N}^{s_{\alpha}g^*}-$ closed which implies $(\mathcal{G}^c)^*_{ns_{\alpha}g}\subset\mathcal{G}^c$ and hence $\mathcal{G}\subset\left((\mathcal{G}^c)^*_{ns_{\alpha}g}\right)^c$.Thus if $\gamma\in\mathcal{G}$, $\gamma\notin(\mathcal{G}^c)^*_{ns_{\alpha}g}$, then there exists $\mathcal{F}\in\mathcal{N}^{s_{\alpha}g}(\gamma)$ such that $\mathcal{F}\cap\mathcal{G}^c\in\mathcal{I}$. Hence, let $\mathcal{J}=\mathcal{F}\cap\mathcal{G}^c$ and we have $\gamma\in\mathcal{F}\setminus\mathcal{J}\subset\mathcal{G}$ where $\mathcal{F}\in\mathcal{N}^{s_{\alpha}g}(\gamma)$ and $\mathcal{J}\in\mathcal{I}$. So the basis for $\mathcal{N}^{s_{\alpha}g^*}$ is $\mathcal{B}(\mathcal{I},\mathcal{N})=\{\mathcal{F}\setminus\mathcal{J}:\mathcal{F}\in\mathcal{N}^{s_{\alpha}g}(\gamma),\mathcal{J}\in\mathcal{I}\}$.

Lemma 5.12 Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a $n\mathcal{I}-$ topological space. Then $\mathcal{B}(\mathcal{I},\mathcal{N})$ is a basis for $\mathcal{N}^{s_{\alpha}g^*}$.

Proof: Since $\emptyset\in\mathcal{I}$, $\mathcal{F}=\mathcal{F}\setminus\emptyset\in\mathcal{N}^{s_{\alpha}g}(\gamma)$ and $\mathcal{N}^{s_{\alpha}g}(\gamma)\subset\mathcal{G}$ from which it follows that $\Gamma=\cup\mathcal{B}$. Also, $\mathcal{B}_1,\mathcal{B}_2\in\mathcal{B}$ and $\mathcal{J},\mathcal{J}'\in\mathcal{I}$ we have $\mathcal{B}_1=\mathcal{F}_1\setminus\mathcal{J}$ and $\mathcal{B}_2=\mathcal{F}_2\setminus\mathcal{J}'$ where $\mathcal{F}_1,\mathcal{F}_2\in\mathcal{N}^{s_{\alpha}g}(\gamma)$.Then $\mathcal{B}_1\cap$

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$\mathcal{B}_2 = (\mathcal{F}_1 \setminus \mathcal{I}) \cap (\mathcal{F}_2 \setminus \mathcal{I}') = (\mathcal{F}_1 \cap \mathcal{I}^c) \cap (\mathcal{F}_2 \cap \mathcal{I}'^c) = (\mathcal{F}_1 \cap \mathcal{F}_2) \setminus (\mathcal{I} \cup \mathcal{I}') \in \mathcal{B}$ where $\mathcal{F}_1, \mathcal{F}_2 \in \mathcal{N}^{\delta\alpha\mathcal{g}}(\gamma)$, $\mathcal{I} \cup \mathcal{I}' \in \mathcal{I}$.

Remark 5.13 $\mathcal{B}(\mathcal{I}, \mathcal{N})$ is not a topology in general.

For instance, consider the $n\mathcal{I} -$ topological space $(\Delta, \mathcal{N}, \mathcal{I})$ as follows: $\Delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta/\mathcal{R} = \{\{\delta_1\}, \{\delta_2, \delta_3\}, \{\delta_4\}\}$; $\mathcal{X} = \{\delta_1, \delta_3\}$; $\mathcal{N} = \{\emptyset, \Delta, \{\delta_1\}, \{\delta_1, \delta_2, \delta_3\}, \{\delta_2, \delta_3\}\}$; $\mathcal{I} = \{\emptyset, \{\delta_1\}, \{\delta_3\}, \{\delta_4\},$

$\{\delta_1, \delta_3\}, \{\delta_3, \delta_4\}, \{\delta_1, \delta_4\}, \{\delta_1, \delta_3, \delta_4\}\}$. $\mathcal{B}(\mathcal{I}, \mathcal{N}) = \{\emptyset, \Delta, \{\delta_1\}, \{\delta_2\}, \{\delta_3\}, \{\delta_1, \delta_2\}, \{\delta_2, \delta_3\}, \{\delta_2, \delta_4\},$

$\{\delta_1, \delta_2, \delta_3\}, \{\delta_1, \delta_2, \delta_4\}, \{\delta_2, \delta_3, \delta_4\}\}$. Here, $\{\delta_1\} \cup \{\delta_3\} \notin \mathcal{B}(\mathcal{I}, \mathcal{N})$. Hence, it is not a topology.

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