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INVENTORY CONTROL MODEL FOR PHARMACEUTICAL ITEMS WITH EXPONENTIAL DEMAND AND HOLDING COST VARYING BASED ON TIME

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ABSTRACT:

Supply chain is the strategy of collaboration between suppliers, merchants, distributers, consumers etc. To fulfil the consumers' demand in existing uncertain business, significant data needs to be shared through the stock network. Association between merchants and consumers implementation is necessary for existing business situation. In supply chain inventory, consumers deal with merchant to purchase products for gaining more profit. The consumers may adopt the policy at regular basis and purchase goods from merchant as long-term agreements.

Importance of inventory control model cannot be ignored for pharmaceutical products such as drugs, healthcare product, surgical equipment's, medicines etc. Consumers who are in need of long terms medicine for the cure of diseases such as heart problems, vitamins and mineral deficiency diseases, liver problems and many more looks for a merchant who could offer him an optimum price. Consumers always look for a medicinal shop that can assure him the availability of the drug and can commit a timely delivery of the product too. And, hence because of these factors proper management of drug inventory is very essential and most demanding that can provide an optimum profit to both consumers and merchant.

The association between consumers and merchant creates greater commitment for quality maintenance. This integration process also develops effective and significant relation between the consumers and merchant across time. The following literatures are developed collaborative inventory model when supply chain considered merchant and consumers with numerous expectations for demand outline like price-dependent, time dependent demand, etc.

Keywords: Pharmaceutical Inventory, Drug Inventory Control, Deterministic Model, Deteriorating Items, Operation Research

1.1 INTRODUCTION:

Healthcare sectors are the most essential, demanding and challenging fields and requires continuous improvement, progression and integration of new age technology to provide optimum services to the consumers without compromising the quality of product and services. Everyday many peoples suffer due to mismanagement of drugs inventory in hospital and medicinal shops. During the present scenario when the world is going through the pandemic situation of COVID, we know how important is the proper and efficient management of drug inventory along with the timely availability of the drug. Yang and Wee (2001) derived the joint inventory model by considering deterioration of units' after they received inventory for multiple consumers and single merchant. Woo et al. (2001) considered supply chain inventory models when multiple consumers and single merchant adopted applications of information technologies and reduction efforts for ordering cost is to be updated and results in highest coordination and mechanization between associated business parties. Zavanella and Zanonia (2009) introduced an analytical model when single merchant and multiple consumers referred the industrial case for combined situation. Shah et al. (2011) derived joint inventory model under supply chain system when several consumers and single merchant considered quadratic demand. Ghiami and Williams (2015) carried two stages creation inventory analysis models with multiple consumers and when decaying items have a fixed production rate, the manufacturer dispatches order quantities to consumers for a set period of time, and the surplus inventory supplies for further deliveries.

In this paper we have considered exponential demand and derived supply chain inventory model when the system considers single merchant-multiple consumers and with deteriorating items for consumers and holding cost is varying based on time for the merchant and consumers.

1.2 NOTATIONS AND ASSUMPTIONS:

NOTATIONS

$D(t) = a_i e^{c_i t}$, where $a_i > 0, 0 < c_i < 1$

$I_{c_i}(t) =$ i^{th} buyer pharmaceutical inventory level at any given time t

$I_m(t) =$ i^{th} merchant pharmaceutical inventory level at any given time t

A_i =Pharmaceutical inventory ordering cost of i^{th} buyer for each order

A_m = Pharmaceutical inventory ordering cost of merchant each order

C_c = Pharmaceutical inventory purchase cost of i^{th} buyer per unit

θ_i = i^{th} consumers deterioration rates

x_{c_i} = i^{th}consumers fixed holding cost for pharmaceutical inventory

y_{c_i} = time-varying holding cost of i^{th} buyer

x_m = Merchant's fixed holding cost for pharmaceutical inventory

y_m =Merchant's varying holding cost for pharmaceutical inventory

p_i = i^{th}consumers selling price per unit for pharmaceutical inventory

n_i =Number of times demand ordered by i^{th} consumers throughout rotation time

N = Number of consumers

T_{P_c} =Total profits of consumers

T_{P_m} =Total profit of merchant

T_P = Integrated total profit for both merchant and consumers

$t_1 = \frac{m_1 T}{n_i}$

T = Decision variable of merchant's cycle time

ASSUMPTIONS:

Under the following assumptions the inventory models are developed:

1. Pharmaceutical inventory product is time dependent and function of exponential distribution.
2. One merchant with multiple consumers is measured.
3. Shortages for pharmaceutical drugs are not permitted.
4. Lead time of pharmaceutical product is not specified.
5. Deteriorated pharmaceutical products cannot be fixed and replaced during the cycle, and consumer inventory deterioration of pharmaceutical products is function of time.
6. Time varying holding cost is considered for consumers and merchant.

1.3 MATHEMATICAL MODEL AND ANALYSIS:

Following Figure-1.1 shown the inventory stock $I_b(t)$ of consumers at time t, where $t \geq 0$ and $t \leq \frac{T}{n}$.

Consumers Inventory of Pharmaceutical Products:

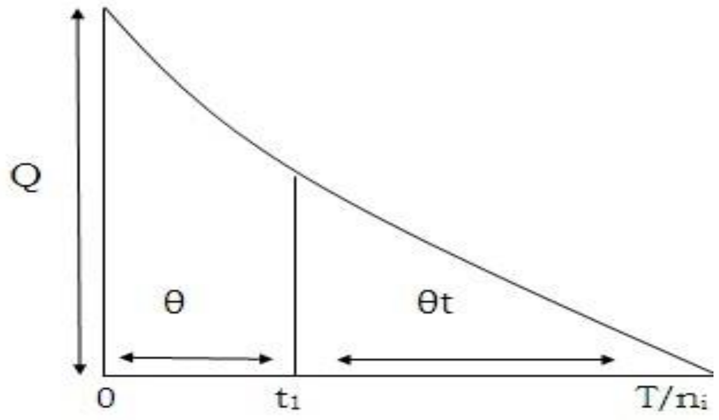


Figure 1.1

In the derived inventory model two situations are explained. The first situation does not consider the merchant and consumers collaboration, while the second situation considers merchant and consumers collaboration. Inventory level for both merchant and consumers are exhausted by exponential function demand. The differential equations of consumers and merchant's inventory are given by:

$$\frac{dI_{c_i}(t)}{dt} + \theta_i I_{c_1}(t) = -a_i e^{c_i t};\ 0 \le t \le \mathrm{t}_1 \qquad 1.1$$

$$\frac{dI_{c_i}(t)}{dt} + \theta_i t I_{c_1}(t) = -a_i e^{c_i t};\ t_1 \le t \le \frac{T}{n_i} \qquad 1.2$$

$$\frac{dI_m(t)}{dt} = -\sum a_i e^{c_i t}{}_{i=1}^{N}\ ;\ 0 \le t \le T \qquad 1.3$$

Where the boundary conditions are considered as:

$$I_{c_i}(0) = \mathrm{Q}, I_{c_i}\left(\frac{T}{n_i}\right) = 0,\ \text{and}\ \mathrm{I_m(T)} = 0$$

Their solutions are given by

$$I_{c_i}(t) = Q(1-\theta_i t) - a_i\left(t + \frac{c_i}{2}t^2 + \frac{\theta_i}{2}t^2 + \frac{c_i\theta_i}{3}t^3\right) + a_i\theta_i t\left(t + \frac{c_i}{2}t^2\right) \qquad 1.4$$

$$I_{c_i}(t) = a_i\left\{\begin{matrix}\left(\frac{T}{n_i} - t\right) + \frac{c_i}{2}\left(\frac{T^2}{n_i^2} - t^2\right) + \frac{\theta_i}{6}\left(\frac{T^3}{n_i^3} - t^3\right) \\ + \frac{c_i\theta_i}{8}\left(\frac{T^4}{n_i^4} - t^4\right) - \frac{\theta_i t^2}{2}\left(\frac{T}{n_i} - t\right) - \frac{c_1\theta_i t^2}{4}\left(\frac{T^2}{n_i^2} - t^2\right)\end{matrix}\right\} \qquad 1.5$$

$$I_{\mathrm{m}}(\mathrm{t}) = \sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{a_i}\left\{(\mathrm{T-t}) + \frac{\mathrm{c_i}}{2}(\mathrm{T^2 - t^2})\right\} \qquad 1.6$$

(By ignoring the higher power of θ_i)

Putting $t - t_1$ in equations (1.4) and (1.5) and simplifying, we get

$$Q = \frac{1}{(1-\theta_i t_1)}\left\{\begin{matrix} a_i\left(\frac{\theta_i}{2}t_1^2 + \frac{c_i\theta_i}{3}t_1^3\right) - a_i\theta_i t_1\left(t_1 + \frac{c_i}{2}t_1^2\right) \\ +a_i\left(\begin{matrix}\frac{T}{n_i} + \frac{c_i}{2}\frac{t^2}{n_i^2} + \frac{\theta_i}{6}\left(\frac{T^3}{n_i^3} - t_1^3\right) \\ + \frac{c_i\theta_i}{8}\left(\frac{T^4}{n_i^4} - t_1^4\right) - \frac{\theta_i}{2}t_1^2\left(\frac{T}{n_i} - t_1\right) \\ - \frac{c_i\theta_i}{4}t_1^2\left(\frac{T^2}{n_i^2} - t_1^2\right)\end{matrix}\right)\end{matrix}\right\} \qquad 1.7$$

Putting the value of Q in equation (1.4) we get

$$I_{c_i}(t)=\frac{(1-\theta_i t)}{(1-\theta_i t_1)}\begin{Bmatrix} a_i\left(\frac{\theta_i}{2}t_1^2+\frac{c_i\theta_i}{3}t_1^3\right)-a_i\theta_i t_1\left(t_1+\frac{c}{2}t_1^2\right) \\ +a_i\begin{pmatrix} \frac{T}{n_i}+\frac{c}{2}\frac{T^2}{n_i^2}+\frac{\theta_i}{6}\left(\frac{T^3}{n_i^3}-t_1^3\right) \\ +\frac{c_i\theta_i}{8}\left(\frac{T^4}{n_i^4}-t_1^4\right)-\frac{\theta_i}{2}t_1^2\left(\frac{T}{n_i}-t_1\right) \\ -\frac{c_i\theta_i}{4}t_1^2\left(\frac{T^2}{n_i^2}-t_1^2\right)\end{pmatrix}\end{Bmatrix} - a_i\left(t+\frac{c_i}{2}t^2+\frac{\theta_i}{2}t^2+\frac{c_i\theta_i}{3}t^3\right)+a_i\theta_i t\left(t+\frac{c_i}{2}t^2\right) \quad 1.8$$

1.4 CONSUMERS RELEVANT COST of PHARMACEUTICAL INVENTORY:

Holding Cost:

$$HC_c=\sum_{i=1}^{N} n_i\left\{x_{c_i}\left(\int_0^{t_1} I_{c_i(t)dt}+\int_{t_1}^{\frac{T}{n_i}} I_{c_i(t)dt}\right)+y_{c_i}\left(\int_0^{t_1} tI_{c_i(t)dt}+\int_{t_1}^{\frac{T}{n_i}} tI_{c_i(t)dt}\right)\right\} \quad 1.9$$

Deterioration Cost:

$$\mathrm{DC_c}=\sum_{i=1}^{N} n_i C_c\theta_i\left\{t_1 I_i(t)dt+\int_{t_1}^{\frac{T}{n_i}}\mathrm{t}\mathrm{I}_{c_i}(t)dt\right\} \quad 1.10$$

Ordering Cost:

$$OC_c=\sum n_i A_{i_{i=1}}^{N} \quad 1.11$$

Sales Revenue:

$$\begin{aligned}\mathrm{SR_c} &= \sum \mathrm{n_i p_i}_{\mathrm{i=1}}^{\mathrm{N}}\int \mathrm{D(t)dt}_{0}^{\frac{\mathrm{T}}{\mathrm{n_i}}} \\ &= \sum \mathrm{n_i p_i}_{\mathrm{i=1}}^{\mathrm{N}}\int a_{\mathrm{i}}\mathrm{e}^{\mathrm{c_i t}}\mathrm{dt}_{0}^{\frac{\mathrm{T}}{\mathrm{n}}}\end{aligned} \quad 1.12$$

(By neglecting higher exponents of $\mathrm{c_i}$ and θ_i)

Total profit for consumers: $\mathrm{TP_c}=\frac{1}{\mathrm{T}}[\mathrm{SR_c}-\mathrm{HC_c}-\mathrm{DC_c}-\mathrm{OC_c}]$ 1.13

1.5 MERCHANT'S RELEVANT COST of PHARMACEUTICAL INVENTORY:

Holding Cost:

$$\begin{aligned}HC_m &= x_m\left\{\int_0^T I_m(t)dt-\sum_{i=1}^{N} n_i\left(\int_0^{t_1} I_{c_i}(t)dt+\int_{t_1}^{\frac{T}{n_i}} I_{c_i}(t)dt\right)\right\} \\ &+ y_m\left\{\int_0^T tI_m(t)dt-\sum_{i=1}^{N} n_i\left(\int_0^{t_1} tI_{c_i}(t)dt+\int_{t_1}^{\frac{T}{n_i}} tI_{c_i}(t)dt\right)\right\}\end{aligned} \quad 1.14$$

Ordering Cost:

$$\mathrm{OC_m}=\mathrm{A_m} \quad 1.15$$

Sales Revenue:

$$SR_m = C_c \sum_{i=1}^{N} \int_0^T D(t)dt$$
$$= C_c \sum_{i=1}^{N} \int_0^T a_i e^{c_1 t} dt \qquad 1.16$$

(By neglecting higher exponents of c_i and θ_i)

Total Profit for merchant:

$$TP_m = \frac{1}{T}[SR_m - HC_m - OC_m] \qquad 1.17$$

1.6 WITHOUT COLLABORATION AND WITH COLLABORATION DECISION:

In this section, we discuss two situations: when buyer and merchant take decisions independently and jointly under supply chain strategy.

SITUATION-I: CONSUMERS WITH MERCHANT TAKE DECISIONS

WITHOUT ANY MUTUAL COLLABORATION

Here the consumers and merchant do decision independently.

Consumers maximum profit TP_bcan be determined by following conditions:

$$\frac{dTP_c}{dT_c} = 0, \text{ where } T_c = \frac{T}{n_i}, \qquad 1.18$$

provided it satisfies the condition

$$\frac{d^2TP_b}{dT_c^2} < 0 \qquad 1.19$$

This solution (n, T) maximizesTP_m.

Then the over-all revenue without association is specified by;

$$TP = \max(TP_c + TP_m) \qquad 1.20$$

SITUATION-II: CONSUMERS AND MERCHANT TAKE DECISIONS

WITH COLLABORATION

Here the consumers and the merchant jointly make decision: Moreover, the optimal value of T essentially content the following situations, which maximize over-all revenue (TP) when buyer and merchant take collaborative decision:

$$\frac{dTP}{dT} = 0, \text{ for T} \qquad 1.21$$

provided it satisfies the condition.

$$\frac{d^2TP}{dT^2} < 0 \qquad 1.22$$

Where over-all revenue (TP) with association is specified by;

$$TP = TP_c + TP_m \qquad 1.23$$

1.7 NUMERICAL EXAMPLE:

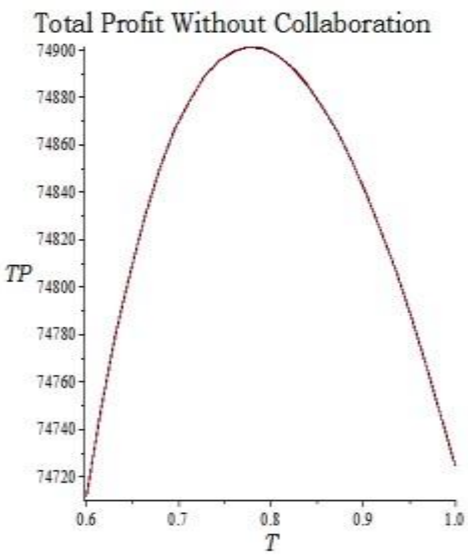
Numerical analysis considers the values of various parameters in appropriate units of a pharmaceutical inventories, $a_1 = 450$, $a_2 = 550, c_1 = 0.045, c_2 = 0.055, \theta_1 = 0.06, \theta_2 = 0.04, x_{c_1} = 9.5, y_{c_1} = 0.025, x_{c_2} = 10.5, y_{c_2} = 0.035, x_m = 8, y_m = 0.01, A_1 = 60, A_2 = 90, A_m = 1500, C_c = 35, p_1 = 43, p_2 = 47, m_1 = 0.4, N = 2$. The optimum numeric of T and revenues intended for consumer and mercantile are provided in Table-1.1. The optimal total profit $TP = \text{Rs.}\,74885.99$ at $n_1 = 4$ and $n_2 = 4$ for consumers' profit $TP_c^* = \text{Rs.}\,43556.17, T^* = 0.7238$ and $TP_m = \text{Rs.}\,31329.82$

when consumers and merchant take independent decision. Though after consumers and merchant plans combined verdict then the optimal overall profit $TP^* = \text{Rs.}\,75199.36$ at $n_1 = 2$ and $n_2 = 2$ and $T^* = 0.7057$ with consumers' profit $TP_c = \text{Rs.}\,43159.52$ and $TP_m = \text{Rs.}\,32039.84$.

The additional demand situations are also proved in mathematical expression (1.19) and mathematical expression (1.22). The graphical interpretation of the concaveness of the profits of self-governing and dual decisions are also given (Figures-1.2 and 1.3).

Table-1 The optimum solutions

	without association	with association
N	$n1 = 4$ $n2 = 4$	$n1 = 2$ $n2 = 2$
T	0.7238	0.7057
Consumers Profit	43556.17	43159.52
Merchant’s Profit	31329.82	32039.84
Total Profit	74885.99	75199.36



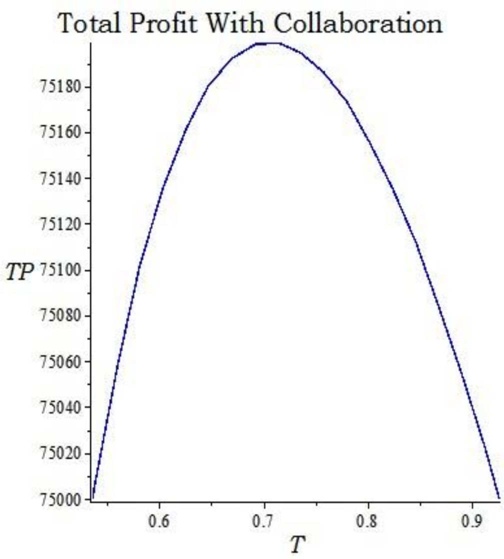


Figure-1.2 Figure-1.3

1.8 SENSITIVITY ANALYSIS:

Table-2 Sensitivity Analysis

%	Para - meters	without collaboration			With collaboration		
		TP_c	TP_m	TP	TP_c	TP_m	TP
20%	a_1, a_2	52199.50	37979.80	90179.30	51028.87	39504.49	90533.36
10%		47775.60	34647.90	82423.40	46654.69	36105.44	82760.13
-10%		38938.90	28008.70	66947.60	37924.75	29322.32	67247.06
-20%		34447.20	24807.60	59254.80	33571.03	25939.93	59510.95
20%	A_1, A_2	43200.10	31402.40	74602.50	42199.94	32736.08	74936.02
10%		43277.40	31362.90	74640.30	42242.90	32723.72	74966.70
-10%		43440.20	31297.60	74737.90	43012.70	32021.47	75034.20
-20%		43530.10	31256.00	74786.10	43066.40	32010.94	75077.30
20%	xc_1, xc_2	43182.00	31265.10	74447.20	42700.20	31945.54	74645.78
10%		43266.50	31297.70	74564.20	42827.00	31988.85	74815.82
-10%		43451.80	31373.40	74825.20	41917.80	33400.61	75318.40
-20%		43551.20	31428.20	74979.40	42074.20	33606.64	75680.85
20%	θ_1, θ_2	43333.50	31318.00	74651.50	42926.30	32020.32	74946.65
10%		43344.30	31321.00	74665.30	42943.00	32025.98	74969.01
-10%		43366.10	31326.50	74692.60	42305.40	32727.80	75033.20
-20%		43380.20	31327.90	74708.10	41801.50	33270.40	75071.95
20%	x_m	43358.20	30959.00	74317.30	41763.70	33219.91	74983.60
10%		43358.20	31179.40	74537.60	41766.00	33215.86	74981.89
-10%		43355.20	31491.40	74846.60	42932.40	32169.49	75101.90
-20%		43355.20	31770.90	75126.10	42881.20	32410.45	75291.69
20%	A_m	43355.20	30909.20	74264.30	42861.70	31719.89	74581.60
10%		43355.20	31116.50	74471.70	42910.40	31872.34	74782.70
-10%		43358.20	31557.50	74915.70	42380.80	32974.81	75228.60
-20%		43358.20	31803.40	75161.60	42478.30	32992.23	75470.60

Sensitive investigation is executing by altering the values of single parameter at the same time from given parameters $a_i, A_{c_i}, A_m, x_{c_i}, x_m,$ and θ_i respectively, and kept reaming parameters constant. From Table-1.2 we observed that total profits increase when merchant and consumers take decision with collaboration instead of without collaboration. When parameter a upsurges/diminutions then overall profit will growth/decline, though if $A_{c_i}, x_{c_i}, x_m, A_m,$ and θ_i rise/reduction then over-all revenue will reduction/rise in self-determining and combined conclusion.

1.9 CONCLUSION:

The mathematical model developed in this paper is very practical and applicable to the pharmaceutical inventory. In this paper demand rate and holding cost are being treated as varying based on time. Sensitivity analysis clearly give us the idea of the stability of this mathematical model with respect to the various parametres. In this research paper, two situations were explained. One with combined approach of seller and buyer and the other without any proper collaboration among them. In both the cases demand of inventory being kept exponentially. The mathematical model for various parameters of the inventory were formulated and solutions of the differential equation formed were derived. Various cost associated with this inventory model were evaluated and the computation of total profit for consumer and merchant were discussed. It has been observed that the total profit for seller and buyer increases significantly when they both do collaborative approach rather that independent decision taken by them. Sensitivity analysis shows that when decision with collaboration instead of without collaboration were consider, the total profit in terms of overall revenue increases significantly.

The outcome illustrate that the optimum rotation period is significantly reduced and over-all revenue significantly improved when consumers and merchant yield collaborative decision as compared to self-determining conclusion taken by consumers and merchant. We also detect that the merchant's revenue is improved and number of times order placed by buyer during cycle time is decreased when consumers and merchant take collaborative decision.

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