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EPQ Model with Mixture of Weibull Production Having Exponential Decay and Time Dependent Demand

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Abstract - This paper deals with economic production quantity(EPQ) model with production process characterized by two component mixture of Weibull distribution. Demand is considered to be a power function of time. Furthermore, it is assumed that the commodity's lifespan is random and follows an exponential distribution. Assuming that demand is a function of time, the production quantity can be derived. The optimal ordering policies and optimal production quantity are obtained. The production rate distribution parameters and the deterioration rate distribution parameters have a significant impact on the model's optimal production schedule, according to sensitivity analysis. This model is extended to the case of without shortages. This model can be used to analyze heterogeneous production systems.

Index Terms - Mixture of Weibull rate of production, Exponential rate of decay, Heterogeneous production, Time dependent demand, Stochastic production scheduling model.

INTRODUCTION

Stochastic production quantity models plays a significant role in determining the optimal production schedules and production quantity. Recently the researchers in operations research are working more on random production and lifetime of the commodities. Shah and Jaiswal (1977), Nahmias (1982), Dave and Shah (1982), Nirupama Devi, et al (2001), Srinivasa Rao, et al (2009), Biswajit Sarkar (2012) developed inventory models for deteriorating items with time dependent demand. The replenishment (production) is considered to be instantaneous in all of these articles. But in reality replenishment depends on time and is a variable. It depends on several factors such as change in supply of raw materials, change in ordering quantity, change in storage space, transportation etc. In the literature, very little work has been published on inventory models with time-dependent demand having random replenishment.

Recently Sridevi, et al (2010), Srinivasa Rao, et al (2010), Lakshamana Rao, et al (2016), Srinivasa Rao, et al (2020) and Madhulatha, et al (2021) developed and analyzed Economic production quantity models assuming that the production is random and their is a variable rate of production. They assumed that the production process is homogeneous in nature in all of these articles. Hence, in this article, we created and analyzed production quantity models based on the assumption that production is random and follows a mixture of two parameter Weibull distribution with heterogeneous production processes having exponential rate of deterioration. Furthermore, demand is assumed to be a power function of time.

The total cost function is obtained using differential calculus. By minimizing the total cost function the optimal production downtime, optimal production uptime and optimal production quantity are obtained. Through sensitivity analysis the effect of change in parameters and costs is examined. This model is extended to the case of without shortages.

ASSUMPTIONS

For developing the model the following assumptions are made:

1. The demand rate is a power function of time. i.e.,

$$\lambda(t) = \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \qquad (1)$$

where 'n' is the indexing parameter, 'T' is the cycle length and 'r' is the total demand.

2. The production is finite and follows a two parameter Weibull distribution.

The instantaneous rate of production is:

$$R(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}};$$

$$\alpha_1, \alpha_2 > 0, \beta_1, \beta_2 > 0, 0 \le p \le 1 \qquad (2)$$

3. Lead time is zero.
4. Cycle length is T. It is known and fixed.
5. Shortages are allowed and fully backlogged.
6. A deteriorated unit is lost.
7. The lifetime of the item is random and follows a exponential distribution with probability density function

$f(t) = \theta e^{-\theta t};\ \theta > 0, t > 0$

Therefore the instantaneous rate of deterioration is $h(t) = \theta;\ \theta > 0$ (3)

The following notations are used for developing the model.

Q is the production quantity

A is setup cost

C is cost per unit

h Inventory holding cost per unit per unit time

π Shortages cost per unit per unit time

EPQ MODEL WITH SHORTAGES

Consider a production system with zero stock at time t = 0. Because of production after meeting demand and deterioration, the stock level rises during the period $(0, t_1)$. When the stock level reaches S, production ends at time t_1. The inventory continues to decline due to demand and deterioration in the interval (t_1, t_2). Back orders accumulate during the interval (t_2, t_3) as inventory reaches zero at time t_2. At time t_3, replenishment begins again, and after fulfilling demand, it completes the backlog. Backorders are fulfilled during (t_3, T) and inventory levels drop to zero at the end of cycle T. Figure 1 depicts a schematic diagram of the instantaneous state of inventory.

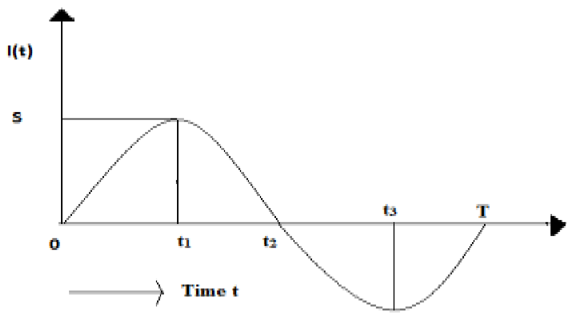


FIGURE 1

SCHEMATIC DIAGRAM DEPICTING THE INVENTORY LEVEL

Let *I(t)* be the inventory level of the system at time '*t*'*(0 ≤ t ≤ T).* The differential equations governing the instantaneous state of inventory *I(t)* over the cycle of length *T* are

$$\frac{d}{dt}I(t) + h(t)I(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\ 0 \le t \le t_1 \quad (4)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\ t_1 \le t \le t_2 \quad (5)$$

$$\frac{d}{dt}I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\ t_2 \le t \le t_3 \quad (6)$$

$$\frac{d}{dt}I(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}$$

$$-\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\ t_3 \le t \le T \tag{7}$$

Where, *h(t)* is as given in equation (3), with the initial conditions $I(0) = 0$, $I(t_1) = S$, $I(t_2) = 0$ and $I(T) = 0$

Substituting *h*(*t*) given in equation (3) in equations (4) and (5) and solving the differential equations, the on hand inventory at time '*t*' is obtained as

$$I(t) = Se^{\theta(t_1-t)} - e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}} - \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{u\theta}du;\ 0 \le t \le t_1 \tag{8}$$

$$I(t) = Se^{\theta(t_1-t)} - \frac{re^{-t\theta}}{nT^{\frac{1}{n}}}\int_{t_1}^{t} u^{\frac{1}{n}-1}e^{u\theta}\,du;\ t_1 \le t \le t_2 \tag{9}$$

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left(t_2^{\frac{1}{n}} - t^{\frac{1}{n}}\right);\ t_2 \le t \le t_3 \tag{10}$$

$$I(t) = \int_t^T \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}dt + \frac{r}{T^{\frac{1}{n}}}\left(T^{\frac{1}{n}} - t^{\frac{1}{n}}\right);\ t_3 \le t \le T \tag{11}$$

Stock loss due to deterioration in the interval *(0,t)* is

$$L(t) = \int_0^t R(t)dt - \int_0^t \lambda(t)dt - I(t);\ 0 \le t \le t_2$$

$$L(t) = \left[\int_0^t \left(\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}\right)dt - r\left(\frac{t}{T}\right)^{\frac{1}{n}} - \left[Se^{\theta(t_1-t)} - e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}du - \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{u\theta}du\right]\right] 0 \le t \le t_1$$

$$\int_0^{t_1}\left(\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}\right)dt - r\left(\frac{t}{T}\right)^{\frac{1}{n}} - \left[Se^{\theta(t_1-t)} - \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}e^{-t\theta}\int_{t_1}^{t} e^{u\theta}\,du\right];\ t_1 \le t \le t_2$$

Stock loss due to deterioration in the cycle of length *T* is

$$L(t) = \int_0^{t_1}\left(\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}\right)dt - r\left(\frac{t_2}{T}\right)^{\frac{1}{n}} \tag{12}$$

Production quantity *Q* in the cycle of length *T* is

$$Q = \int_0^{t_1} R(t)dt + \int_{t_3}^{T} R(t)dt$$

$$= \int_0^{t_1} \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}dt$$

$$+\int_{t_3}^{T}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt \quad (13)$$

From equation (8) and using the initial condition *I(0) = 0*,we obtain the value of '*S*' as

$$S=e^{-\theta t_1}\int_0^{t_1}\left(\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)e^{u\theta}du \quad (14)$$

When $t = t_3$, then equations (10) and (11) becomes

$$I(t_3)=\frac{r}{T^{\frac{1}{n}}}\left(t_2^{\frac{1}{n}}-t_3^{\frac{1}{n}}\right) \quad (15)$$

$$I(t_3)=\frac{r}{T^{\frac{1}{n}}}\left(T^{\frac{1}{n}}-t_3^{\frac{1}{n}}\right)+\int_{t_3}^{T}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}du \quad (16)$$

$$t_2=T\left[1+\frac{1}{r}\times\int_{t_3}^{T}\left(\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}\right)du\right]^n = x(t_3) \quad (17)$$

Let $K(t_1,t_2,t_3)$ be the total production cost per unit time. Since the total production cost is the sum of the set up cost, cost of the units, the inventory holding cost. Hence the total production cost per unit time become

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left[\int_0^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt\right]+\frac{\pi}{T}\left[\int_{t_2}^{t_3}-I(t)dt+\int_{t_3}^{T}-I(t)dt\right] \quad (18)$$

Substituting the values of *I(t)* given in equations (8),(9),(10) and (11) and *Q* given in equation (13) in equation (18) one can obtain $K(t_1,t_2,t_3)$ as:

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{C}{T}\left[\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt+\int_{t_3}^{T}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\left[\int_0^{t_1}\left[Se^{\theta(t_1-t)}-e^{-t\theta}\times\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{u\theta}du\right]dt+\int_{t_1}^{t_2}\left[Se^{\theta(t_1-t)}-e^{-t\theta}\int_{t_1}^{t}\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}e^{u\theta}\,du\right]dt\right]$$

$$-\frac{\pi}{T}\left[\frac{r}{T^{\frac{1}{n}}}\int_{t_2}^{t_3}\left(t_2^{\frac{1}{n}}-t^{\frac{1}{n}}\right)dt\right.$$

$$+\int_{t_3}^{T}\left[\int_{t}^{T}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}}+(1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}}+(1-p)e^{-\alpha_2u^{\beta_2}}}du\right]dt$$

$$\left.+\frac{r}{T^{\frac{1}{n}}}\int_{t_3}^{T}\left(T^{\frac{1}{n}}-t^{\frac{1}{n}}\right)dt\right] \tag{19}$$

On integration and simplification one can get

$$K(t_1,t_2,t_3)=\frac{A}{T}$$

$$+\frac{C}{T}\left[\int_0^{t_1}\frac{p\alpha_1\beta_1t^{\beta_1-1}e^{-\alpha_1t^{\beta_1}}+(1-p)\alpha_2\beta_2t^{\beta_2-1}e^{-\alpha_2t^{\beta_2}}}{pe^{-\alpha_1t^{\beta_1}}+(1-p)e^{-\alpha_2t^{\beta_2}}}dt\right.$$

$$\left.+\int_{t_3}^{T}\frac{p\alpha_1\beta_1t^{\beta_1-1}e^{-\alpha_1t^{\beta_1}}+(1-p)\alpha_2\beta_2t^{\beta_2-1}e^{-\alpha_2t^{\beta_2}}}{pe^{-\alpha_1t^{\beta_1}}+(1-p)e^{-\alpha_2t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\left[\frac{S}{\theta}e^{t_1\theta}\left(1-e^{-t_2\theta}\right)-\int_0^{t_1}e^{-t\theta}\right.$$

$$\times\left[\int_t^{t_1}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}}+(1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}}+(1-p)e^{-\alpha_2u^{\beta_2}}}e^{u\theta}du\right.$$

$$\left.\left.-\frac{r}{nT^{\frac{1}{n}}}\int_t^{t_1}u^{\frac{1}{n}-1}e^{u\theta}du\right]dt-\int_{t_1}^{t_2}e^{-t\theta}\frac{r}{nT^{\frac{1}{n}}}\left(\int_{t_1}^{t}u^{\frac{1}{n}-1}e^{u\theta}du\right)dt\right]$$

$$-\frac{\pi}{T}\left[\frac{r\left[t_3(1+n)\left(t_2^{\frac{1}{n}}-T^{\frac{1}{n}}\right)+T^{\frac{1+n}{n}}-t_2^{\frac{1+n}{n}}\right]}{T^{\frac{1}{n}}(n+1)}\right.$$

$$\left.+\int_{t_3}^{t}\left[\int_t^T\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}}+(1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}}+(1-p)e^{-\alpha_2u^{\beta_2}}}du\right]dt\right] \tag{20}$$

Substituting the value of S given in equation (14) in the total production cost equation (20), we obtain

$$K(t_1,t_2,t_3)=\frac{A}{T}$$

$$+\frac{C}{T}\left[\int_0^{t_1}\frac{p\alpha_1\beta_1t^{\beta_1-1}e^{-\alpha_1t^{\beta_1}}+(1-p)\alpha_2\beta_2t^{\beta_2-1}e^{-\alpha_2t^{\beta_2}}}{pe^{-\alpha_1t^{\beta_1}}+(1-p)e^{-\alpha_2t^{\beta_2}}}dt\right.$$

$$\left.+\int_{t_3}^{T}\frac{p\alpha_1\beta_1t^{\beta_1-1}e^{-\alpha_1t^{\beta_1}}+(1-p)\alpha_2\beta_2t^{\beta_2-1}e^{-\alpha_2t^{\beta_2}}}{pe^{-\alpha_1t^{\beta_1}}+(1-p)e^{-\alpha_2t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\left[\left[\frac{1}{\theta}\int_0^{t_1}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}}+(1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}}+(1-p)e^{-\alpha_2u^{\beta_2}}}e^{u\theta}du\right.\right.$$

$$\left.-\frac{r}{\theta nT^{\frac{1}{n}}}\int_0^{t_1}u^{\frac{1}{n}-1}e^{u\theta}du\right]\times\left(1-e^{-\theta t_2}\right)$$

$$-\int_0^{t_1}e^{-t\theta}\left(\int_t^{t_1}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}}+(1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}}+(1-p)e^{-\alpha_2u^{\beta_2}}}e^{u\theta}du\right.$$

$$-\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{t_1} u^{\frac{1}{n}-1}e^{u\theta}du\Bigg) - \int_{t_1}^{t_2} \frac{re^{-t\theta}}{nT^{\frac{1}{n}}}\left(\int_{t_1}^{t} u^{\frac{1}{n}-1}e^{-u\theta}du\right)dt\Bigg]$$

$$-\frac{\pi}{T}\left[\frac{r\left[t_3(1+n)\left(t_2^{\frac{1}{n}} - T^{\frac{1}{n}}\right) + T^{\frac{1+n}{n}} - t_2^{\frac{1+n}{n}}\right]}{T^{\frac{1}{n}}(n+1)}\right.$$

$$\left.+\int_{t_3}^{t}\left[\int_{t}^{T}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}} + (1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}} + (1-p)e^{-\alpha_2u^{\beta_2}}}du\right]dt\right] \tag{21}$$

Substituting the value of t_2 given in equation (17) in the total production cost equation (21), we obtain

$$K(t_1, t_3) = \frac{A}{T}$$

$$+\frac{C}{T}\left[\int_{0}^{t_1}\frac{p\alpha_1\beta_1t^{\beta_1-1}e^{-\alpha_1t^{\beta_1}} + (1-p)\alpha_2\beta_2t^{\beta_2-1}e^{-\alpha_2t^{\beta_2}}}{pe^{-\alpha_1t^{\beta_1}} + (1-p)e^{-\alpha_2t^{\beta_2}}}dt\right.$$

$$\left.+\int_{t_3}^{T}\frac{p\alpha_1\beta_1t^{\beta_1-1}e^{-\alpha_1t^{\beta_1}} + (1-p)\alpha_2\beta_2t^{\beta_2-1}e^{-\alpha_2t^{\beta_2}}}{pe^{-\alpha_1t^{\beta_1}} + (1-p)e^{-\alpha_2t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\Bigg[\Bigg[\frac{1}{\theta}\int_{0}^{t_1}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}} + (1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}} + (1-p)e^{-\alpha_2u^{\beta_2}}}e^{u\theta}du$$

$$-\frac{r}{\theta nT^{\frac{1}{n}}}\int_{0}^{t_1} u^{\frac{1}{n}-1}e^{u\theta}du\Bigg] \times \left(1 - e^{-\theta x(t_3)}\right) - \int_{0}^{t_1} e^{-t\theta}$$

$$\times\Bigg(\int_{t}^{t_1}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}} + (1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}} + (1-p)e^{-\alpha_2u^{\beta_2}}}e^{u\theta}du$$

$$-\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{t_1} u^{\frac{1}{n}-1}e^{u\theta}du\Bigg)dt - \int_{t_1}^{x(t_3)} \frac{re^{-t\theta}}{nT^{\frac{1}{n}}}\left(\int_{t_1}^{t} u^{\frac{1}{n}-1}e^{-u\theta}du\right)dt\Bigg]$$

$$-\frac{\pi}{T}\left[\frac{r\left[t_3(1+n)\left[[x(t_3)]^{\frac{1}{n}} - T^{\frac{1}{n}}\right] + T^{\frac{1+n}{n}} - [x(t_3)]^{\frac{1+n}{n}}\right]}{T^{\frac{1}{n}}(n+1)}\right.$$

$$\left.+\int_{t_3}^{t}\left[\int_{t}^{T}\frac{p\alpha_1\beta_1u^{\beta_1-1}e^{-\alpha_1u^{\beta_1}} + (1-p)\alpha_2\beta_2u^{\beta_2-1}e^{-\alpha_2u^{\beta_2}}}{pe^{-\alpha_1u^{\beta_1}} + (1-p)e^{-\alpha_2u^{\beta_2}}}du\right]dt\right] \tag{22}$$

OPTIMAL ORDERING POLICIES OF THE MODEL

In this section, we determine the system's optimal policies. We obtain the first order partial derivatives of $K(t_1, t_3)$ given in equation (22) with respect to t_1 and t_3 equate them to zero to obtain the optimal values of t_1 and t_3. The minimizing condition for $K(t_1, t_3)$ is

$$D = \begin{vmatrix} \frac{\partial^2 K(t_1,t_3)}{\partial t_1^2} & \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} \\ \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} & \frac{\partial^2 K(t_1,t_3)}{\partial {t_3}^2} \end{vmatrix} > 0$$

where D is the Hessian matrix.

Differentiating $K(t_1, t_3)$ given in equation (22) with respect to t_1 and equating to zero, we get

$$\Bigg\{\frac{C}{T}\left[\frac{p\alpha_1\beta_1{t_1}^{\beta_1-1}e^{-\alpha_1{t_1}^{\beta_1}} + (1-p)\alpha_2\beta_2{t_1}^{\beta_2-1}e^{-\alpha_2{t_1}^{\beta_2}}}{pe^{-\alpha_1{t_1}^{\beta_1}} + (1-p)e^{-\alpha_2{t_1}^{\beta_2}}}\right]$$

$$+\frac{h}{T}\left[\frac{1-e^{-x(t_3)\theta}e^{t_1\theta}}{\theta}\right.$$

$$\times\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right.$$

$$\left.\left.\left.-\frac{r{t_1}^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]\right]\right\}=0 \qquad (23)$$

Differentiating $K(t_1, t_3)$ given in equation (22) with respect to t_3 and equating to zero, we get

$$\left\{-\frac{C}{T}\left[\frac{p\alpha_1\beta_1 {t_3}^{\beta_1-1}e^{-\alpha_1 {t_3}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_3}^{\beta_2-1}e^{-\alpha_2 {t_3}^{\beta_2}}}{pe^{-\alpha_1 {t_3}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_3}^{\beta_2}}}\right]\right.$$

$$+\frac{h}{T}\left[e^{-\theta x(t_3)}y(t_3)\right.$$

$$\times\left[\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}\right]e^{u\theta}\,du\right.$$

$$\left.\left.+\frac{r}{nT^{\frac{1}{n}}}\left[\theta\int_{t_1}^{x(t_3)}u^{\frac{1}{n}-1}e^{u\theta}du-\int_0^{t_1}u^{\frac{1}{n}-1}e^{u\theta}du\right]\right]\right]$$

$$-\frac{\pi}{T}\left[\left[\frac{p\alpha_1\beta_1 {t_3}^{\beta_1-1}e^{-\alpha_1 {t_3}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_3}^{\beta_2-1}e^{-\alpha_2 {t_3}^{\beta_2}}}{pe^{-\alpha_1 {t_3}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_3}^{\beta_2}}}\right]\right.$$

$$\times(x(t_3)-1)-r]$$

$$\left.\left.\left.-\int_{t_3}^{T}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}du\right]\right]\right\}=0 \qquad (24)$$

Solving the equations (23) and (24) simultaneously, we obtain the optimal time at which production is stopped t_1^* of t_1 and the optimal time t_3^* of t_3 at which the production is restarted after accumulation of backorders.

The optimum production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1^*, t_3^* in equation (13) as

$$Q^*=\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt$$

$$+\int_{t_3}^{T}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt$$

(25)

NUMERICAL ILLUSTRATION

In this section, we study how the model works with a numerical example to obtain the production uptime, downtime, optimum production quantity, and profit of an inventory system. It is presumed that the commodity is deteriorating in nature, and shortages are permitted and fully backlogged. The following parameters are used to demonstrate the model's solution procedure:

$A = 100, 105, 110, 115;$

$C = 10, 10.5, 11, 11.5;$

$h = 0.3, 0.315, 0.33, 0.345;$

$\pi = 0.5, 0.525, 0.55, 0.575;$

$\alpha_1 = 11, 11.55, 12.1, 12.65;$

$\alpha_2 = 15, 15.75, 16.5, 17.25;$

$\beta_1 = 0.45, 0.473, 0.495, 0.518;$

$\beta_2 = 3, 3.15, 3.3, 3.45;$

$\theta = 3, 3.15, 3.3, 3.45;$

$r = 100, 105, 110, 115;$

$n = 1, 1.05, 1.1, 1.15;$

$p = 0.5, 0.525, 0.55, 0.575;$

and $T = 12$ months

Substituting these values the optimal production quantity Q^*,the production uptime, production downtime and total production cost are computed and presented in Table 1.

From Table 1 it is observed that the deterioration parameter and production parameters have a tremendous influence on the optimal values of production times, production quantity and total production cost function.

It is found that as the ordering cost "*A*" goes up from 100 to 115, the optimal production down time $t_1{}^*$, the optimal production uptime $t_3{}^*$, the optimal production quantity Q^* remains constant, the total production cost per unit time K^* rises from 72.608 to 73.857.

It is found that as the cost per unit "C" goes up from 10 to 11.5, the optimal production downtime $t_1{}^*$ rises from 1.268 to 1.271, the optimal production uptime $t_3{}^*$ remains constant, the optimal production quantity Q^* rises from 24.969 to 24.974, the total production cost per unit time K^* rises from 72.608 to 74.229.

It is found that as the holding cost "h" goes up from 0.3 to 0.345, the optimal production downtime $t_1{}^*$ and the optimal production uptime $t_3{}^*$ remains constant, the optimal production quantity Q^* drops from 24.969 to 24.968, the total production cost per unit time K^* drops from 72.608 to 72.485.

It is found that as the shortage cost "π' goes up from 0.5 to 0.575, the optimal production downtime $t_1{}^*$ remains constant, the optimal production uptime $t_3{}^*$ rises from 4.488 to 4.496, the optimal production quantity Q^* drops from 24.969 to 24.951, the total production cost per unit time K^* rises from 72.608 to 80.621.

It is found that as the production parameter "α_1" goes up from 11 to 12.65, the optimal production downtime $t_1{}^*$ rises from 1.268 to 1.271, the optimal production uptime $t_3{}^*$ drops from 4.488 to 4.484, the optimal production quantity Q^* rises from 24.969 to 28.611, the total production cost per unit time K^* rises from 72.608 to 82.781.

It is found that as the production parameter "α_2" goes up from 15 to 17.25, the optimal production down time $t_1{}^*$, the optimal production uptime $t_3{}^*$, the optimal production quantity Q^* and the total production cost per unit time K^* remains constant.

It is found that as the production parameter "β_1" goes up from 0.45 to 0.518, the optimal production downtime $t_1{}^*$ rises from 1.268 to 1.271, the optimal production uptime $t_3{}^*$ rises from 4.488 to 4.494, the optimal production quantity Q^* rises from 24.969 to 29.036 and the total production cost per unit time K^* rises from 72.608 to 91.594.

It is found that as the production parameter "β_2" goes up from 3 to 3.45, the optimal production down time $t_1{}^*$, the optimal production uptime $t_3{}^*$ and the optimal production quantity Q^* and the total production cost per unit time K^* remains constant.

It is found that as the production parameter "p" goes up from 0.5 to 0.575, the optimal production down time $t_1{}^*$ and the optimal production uptime $t_3{}^*$ remains constant, the optimal production quantity Q^* drops from 24.969 to 24.83 and the total production cost per unit time K^* drops from 72.608 to 72.49.

It is found that as the deterioration parameter "ɵ" goes up from 3 to 3.45, the optimal production down time $t_1{}^*$, the optimal production uptime $t_3{}^*$ and the optimal production quantity Q^* remains constant and the total production cost per unit time K^* rises from 72.608 to 72.71.

It is found that as the demand parameter "r" goes up from 100 to 115, the optimal production down time $t_1{}^*$ remains constant, the optimal production uptime $t_3{}^*$ rises from 4.488 to 4.489, the optimal production quantity Q^* drops from 24.969 to 24.965 and the total production cost per unit time K^* drops from 72.608 to 72.098.

It is found that as the demand parameter "n" goes up from 1 to 1.15, the optimal production down time $t_1{}^*$ remains constant, the optimal production uptime $t_3{}^*$ rises from 4.488 to 4.489, the optimal production quantity Q^* drops from 24.969 to 24.966 and the total production cost per unit time K^* drops from 72.608 to 77.449.

TABLE 1

NUMERICAL ILLUSTARTION

A	*C*	*h*	*π*	*T*	*α₁*	*α₂*	*β₁*	*β₂*	*ϴ*	*r*	*n*	*p*	t_1^*	t_3^*	Q^*	K^*
100	10	0.3	0.5	12	11	15	0.45	3	3	100	1	0.5	1.268	4.488	24.969	72.608
105													1.268	4.488	24.969	73.024
110													1.268	4.488	24.969	73.441
115													1.268	4.488	24.969	73.857
	10.5												1.269	4.488	24.97	73.148
	11												1.27	4.488	24.972	73.688
	11.5												1.271	4.488	24.974	74.229
		0.315											1.268	4.488	24.969	72.568
		0.33											1.268	4.488	24.968	72.528
		0.345											1.268	4.488	24.968	72.485
			0.525										1.268	4.491	24.963	75.278
			0.55										1.268	4.493	24.957	77.95
			0.575										1.268	4.496	24.951	80.621
					11.55								1.269	4.487	26.189	76.003
					12.1								1.27	4.485	27.411	79.416
					12.65								1.271	4.484	28.611	82.781
						15.75							1.268	4.488	24.969	72.608
						16.5							1.268	4.488	24.969	72.608
						17.25							1.268	4.488	24.969	72.608
							0.473						1.269	4.49	26.256	78.52
							0.495						1.27	4.492	27.57	84.649
							0.518						1.271	4.494	29.036	91.594
								3.15					1.268	4.488	24.969	72.607
								3.3					1.268	4.488	24.969	72.607
								3.45					1.268	4.488	24.969	72.607
									3.15				1.268	4.488	24.969	72.645
									3.3				1.268	4.488	24.969	72.681
									3.45				1.268	4.488	24.969	72.71
										105			1.268	4.489	24.967	72.414
										110			1.268	4.489	24.966	72.234
										115			1.268	4.489	24.965	72.098
											1.05		1.268	4.488	24.968	74.212
											1.1		1.268	4.489	24.967	75.826
											1.15		1.268	4.489	24.966	77.449
												0.525	1.268	4.488	24.92	72.567
												0.55	1.268	4.488	24.874	72.528
												0.575	1.268	4.488	24.83	72.49

SENSITIVITY ANALYSIS OF THE MODEL

By changing each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) for the model under study at a time, Sensitivity analysis is used to see how changes in model parameters and costs effect the optimal policies.

The results are presented in Table 2. Figure 2 illustrates the relationship between the parameters and the replenishment schedule's optimal values.

TABLE 2

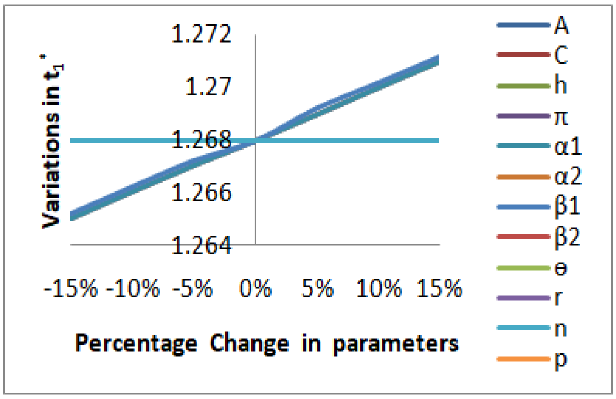
SENSITIVITY ANALYSIS OF THE MODEL - WITH SHORTAGES

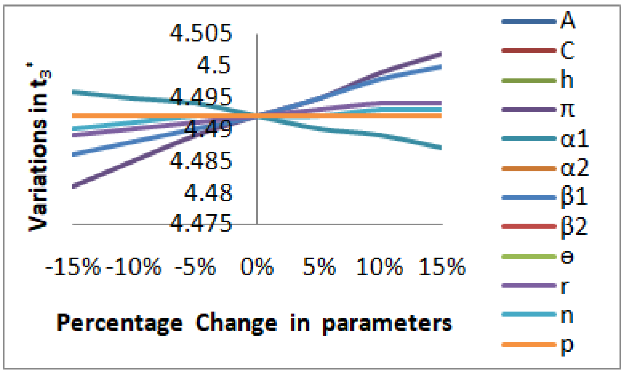
Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	24.968	24.968	24.968	24.969	24.969	24.969	24.969
	K^*	71.358	71.775	72.025	72.608	73.024	73.441	73.857
C	t_1^*	1.265	1.266	1.267	1.268	1.269	1.27	1.271
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	24.963	24.965	24.967	24.969	24.97	24.972	24.974
	K^*	70.988	71.528	72.068	72.608	73.148	73.688	74.229
h	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	24.969	24.969	24.969	24.969	24.969	24.968	24.968
	K^*	72.728	72.688	72.648	72.608	72.568	72.528	72.485
Π	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.48	4.482	4.485	4.488	4.491	4.493	4.496
	Q^*	24.987	24.981	24.975	24.969	24.963	24.957	24.951
	K^*	64.599	67.268	69.938	72.608	75.278	77.95	80.621
α_1	t_1^*	1.265	1.266	1.267	1.268	1.269	1.27	1.271
	t_3^*	4.492	4.491	4.489	4.488	4.487	4.485	4.484
	Q^*	21.311	22.529	23.749	24.969	26.189	27.411	28.611
	K^*	62.517	65.865	69.228	72.608	76.003	79.416	82.781
α_2	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	24.969	24.969	24.969	24.969	24.969	24.969	24.969
	K^*	72.608	72.608	72.608	72.608	72.608	72.608	72.608
β_1	t_1^*	1.265	1.266	1.267	1.268	1.269	1.27	1.271
	t_3^*	4.485	4.486	4.487	4.488	4.49	4.492	4.494
	Q^*	21.767	22.686	23.765	24.969	26.256	27.57	29.036
	K^*	58.29	62.348	67.16	72.608	78.52	84.649	91.594
β_2	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	24.969	24.969	24.969	24.969	24.969	24.969	24.969
	K^*	72.608	72.608	72.608	72.608	72.607	72.607	72.607
Θ	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	24.968	24.968	24.969	24.969	24.969	24.969	24.969
	K^*	72.471	72.522	72.567	72.608	72.645	72.681	72.71

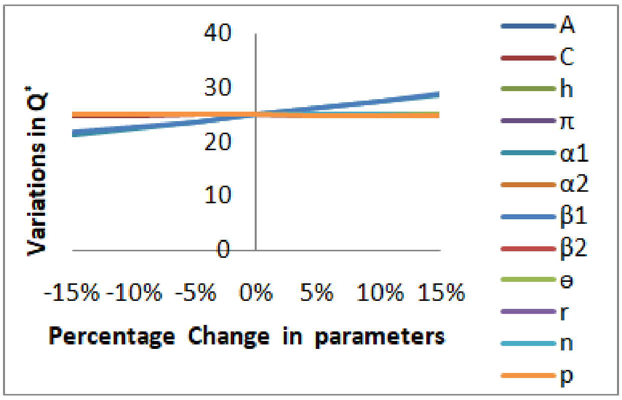
r	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.486	4.487	4.487	4.488	4.489	4.489	4.489
	Q^*	24.973	24.972	24.97	24.969	24.967	24.966	24.965
	K^*	73.298	73.047	72.818	72.608	72.414	72.234	72.098
n	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.486	4.487	4.487	4.488	4.488	4.489	4.489
	Q^*	24.972	24.971	24.97	24.969	24.968	24.967	24.966
	K^*	67.849	69.426	71.012	72.608	74.212	75.826	77.449
p	t_1^*	1.268	1.268	1.268	1.268	1.268	1.268	1.268
	t_3^*	4.488	4.488	4.488	4.488	4.488	4.488	4.488
	Q^*	25.155	25.075	25.02	24.969	24.92	24.874	24.83
	K^*	72.765	72.695	72.651	72.608	72.567	72.528	72.49

The major observations drawn from the numerical study of the Table 2 are

- t_1^* and t_3^* are less sensitive, Q^* is slightly sensitive and K^* is moderately sensitive to the changes in ordering cost 'A'.
- t_1^* and Q^* are slightly sensitive, t_3^* is less sensitive and K^* is moderately sensitive to the changes in cost per unit 'C'.
- t_1^* and t_3^* are less sensitive, Q^* and K^* are slightly sensitive to the changes in holding cost 'h'.
- t_1^* is less sensitive, t_3^* and Q^* are slightly sensitive and K^* is moderately sensitive to the changes in shortage cost 'π'.
- t_1^* and t_3^* are slightly sensitive, Q^* and K^* are moderately sensitive to the change in the production parameter 'α_1'.
- t_1^*, t_3^*, Q^* and K^* are less sensitive to the change in the production parameter 'α_2'.
- t_1^* and t_3^* are slightly sensitive, Q^* and K^* are moderately sensitive to the change in the production parameter 'β_1'.
- t_1^*, t_3^* and Q^* are less sensitive, K^* is slightly sensitive to the change in the production parameter 'β_2' .
- t_1^* and t_3^* are less sensitive, Q^* is moderately sensitive and K^* is slightly sensitive to the change in the production parameter 'p'.
- t_1^* and t_3^* are less sensitive, Q^* and K^* are slightly sensitive to the change in the deterioration parameter 'ɵ'.
- t_1^* is less sensitive, t_3^* and Q^* are slightly sensitive, K^* is moderately sensitive to the change in the demand parameter 'r'.
- t_1^* is less sensitive, t_3^* and Q^* are slightly sensitive, K^* is moderately sensitive to the change in the demand parameter 'n'.







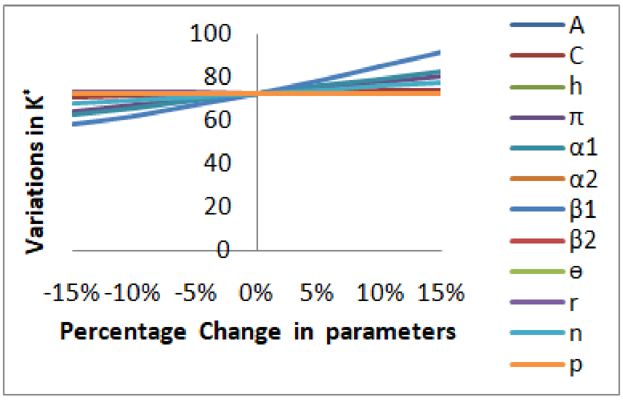


FIGURE 2

RELATIONSHIP BETWEEN PARAMETERS AND OPTIMAL VALUES WITH SHORTAGES

EPQ MODEL WITHOUT SHORTAGES

The inventory model for decaying products without shortages is established and studied in this part. At time $t = 0$, it is assumed that shortages are not permitted and that the stock level is zero. Due to excess production after meeting demand and deterioration, the stock level rises during the period $(0, t_1)$. When the stock level reaches S, production ends at time t_1. In the interval (t_1, T), the inventory reduces gradually because of demand and deterioration. The inventory reaches zero at time T. Figure 3 depicts a schematic diagram of the instantaneous state of inventory.

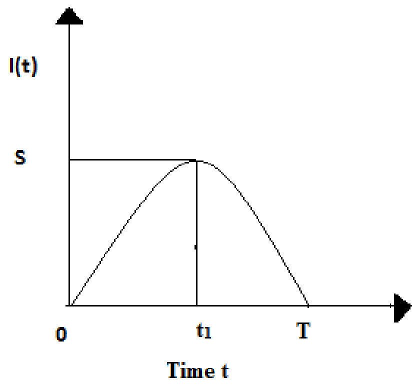


FIGURE 3

SCHEMATIC DIAGRAM DEPICTING THE INVENTORY LEVEL

Let $I(t)$ be the inventory level of the system at time *'t'*

$(0 \leq t \leq T)$. Then the differential equations governing the instantaneous state of $I(t)$ over the cycle of length T are

$$\frac{d}{dt}I(t) + h(t)I(t) =$$

$$\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}$$

$$-\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\ 0\le t\le t_1 \qquad (26)$$

$$\frac{d}{dt}I(t)+h(t)I(t)=-\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};t_1\le t\le T \qquad (27)$$

where, $h(t)$ is as given in equation (3), with the initial conditions $I(0) = 0$, $I(t_1) = S$ and $I(T) = 0$.

Substituting $h(t)$ given in equation (3) in equations (26) and (27) and solving the differential equations, the on hand inventory at time 't' is obtained as :

$$I(t)=Se^{\theta(t_1-t)}$$

$$-e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}\right.$$

$$\left.-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{u\theta}du;0\le t\le t_1 \qquad (28)$$

$$I(t)=Se^{\theta(t_1-t)}-\frac{re^{-t\theta}}{nT^{\frac{1}{n}}}\int_{t_1}^{t}u^{\frac{1}{n}-1}e^{u\theta}\,du;t_1\le t\le T \qquad (29)$$

Stock loss due to deterioration in the interval $(0, t)$ is

$$L(t)=\int_0^t R(t)dt-\int_0^t \lambda(t)dt-I(t);0\le t\le T \qquad (30)$$

This implies

$$L(t)=\left[\int_0^t\left(\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}\right)dt\right.$$

$$-r\left(\frac{t}{T}\right)^{\frac{1}{n}}$$

$$-\left[Se^{\theta(t_1-t)}-e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}\right.\right.$$

$$\left.\left.-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{u\theta}du\right]\ 0\le t\le t_1$$

$$\int_0^{t_1}\left(\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}\right)dt$$

$$-r\left(\frac{t}{T}\right)^{\frac{1}{n}}$$

$$-\left[Se^{\theta(t_1-t)}-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}e^{-t\theta}\int_{t_1}^{t}e^{u\theta}\,du;\right];\ t_1\le t\le T \qquad (31)$$

Production quantity Q in the cycle of length T is

$$Q=\int_0^{t_1}R(t)dt$$

$$=\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt \qquad (32)$$

From equation (28) and using the initial conditions $I(0) = 0$, we obtain the value of 'S' as

$$S=e^{-\theta t_1}$$

$$\int_0^{t_1}\left(\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}\right.$$
$$\left.-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)e^{u\theta}du \qquad (33)$$

Let $K(t_1)$ be the total production cost per unit time. Since the total production cost is the sum of the set up cost, cost of the units, the inventory holding cost. Therefore the total production cost per unit time becomes

$$K(t_1)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left[\int_0^{t_1} I(t)dt+\int_{t_1}^{T} I(t)dt\right] \qquad (34)$$

Substituting the values of $I(t)$ and Q from equation's (28),(29) and (32) in equation (34), we obtain $K(t_1)$ as

$$K(t_1)=\frac{A}{T}$$
$$+\frac{C}{T}\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}\right]dt$$
$$+\frac{h}{T}\left[\int_0^{t_1}\left[Se^{\theta(t_1-t)}\right.\right.$$
$$-e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}\right.$$
$$\left.\left.-\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{u\theta}du\right]dt$$
$$\left.+\int_{t_1}^{T}\left[Se^{\theta(t_1-t)}-\frac{re^{-t\theta}}{nT^{\frac{1}{n}}}\int_{t_1}^{t}u^{\frac{1}{n}-1}e^{u\theta}\,du\right]dt\right] \qquad (35)$$

0n integration and simplification we get

$$K(t_1)=\frac{A}{T}$$
$$+\frac{C}{T}\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}\right]dt$$
$$+\frac{h}{T}\left[\frac{S}{\theta}e^{t_1\theta}\left(1-e^{-\theta T}\right)-\int_0^{t_1}e^{-t\theta}\right.$$
$$\times\left[\int_t^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du\right.$$
$$\left.-\frac{r}{nT^{\frac{1}{n}}}\int_t^{t_1}u^{\frac{1}{n}-1}e^{u\theta}du\right]dt$$
$$\left.-\int_{t_1}^{T}e^{-t\theta}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t_1}^{t}u^{\frac{1}{n}-1}e^{u\theta}du\right]dt\right] \qquad (36)$$

Substituting the value of S given in equation (33) in the total cost equation (36), we obtain

$$K(t_1)=\frac{A}{T}$$

$$+\frac{C}{T}\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}\right]dt$$

$$+\frac{h}{T}\left[\left(1-e^{-\theta T}\right)\left[\frac{1}{\theta}\right.\right.$$

$$\times\int_0^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du$$

$$\left.-\frac{r}{\theta n T^{\frac{1}{n}}}\int_0^{t_1}u^{\frac{1}{n}-1}e^{u\theta}du\right]$$

$$-\int_0^{t_1}e^{-t\theta}$$

$$\times\left[\int_t^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du\right.$$

$$\left.-\frac{r}{nT^{\frac{1}{n}}}\int_t^{t_1}u^{\frac{1}{n}-1}e^{u\theta}du\right]dt$$

$$\left.-\int_{t_1}^{T}e^{-t\theta}\left(\frac{r}{nT^{\frac{1}{n}}}\int_{t_1}^{t}u^{\frac{1}{n}-1}e^{u\theta}du\right)dt\right] \qquad (37)$$

OPTIMAL ORDERING POLICIES OF THE MODEL

To find the optimal values of t_1, we equate the first order partial derivatives of $K(t_1)$ with respect to t_1 equate them to zero. The condition for minimum of $K(t_1)$ is

$$\frac{\partial^2 K(t_1)}{\partial {t_1}^2}>0$$

Differentiating $K(t_1)$ with respect to t_1 and equating to zero, we get

$$\left\{\frac{C}{T}\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right]\right.$$

$$+\frac{h}{T}\left[\frac{\left(1-e^{-\theta T}\right)e^{t_1\theta}}{\theta}\right.$$

$$\times\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right.$$

$$\left.\left.\left.-\frac{r{t_1}^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]\right]\right\}=0 \qquad (38)$$

Solving the equation (38), we obtain the optimal time t_1^* of $t_1$at which the production is to be stopped.

The optimal production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 in equation (32) as

$$Q^*=\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt$$

(39)

NUMERICAL ILLUSTRATION

In this section, we study how the model works with a numerical example to obtain the production time, optimum production quantity and profit of an inventory system. The following parameters are used to illustrate the model's solution procedure:

$A = 300, 315, 330, 345;$

$C = 10, 10.5, 11, 11.5;$

$h = 0.2, 0.21, 0.22, 0.23;$

$\alpha_1 = 20, 21, 22, 23;$

$\alpha_2 = 50, 52.5, 55, 57.5;$

$\beta_1 = 0.55, 0.578, 0.605, 0.633;$

$\beta_2 = 2, 2.1, 2.2, 2.3;$

$\theta = 3, 3.15, 3.3, 3.45;$

$r = 50, 52.5, 55, 57.5;$

$n = 5, 5.25, 5.5, 5.75;$

$p = 0.5, 0.525, 0.55, 0.575;$

and $T = 12$ months

Substituting these values the optimal production quantity Q^*, the production time and total production cost are computed and presented in Table 3

It is found that as the ordering cost "A" goes up from 300 to 345, the optimal production downtime t_1^* remains constant, the optimal production quantity Q^* drops from 52.287 to 52.285 and the total production cost per unit time K^* rises from 68.596 to 72.345.

It is found that as the cost per unit "C" goes up from 10 to 11.5, the optimal production downtime t_1^* rises from 5.601 to 5.613, the optimal production quantity Q^* rises from 52.287 to 52.346 and the total production cost per unit time K^* rises from 68.596 to 75.189.

It is found that as the holding cost "h" goes up from 0.2 to 0.23, the optimal production downtime t_1^* and the optimal production quantity Q^* remains constant and the total production cost per unit time K^* rises from 68.596 to 68.599.

It is found that as the production parameter "α_1" goes up from 20 to 23, the optimal production downtime t_1^* rises from 5.601 to 5.619, the optimal production quantity Q^* rises from 52.287 to 60.128 and the total production cost per unit time K^* rises from 68.596 to 75.184.

It is found that as the production parameter "α_2" goes up from 50 to 57.5, the optimal production downtime t_1^* rises from 5.601 to 5.603, the optimal production quantity Q^* rises from 52.287 to 52.293 and the total production cost per unit time K^* rises from 68.596 to 68.624.

It is found that as the production parameter "β_1" goes up from 0.55 to 0.633, the optimal production downtime t_1^* rises from 5.601 to 5.651, the optimal production quantity Q^* rises from 52.287 to 60.549 and the total production cost per unit time K^* rises from 68.596 to 75.541.

It is found that as the production parameter "β_2" goes up from 2 to 2.3, the optimal production downtime t_1^* drops from 5.601 to 5.597, the optimal production quantity Q^* drops from 52.287 to 52.262 and the total production cost per unit time K^* drops from 68.596 to 68.577.

TABLE 3

NUMERICAL ILLUSTRATION

A	C	h	T	α_1	α_2	β_1	β_2	θ	r	n	p	t_1^*	Q^*	K^*
300	10	0.2	12	20	50	0.55	2	3	50	5	0.5	5.601	52.287	68.596
315												5.601	52.286	69.845
330												5.601	52.286	71.095
345												5.601	52.285	72.345
	10.5											5.605	52.306	70.792
	11											5.611	52.334	73.869
	11.5											5.613	52.346	75.189
		0.21										5.601	52.287	68.597
		0.22										5.601	52.287	68.598
		0.23										5.601	52.287	68.599
				21								5.608	54.899	70.779
				22								5.613	57.51	72.971
				23								5.619	60.128	75.184
					52.5							5.602	52.291	68.607
					55							5.603	52.292	68.616
					57.5							5.603	52.293	68.624
						0.578						5.617	54.924	70.813
						0.605						5.633	57.61	73.071
						0.633						5.651	60.549	75.541
							2.1					5.6	52.278	68.58
							2.2					5.597	52.266	68.577
							2.3					5.597	52.262	68.577
								3.15				5.601	52.286	68.591
								3.3				5.601	52.286	68.587
								3.45				5.601	52.286	68.583
									52.5			5.601	52.287	68.584
									55			5.601	52.287	68.573
									57.5			5.601	52.287	68.562
										5.25		5.601	52.287	68.6
										5.5		5.601	52.287	68.604
										5.75		5.601	52.287	68.608
											0.525	5.602	52.239	68.557
											0.55	5.602	52.193	68.52
											0.575	5.602	52.15	68.485

It is found that as the production parameter “p” goes up from 0.5 to 0.575, the optimal production downtime t_1^* rises from 5.601 to 5.602, the optimal production quantity Q^* drops from 52.287 to 52.15 and the total production cost per unit time K^* drops from 68.596 to 68.545.

It is found that as the deterioration parameter “ɵ” goes up from 3 to 3.45, the optimal production downtime t_1^* remains constant, the optimal production quantity Q^* drops from 52.287 to 52.286 and the total production cost per unit time K^* drops from 68.596 to 68.583.

It is found that as the demand parameter "r" goes up from 50 to 57.5, the optimal production downtime t_1^* and the optimal production quantity Q^* remains constant, the total production cost per unit time K^* drops from 68.596 to 68.562.

It is found that as the demand parameter "n" goes up from 5 to 5.75, the optimal production downtime t_1^* and the optimal production quantity Q^* remains constant, the total production cost per unit time K^* rises from 68.596 to 68.608.

SENSITIVITY ANALYSIS OF THE MODEL

By changing each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) for the model under study at a time, Sensitivity analysis is used to see how changes in model parameters and costs effect the optimal policies. The results are presented in Table 4. Figure 4 illustrates the relationship between the parameters and the production schedule's optimal values.

It is observed that production parameters and deterioration parameters are having significant influence on optimal production quantity and total cost.

TABLE 4

SENSITIVITY ANALYSIS OF THE MODEL - WITHOUT SHORTAGES

Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	5.602	5.602	5.602	5.601	5.601	5.601	5.601
	Q^*	52.288	52.287	52.287	52.287	52.286	52.286	52.285
	K^*	64.847	66.096	67.346	68.596	69.845	71.095	72.345
C	t_1^*	5.59	5.594	5.598	5.601	5.605	5.611	5.613
	Q^*	52.227	52.247	52.267	52.287	52.306	52.334	52.346
	K^*	62.017	64.208	66.401	68.596	70.792	73.869	75.189
h	t_1^*	5.601	5.601	5.601	5.601	5.601	5.601	5.601
	Q^*	52.286	52.286	52.287	52.287	52.287	52.287	52.287
	K^*	68.592	68.593	68.594	68.596	68.597	68.598	68.599
α_1	t_1^*	5.581	5.588	5.595	5.601	5.608	5.613	5.619
	Q^*	44.717	47.064	49.675	52.287	54.899	57.51	60.128
	K^*	62.272	64.234	66.415	68.596	70.779	72.971	75.184
α_2	t_1^*	5.599	5.599	5.6	5.601	5.602	5.603	5.603
	Q^*	52.274	52.274	52.281	52.287	52.291	52.292	52.293
	K^*	68.587	68.589	68.593	68.596	68.607	68.616	68.624
β_1	t_1^*	5.561	5.574	5.587	5.601	5.617	5.633	5.651
	Q^*	45.26	47.507	49.704	52.287	54.924	57.61	60.549
	K^*	62.688	64.577	66.424	68.596	70.813	73.071	75.541
β_2	t_1^*	5.603	5.603	5.602	5.601	5.6	5.597	5.597
	Q^*	52.293	52.292	52.291	52.287	52.278	52.266	52.262
	K^*	68.641	68.627	68.613	68.596	68.58	68.577	68.577
Θ	t_1^*	5.602	5.602	5.601	5.601	5.601	5.601	5.601
	Q^*	52.288	52.287	52.287	52.287	52.286	52.286	52.286
	K^*	68.61	68.605	68.6	68.596	68.591	68.587	68.583
r	t_1^*	5.601	5.601	5.601	5.601	5.601	5.601	5.601
	Q^*	52.287	52.287	52.287	52.287	52.287	52.287	52.287
	K^*	68.63	68.618	68.607	68.596	68.584	68.573	68.562
n	t_1^*	5.601	5.601	5.601	5.601	5.601	5.601	5.601
	Q^*	52.287	52.287	52.287	52.287	52.287	52.287	52.287
	K^*	68.583	68.587	68.591	68.596	68.6	68.604	68.608
p	t_1^*	5.601	5.601	5.601	5.601	5.602	5.602	5.602
	Q^*	52.448	52.391	52.326	52.287	52.239	52.193	52.15
	K^*	68.727	68.681	68.628	68.596	68.557	68.52	68.485

The major observations drawn from the numerical study of the Table 4 are:

- t_1^* and Q^* are slightly sensitive and K^* is moderately sensitive to the changes in ordering cost 'A'.
- t_1^* and Q^* are slightly sensitive and K^* is moderately sensitive to the changes in cost per unit 'C'.
- t_1^* is less sensitive, Q^* and K^* are slightly sensitive to the changes in holding cost 'h'.
- t_1^* is slightly sensitive, Q^* and K^* are moderately sensitive to the change in the production parameter 'α_1'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the production parameter 'α_2'.
- t_1^* is slightly sensitive, Q^* and K^* are moderately sensitive to the change in the production parameter 'β_1'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the production parameter 'β_2'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the production parameter 'p'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the deterioration parameter 'θ'.

- t_1^* and Q^* are less sensitive and K^* is slightly sensitive to the change in the demand parameter 'r'.
- t_1^* and Q^* are less sensitive and K^* is slightly sensitive to the change in the demand parameter 'n'.

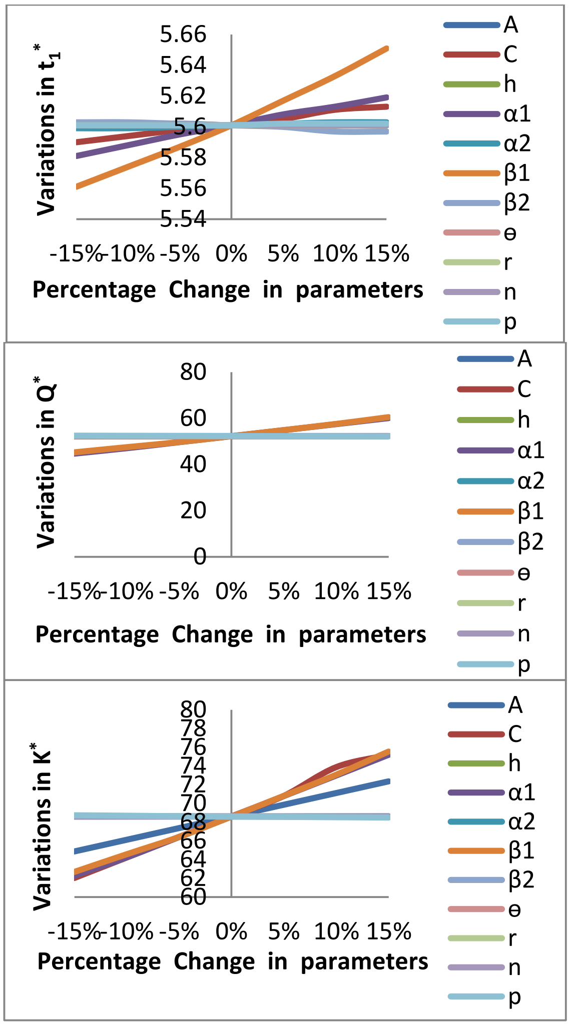


FIGURE 4

RELATIONSHIP BETWEEN PARAMETERS AND OPTIMAL VALUES WITHOUT SHORTAGES

CONCLUSIONS

This paper addresses the derivation of optimal ordering policies of an EPQ model with mixture of two parameter Weibull production for deteriorating items. It is assumed that the lifetime of the commodity is random and follows an exponential distribution. Further it is assumed that the production rate increases with increase in time. The instantaneous state of inventory is derived by considering the power pattern demand. The optimal production schedules and production quantity are derived. Through sensitivity analysis it is observed that the demand function parameters and lifetime distribution parameters have significant influence on operating policies of the model. This model also includes some of the earlier models as particular cases. This model is useful in situations prevailing at places like vegetable or fruits markets, sea food industries and pharmaceutical industries in which the production manager can develop optimal production schedules with the historical data on production. It can also be used in supply chain management. This model also includes some of the earlier EPQ models as particular cases for specific values of the parameters. This model can be extended to the case of multi commodity production processes where the production is random and follows a mixture of Weibull distribution, which will be published elsewhere.

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