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Under Rotating Buckling Loading, An Evaluation of Euler and Rankin Buckling Theories

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Abstract:
In the present research, rotating buckling columns made of 3o4 stainless steel were used. The experimental result showed that when the slenderness ratio (SR) more than 115 the columns behave as long columns and when its (SR) less than 115 buckled as intermediate columns. the applications of Euler and Rankin to the columns data observed that the predictions of Critical buckling loads showed satisfactory estimation considering a resendable factor of safety. Also, the comparison between experimental and numerical results has been achieved and agreed well takin into consideration of factor of safety more than (1.5).
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1. Introduction

The computation of a structure's buckling loads is crucial owing to the risk of the structure failing suddenly if the critical buckling load is achieved. When the buckling load is met, some buildings may lose all stability, putting people in danger. If a roof or other similar structures loses all stability, people may be at risk. [1].

The present manufacturing industry puts a focus on structural parts that are low in weight yet strong enough to absorb a lot of energy and carry a lot of weight. Shell constructions and shallow trusses make up these components. In terms of weight bearing capability, shell structures have a major advantage. When a load is applied to a shell, various internal forces are produced, which can result in bending, twisting, transverse shearing, and buckling. [2].

1-1 Factors Affecting column buckling phenomena:

The form and dimensions of a column's cross-section, as well as its length and the method in which it is attached to neighboring members or supports, all influence its tendency to buckle.

*** The following are key cross-sectional properties:[3]**

a- The area of the cross-section A.

b-The moment of inertia of the cross-section, I , with respect to the axis about which the value of I is minimum.

c- The smallest radius of gyration of the cross-section R. R is calculated as follows:

$$r_{min}=\sqrt{\frac{I}{A}} \quad \text{..................}(1)$$

r_{min}: smallest radius of gyration

I: moment of inertia

A: cross-sectional area

*** The end fixity and effective length**

The way a column's ends are sustained is referred to as end fixity. Pinned, fixed, and free end restraints are the three types of end restraint. The varieties of end fixity are depicted in Figure 1. The way both ends of the column are supported has an impact on the column's effective length. The following is a definition of effective length (Le):

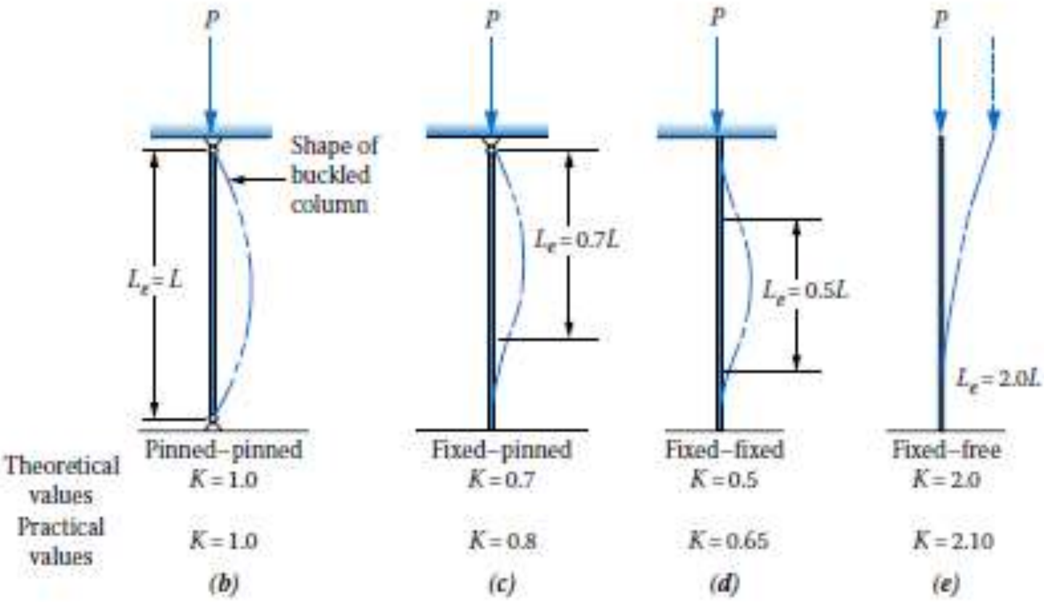
$$L_e = KL \text{..................} (2)$$

Were

L= The length between both the supports of the column.

K= constant of end fixity the theoretical and experimental values of K is shown in Figure 1.

Figure (1) Types of end conditions of columns. [3]



***The Slenderness Ratio (SR).[3]**

SR is the ratio of the L_e of column to its least radius of gyration:

SR = KL / r_{min} = L_e / r_{min} ……...…...…………..…………...… (3)

Were

L_e: is the effective length

r_{min}: smallest radius of gyration

*** Column Constant (Cc).[3]**

The choice of which method is used depends on the actual value of (SR) and (Cc).
Cc is defined by:

Cc = $\sqrt{\frac{2\pi^2 E}{\sigma_y}}$…………………………………… (4)

Were
E: is the modulus of elasticity of the column material
σ_y: is yield stress of column material (in tension)

The column constant is clearly dependent on the mechanical characteristics of the material employed.
Short columns, large columns, & the three kinds of columns are columns of moderate length. Table1 shows the three types of columns for various materials based on the slenderness ratio (S.R.). [4]

Table (1) Slenderness ratio of columns for different materials [5].

Material	Long column (Elastic stability limit)	Intermediate column (Inelastic stability limit)	Short column (strength limit)
Structural steel	SR>150	40< SR< 150	SR<40
Aluminum Alloy	SR> 55	12< SR<55	SR<12
Wood	(18-30) <SR< 50	11 < SR < (18-30)	SR<11

1-2 The Euler formulas.

The Euler formula is used to analyze a lengthy column. [3].

$$P_{cr}=\frac{\pi^2 EI}{(KL)^2} \quad \text{…….….. ……..…………...…} (5)$$

More information on the derivation and assumptions used to arrive at the above equation may be found elsewhere.[1]

The buckling load (Pcr) is completely determined by the geometry of the column (length and cross section) and the stiffness of the material (modulus of elasticity). The strength of the material isn't taken into account at all. As a result, in a long column application, specifying a high-strength material is rarely beneficial.[3]

1-3 Rankine or Rankine-Gordon Formula

The Euler theory yields proper answers only for long columns, while this formula is applicable to columns of all lengths, from extremely long to intermediate, and it yields unreliable results [6].

For a strut, the Rankine formula combines the Euler and crushing loads [7]:

$$\frac{1}{P_R}=\frac{1}{P_e}+\frac{1}{P_c} \quad \text{…………………………………} (6)$$

Where

PR: is the Rankine load

Pc: is the ultimate crushing load for the column

PE: is the Euler critical load

Struts with relatively short struts Pe is a so large, so, 1/Pe can be ignored, resulting in PR = Pc. Pe is very small for very long columns, and 1/Pe is quite large, hence l/Pc can be ignored. As a result, PR = Pe For extreme levels of L/k, the Rankine formula holds true. For the intermediate values in the range under examination, it is also determined to be fairly accurate.

The final formula is [8]. Full derivation of Rankine formula:

$$PR = \frac{\sigma_y A}{1+a(\frac{L}{K})^2} \text{……………………………} (7)$$

Where

σy: is the yield stress in compression

a: Rankine's constant = $\sigma y/\pi^2 E$

1-4 solid works software

Solid works software 2018 was used to create a numerical model, which was then compared to the experimental data. Tables 6 illustrate the numerical results of critical buckling under dynamic increasing load without a factor of safety (F.S). The table below shows the percentage discrepancy between experimental and numerical findings. The disparity may be explained by the possibility that some error in interpreting the experimental data may occur as a result of the assumptions made in the solid works program 1and the difficulties associated with regulating the measurement in the experimental work.

2 Experimental Work.

This section covers the mechanical characteristics of 304 stainless steel as well as the specifics of the specimens utilized. Table 2 lists the mechanical characteristics.

Table (2) Mechanical properties of 304stainless steel

304 stainless steel	UTS (Mpa)	(Mpa)Ys 0.2% Proof Stress	E	G	µ Poi.ratio	ϵ% Elongation
Standard ASTM A370	**621**	**290**	**193-200**	**74-77**	**0.30**	**55**
Experimental	**625**	**305**	**198**	**76**	**0.33**	**50**

For columns design, Table 3, shows the size of the sample that was utilized for 304 stainless steel.

Table (3) columns buckling dimension

NO.	**L (mm)**	**L_e (mm)**	**D (mm)**	**A (mm^2)**	**S.R**	**C_c**	**TYPE OF COLUMN**
1	500	400	5	19.634	160	115	long
2	500	400	5	19.634	160	115	long
3	500	400	5	19.634	160	115	long
4	400	300	5	19.634	110	115	intermediate
5	400	300	5	19.634	110	115	intermediate
6	400	300	5	19.634	110	115	intermediate

Also, Table No. (4) Show the Initial deflection of the specimen the columns of stainless steel 304.

K is set to 0.7 (Fixed – Pinned) in the tables above (notice Fig.1).

The Failure Definition is defined as the point at which the specimen buckles to about 1% of its overall length. [9] [10].

The test equipment and experimental methodologies are described in full in [5] [9].

3-Experimental Results.

All specimens are subjected to increasing compressive loads and rotational speeds of **50 RPM**. The applied shear stress was assumed to be a constant value (small value which was neglected) [9].

Each specimen exhibits an initial deviation caused by manufacture.

Table 4 and5 show experimental data obtained straight from the test equipment.

Table (4) displays the outcomes of columns subjected to buckling alone or in combination with buckling shot peening for both long and intermediate columns.

NO.	**L (mm)**	**L_e (mm)**	**D (mm)**	**A (mm^2)**	**S.R**	**P_{cr} (N)**	**C_c**	**TYPE OF COLUMN**
1	500	400	5	19.634	160	198	115	long
2	500	400	5	19.634	160	202	115	long
3	500	400	5	19.634	160	196	115	long
4	400	300	5	19.634	110	455	115	intermediate
5	400	300	5	19.634	110	466	115	intermediate
6	400	300	5	19.634	110	469	115	intermediate

Table (5) Initial deflection of specimens

NO.	**Ơin (mm)**	**Ơcr (mm)**
1	0.31	4.8
2	0.26	4.31
3	0.27	4.6
4	0.23	3.39
5	0.29	3.41
6	0.16	3.25

4- Discussion:

Equation (5) of Using commercial works software, Euler theory was used to estimate the critical load (Pcr) for long columns while Rankine theory was used to predict the critical load (Pcr) for short columns. The data included in the preceding Tables 7

Table (6) Comparison between (Pcr) using Euler and Rankine and sold works software theories with the experimental results.

.NO	P_{cr} Experimental (N)	P_{cr} Euler (N)	S.F
1	198	489.4	2.44
2	202	489.4	2.39
3	196	489.4	2.46

NO.	P_{cr} Experimental (N)	P_{cr} Rankine (N)	S.F
1	198	408.85	2.06
2	202	408.85	2.02
3	196	408.85	2.08
4	455	690.22	1.51
5	466	690.22	1.48
6	469	690.22	1.47

NO.	P_{cr} Experimental (N)	P_{cr} solid works (N)	S.F
1	198	226	1.14
2	202	226	1.11
3	196	226	1.15
4	455	520	1.14
5	466	520	1.11
6	469	520	1.10

The results of the tests in Table 7 indicate that the Euler and Rankine calculations for the sample employed overestimate the critical buckling loads. The following factors may contribute to this overestimation:

1- In Euler & Rankine theories, the column is perfect (no initial deflection), however this parameter has the most influence on the critical dynamic buckling load in experiments [8].

2. Static buckling columns are subjected to the Euler and Rankine formulae, whereas dynamic buckling loads reduce the critical load [9].
3. When employing the Euler formula, For a short column, the critical load (Pcr) is impacted by both the material's strength and stiffness (E), while for a long column, the material's strength is irrelevant.

A design factor of three is used in the majority of machine design applications.

A smaller factor, such as 2.0, can be employed for columns that are stationary & have a known load and end fixity.

Column analysis and design are concerned with ensuring that the load applied to a column is safe and that it is considerably below the critical buckling load. [3] and [10].

Then

Pa $=\frac{p_{cr}}{N}$ ……………….. (8)

Where

Pa = allowable load.

P_{cr} = critical buckling load.

N = Design Factor

Pa must be smaller than the actual applied load (P).

Equation (8) is applied to the data in Table 6. Table 7 summarizes the findings.

Table 7: Comparison of the theoretically determined safe load (P_a) with the experimentally determined safe load (P_{cr}).

.NO	P_{cr} Experimental (N)	P_{cr} Euler (N)	with N=3
1	198	489.4	163.3
2	202	489.4	163.3
3	196	489.4	163.3

NO.	P_{cr} Experimental (N)	P_{cr} rankine (N)	with N=3
1	198	408.85	136.2
2	202	408.85	136.2
3	196	408.85	136.2
4	455	690.22	230
5	466	690.22	230
6	469	690.22	230

NO.	P_{cr} Experimental (N)	P_{cr} solid works (N)	with N=3
1	198	226	75.3
2	202	226	75.3
3	196	226	75.3
4	455	520	173.3
5	466	520	173.3
6	469	520	173.3

5- Conclusions:

1- With a design factor of 3 or higher, Euler formulae may be utilized to estimate the safe design load under rotational compression loading.
2- The critical buckling load is affected by the initial deflection of columns (**Ơ** initial). Increasing the initial deflection (**Ơ** initial) reduces the critical buckling load.
3- When a reasonable factor of safety is included, the Rankin theory agrees well with the experimental data.
4- The result of unmedical computations gave resendable estimation of buckling load and good agreement has been a achieved when compared with the experimental taking in to account the factor of safety

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