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Solving a Fuzzy Linear System of First Order in n - Dimension Via Fuzzy logic

Samer Thaaban Abaas Alshibley(1,2) Athraa Neamah Albukhuttar(2) Ameera Nema Alkiffai(2)

(1) Department of Mathematics, Faculty of Education, University of Kufa, Najaf, Iraq.

(2) Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq.

Abstract: In this paper, fuzzy Aboodh transform used to solve system of fuzzy linear differential equations of first order in n -dimensions. Moreover, support work with an applied example in one of the institution.

Keywords: fuzzy number; fuzzy Aboodh transform; system of fuzzy linear first order differential equation in n -dimensions.

2. Introduction:

In the recent years, several authors have worked much attention to the study of fuzzy integral transform and fuzzy system of differential equations. We refer to [1, 2, 3, 4], where the reader will find the theoretical results necessary to deal with this kind of integrals and equations.

In the other hand, many researchers have studied to the "fuzzification" of varied approaches that are commonly used in the crisp case , and they have developed some fuzzy version of methods: fuzzy Laplace, Sumudu, Elzaki, ...etc (see [5,6,7,8] and the references therein).

Calculating the costs of certain activities within governmental or private institutions is one of the uses of ambiguous mathematical systems, which help accountants give an approximate future picture to investors in calculating the costs of specific activities for the coming years depending on the relationship between these activities and their representation in a fuzzy mathematical system.

In the past, the accountant used the ABC system to calculate costs, but this system neglects the time component and the relationship between costs [9]. Later, the TDABC system arose, which calculates costs and takes time into account [10]. Samer et al, used fuzzy systems in the second dimension to approximate costs (system research) [11]. In this paper, we develop the slavery technique to obtain the formula of general solutions for a system of first-order fuzzy linear differential equations in n dimensions, as well as an example in dimension three solved using this formula.

2. Basic Concepts

This section introduces several terminology keys and basic ideas.

Definition (2.1) [5]: The mapping Ꜧ: Ɽ → [0,1] is fuzzy number if satisfies

I. Ꜧ is upper semi-continuous.

II. Ꜧ is fuzzy convex, i. e., Ꜧ(ςƀ + (1 − ς)ŧ) ≥ min{Ꜧ(ƀ), Ꜧ(ŧ)},for all ƀ , ŧ ∈ Ɽ and ς ∈ [0,1].

III. Ꜧ is normal i. e., ∃ x̧$_0$ ∈ Ɽ for which Ꜧ(x̧) = 1.

IV. Supp (Ꜧ) = { x̧ ∈ Ɽ;Ꜧ(x̧) > 0 }, and $cl(Supp$ (Ꜧ)) is compact.

Definition (2.2) [12]: Assume that $\Psi, \Phi \in$ Ɽ$_f$.Where there is $\Upsilon \in$ Ɽ$_f$ such that $\Psi = \Phi + \Upsilon$ then ψ is known the H-differential of Ψ and Φ and it is represented by $\Psi \ominus \Phi$.

Note that in this work, the sign $\ominus$ always meant the Ḩ-difference as well as $\Psi \ominus \Phi \neq \Psi + (-1)\Phi$.

Definition (2.3) [13]: A parametrically ordered pair is a fuzzy number $(\underline{\rho}, \overline{\rho})$ of functions $\overline{\rho}(\varsigma)$, $\underline{\rho}(\varsigma)$, $\varsigma \in [0,1]$, which satisfies

I. $\underline{\rho}(\varsigma)$ is a non-decreasing bounded, 0 continuous right, and (0,1] continuous left function.

II. $\overline{\rho}(\varsigma)$ is a non-increasing bounded, 0 continuous right and (0,1] continuous left function.

III. $\underline{\rho}(\varsigma) \leq \overline{\rho}(\varsigma)$, $\varsigma \in [0,1]$.

Theorem (2.1) [14]: Let $Ƕ(x)$ be a fuzzy valued function on $[e,\infty)$ embodied by $((\underline{Ƕ}(x,\varsigma),\overline{Ƕ}(x,\varsigma))$ .For any fixed $\varsigma \in [0,1]$, let $(\underline{Ƕ}(x,\varsigma),\overline{Ƕ}(x,\varsigma)$ are Riemann-integrals on $[e,r]$. For every $r \geq e$, if two positive functions exist $\underline{\theta}(\varsigma)$ and $\overline{\theta}(\varsigma)$ such that $\int_e^r |\underline{Ƕ}(x,\varsigma)| dx \leq \underline{\theta}(\varsigma)$ and $\int_e^r |\overline{Ƕ}(x,\varsigma)| dx \leq \overline{\theta}(\varsigma)$ for every $r \geq e$, then $Ƕ(x)$ is said to be improper fuzzy Riemann-Liouville integrals function on $[e,\infty)$, i.e.

$$\int_e^\infty Ƕ(x)\, dx = [\int_e^\infty (\underline{Ƕ}(x,\varsigma) dx, \int_e^\infty \overline{Ƕ}(x,\varsigma) dx].$$

Definition (2.4) [15]: A function $Ƕ: (e,r): \longrightarrow \mathbb{R}_{\mathcal{F}}$ and $x_0 \in (e,r)$. We say that a mapping $Ƕ$ is strongly generalized differentiable at x_0 if there exists an element $Ƕ'(x_0) \in \mathbb{R}_{\mathcal{F}}$, such that:

i. $\forall\ Ʈ > 0$ that is sufficiently small, there exist $Ƕ(x_0 + Ʈ) \ominus Ƕ(x_0), Ƕ(x_0) \ominus Ƕ(x_0 - Ʈ)$,

where $\lim_{Ʈ \to 0} \frac{Ƕ(x_0+Ʈ)\ominus Ƕ(x_0)}{Ʈ} = \lim_{Ʈ \to 0} \frac{Ƕ(x_0)\ominus Ƕ(x_0-Ʈ)}{Ʈ} = Ƕ'(x_0)$ ***or***

ii. $\forall\ Ʈ > 0$ that is sufficiently small, there exist $Ƕ(x_0) \ominus Ƕ(x_0 + Ʈ), Ƕ(x_0 - Ʈ) \ominus Ƕ(x_0)$

where $\lim_{Ʈ \to 0} \frac{Ƕ(x_0)\ominus Ƕ(x_0+Ʈ)}{-Ʈ} = \lim_{Ʈ \to 0} \frac{Ƕ(x_0-Ʈ)\ominus Ƕ(x_0)}{-Ʈ} = Ƕ'(x_0)$ ***or***

iii. $\forall\ Ʈ > 0$ that is sufficiently small, there exist $Ƕ(x_0 + Ʈ) \ominus Ƕ(x_0), Ƕ(x_0 - Ʈ) \ominus Ƕ(x_0)$

where $\lim_{Ʈ \to 0} \frac{Ƕ(x_0+Ʈ)\ominus Ƕ(x_0)}{Ʈ} = \lim_{\tau \to 0} \frac{Ƕ(x_0-Ʈ)\ominus Ƕ(x_0)}{-Ʈ} = Ƕ'(x_0)$ ***or***

iv. $\forall\ Ʈ > 0$ that is sufficiently small, there exist $Ƕ(x_0) \ominus Ƕ(x_0 + Ʈ), Ƕ(x_0) \ominus Ƕ(x_0 - Ʈ)$

where $\lim_{Ʈ \to 0} \frac{Ƕ(x_0)\ominus Ƕ(x_0+Ʈ)}{-Ʈ} = \lim_{Ʈ \to 0} \frac{Ƕ(x_0)\ominus Ƕ(x_0-Ʈ)}{Ʈ} = Ƕ'(x_0)$.

Theorem (2.2) [5]: Let $Ƕ(x): [e,r] \longrightarrow \mathbb{R}_f$ be a function and represent $Ƕ(x) = ((\underline{Ƕ}(x,\varsigma),\overline{Ƕ}(x,\varsigma))$ in each case for $\varsigma \in [0,1]$. Then:

1. $Ƕ(x)$ is differentiable in form i, then $(\underline{Ƕ}(x,\varsigma)$ and $\overline{Ƕ}(x,\varsigma)$ are differentiable functions and

$Ƕ'(x) = (\underline{Ƕ}'(x,\varsigma), \overline{Ƕ}'(x,\varsigma))$.

2. If $Ƕ(x)$ is differentiable in form ii, then $(\underline{Ƕ}(x,\varsigma)$ and $\overline{Ƕ}(x,\varsigma)$ are differentiable functions and

$Ƕ'(t) = (\overline{Ƕ}'(x,\varsigma), \underline{Ƕ}'(x,\varsigma))$.

Definition (2.5) [8]: Let $Ƕ(x)$ be a continuous fuzzy-valued function. Assume that $\frac{1}{s}Ƕ(x)e^{-sx}$ is an improper fuzzy Rimann-integrable on $[0,\infty)$, then $\frac{1}{s}\int_0^\infty Ƕ(x)e^{-sx}\, dx$ is being called fuzzy Aboodh transform and it is referred by $\widehat{A}[Ƕ(x)] = \frac{1}{s}\int_0^\infty Ƕ(x)e^{-sx}\, dx$, $(s > 0$ and integer$)$. Thus

$$\frac{1}{s}\int_0^\infty Ƕ(x)e^{-sx}\, dx = \left(\frac{1}{s}\int_0^\infty \underline{Ƕ}(x,\varsigma)e^{-sx}\, dx, \frac{1}{s}\int_0^\infty \overline{Ƕ}(x,\kappa)e^{-sx}\, dx\right).$$

Using the definition of classical Aboodh transform,

$A[Ƕ(x,\varsigma)] = \frac{1}{s}\int_0^\infty \underline{Ƕ}(x,\varsigma)e^{-st}\, dt$ and $A[\overline{Ƕ}(x,\varsigma)] = \frac{1}{s}\int_0^\infty Ƕ(x,\varsigma)e^{-sx}\, dx$, then

$\widehat{A}[Ƕ(x)] = (A[\underline{Ƕ}(x,\varsigma)], A[\overline{Ƕ}(x,\varsigma)])$.

Theorem (2.3) [8]: Let $Ƕ(x)$ is the primitive of $Ƕ'(x)$ on $[0,\infty)$ and $Ƕ(x)$ be an integrable fuzzy-valued function. Then:

a. $Ƕ(x)$ is (i)-differentiable and $\widehat{A}[Ƕ'(x)] = s\widehat{A}[Ƕ(x)] \ominus \frac{1}{s}Ƕ(0)$.

b. $Ƕ(x)$ is (ii)-differentiable and $\widehat{A}[Ƕ'(x)] = (-\frac{1}{s}Ƕ(0)) \ominus (-s\widehat{A}[Ƕ(x)]$.

3. General Formula of Solution Sets for Fuzzy Systems Linear Differential Equations in n-Dimensional Fuzzy Aboodh Transform

In this section, fuzzy Aboodh transform technique used for solving the following system:

$$\begin{cases} \text{ҳ}_1'(\text{ƫ}) = \text{Ƕ}_1\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right) \\ \text{ҳ}_2'(\text{ƫ}) = \text{Ƕ}_2\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right) \\ \qquad\vdots \\ \text{ҳ}_n'(\text{ƫ}) = \text{Ƕ}_n\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right) \end{cases} \qquad (1)$$

Where,

$$\begin{cases} \text{ҳ}_1(\text{ƫ}) = \left(\underline{\text{ҳ}_1}(\text{ƫ}, \varsigma), \overline{\text{ҳ}_1}(\text{ƫ}, \varsigma)\right);\ \text{ƫ} \geq 0, 0 \leq \varsigma \leq 1 \\ \text{ҳ}_2(\text{ƫ}) = \left(\underline{\text{ҳ}_2}(\text{ƫ}, \varsigma), \overline{\text{ҳ}_2}(\text{ƫ}, \varsigma)\right);\ \text{ƫ} \geq 0, 0 \leq \varsigma \leq 1 \\ \qquad\vdots \\ \text{ҳ}_\text{n}(\text{ƫ}) = \left(\underline{\text{ҳ}_\text{n}}(\text{ƫ}, \varsigma), \overline{\text{ҳ}_\text{n}}(\text{ƫ}, \varsigma)\right);\ \text{ƫ} \geq 0, 0 \leq \varsigma \leq 1, \end{cases}$$

and $\text{Ƕ}_1\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)$, $\text{Ƕ}_2\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)$, …, $\text{Ƕ}_n\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)$ are linear fuzzy value functions. Apply fuzzy Aboodh transform, to both side of (1) with Theorem (2.3):

$$\begin{cases} \widehat{\text{A}}[\text{Ƕ}_1\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)] = \begin{cases} \text{s}\widehat{\text{A}}[\text{ҳ}_1(\text{ƫ})] \ominus \frac{1}{\text{s}}\text{ҳ}_1(0) & if\ \text{ҳ}_1\ be\ (i)-differentiable \\ \widehat{\text{A}}[\text{ҳ}_1(\text{ƫ})] = -\frac{1}{\text{s}}\text{ҳ}_2(0) \ominus -\text{s}\widehat{\text{A}}[\text{ҳ}_1(\text{ƫ})] & if\ \text{ҳ}_1\ be\ (ii)-differentiable \end{cases} \\ \widehat{\text{A}}[\text{Ƕ}_2\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)] = \begin{cases} \text{s}\widehat{\text{A}}[\text{ҳ}_2(\text{ƫ})] \ominus \frac{1}{\text{s}}\text{ҳ}_2(0) & if\ \text{ҳ}_2\ be\ (i)-differentiable \\ \widehat{\text{A}}[\text{ҳ}_2(\text{ƫ})] = -\frac{1}{\text{s}}\text{ҳ}_2(0) \ominus -\text{s}\widehat{\text{A}}[\text{ҳ}_2(\text{ƫ})] & if\ \text{ҳ}_2\ be\ (ii)-differentiable \end{cases} \\ \qquad\vdots \\ \widehat{\text{A}}[\text{Ƕ}_n\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)] = \begin{cases} \text{s}\widehat{\text{A}}[\text{ҳ}_n(\text{ƫ})] \ominus \frac{1}{\text{s}}\text{ҳ}_n(0) & if\ \text{ҳ}_n\ be\ (i)-differentiable \\ \widehat{\text{A}}[\text{ҳ}_n(\text{ƫ})] = -\frac{1}{\text{s}}\text{ҳ}_n(0) \ominus -\text{s}\widehat{\text{A}}[\text{ҳ}_n(\text{ƫ})] & if\ \text{ҳ}_n\ be\ (ii)-differentiable \end{cases} \end{cases}$$

Depending on the formula of the functions $\text{Ƕ}_1\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right), \text{Ƕ}_2\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)$ and $\text{Ƕ}_n\left(\text{ƫ}, \text{ҳ}_1(\text{ƫ}), \text{ҳ}_2(\text{ƫ}), \dots, \text{ҳ}_n(\text{ƫ})\right)$ the right-hand side can be assumed with the following functions:

$$\begin{cases} \text{A}\left[\underline{\text{ҳ}_1}(\text{ƫ}, \varsigma)\right] = \mathbb{P}_1(\text{s}, \varsigma) \\ \text{A}[\overline{\text{ҳ}_1}(\text{ƫ}, \varsigma)] = \mathbb{N}_1(\text{s}, \varsigma) \\ \text{A}\left[\underline{\text{ҳ}_2}(\text{ƫ}, \varsigma)\right] = \mathbb{P}_2(\text{s}, \varsigma) \\ \text{A}[\overline{\text{ҳ}_2}(\text{ƫ}, \varsigma)] = \mathbb{N}_2(\text{s}, \varsigma) \\ \qquad\vdots \\ \text{A}\left[\underline{\text{ҳ}_n}(\text{ƫ}, \varsigma)\right] = \mathbb{P}_\text{n}(\text{s}, \varsigma) \\ \text{A}[\overline{\text{ҳ}_n}(\text{ƫ}, \varsigma)] = \mathbb{N}_\text{n}(\text{s}, \varsigma) \end{cases}$$

The solution set of system (1) is obtained by taking the inverse Aboodh transform for the previous system.

$$\begin{cases} \left[\underline{x_1}(t,\varsigma)\right] = A^{-1}[\mathbb{P}_1(s,\varsigma)] \\ [\overline{x_1}(t,\varsigma)] = A^{-1}[\mathbb{N}_1(s,\varsigma)] \\ \left[\underline{x_2}(t,\varsigma)\right] = A^{-1}[\mathbb{P}_2(s,\varsigma)] \\ [\overline{x_2}(t,\varsigma)] = A^{-1}[\mathbb{N}_2(s,\varsigma)] \\ \quad \vdots \\ \left[\underline{x_n}(t,\varsigma)\right] = A^{-1}[\mathbb{P}_n(s,\varsigma)] \\ [\overline{x_n}(t,\varsigma)] = A^{-1}[\mathbb{N}_n(s,\varsigma)] \end{cases}$$

4. Application:

To provide an approximate future picture of the size of the costs of three activities within a governmental or private institution, without appointment, which benefits investors in estimating profits and costs in advance, that helps in the success of investment projects. If we consider the following system with the initial cost of each activity.

$$x'(t) = H\left(t, x(t), y(t), z(t)\right) = x(t) + y(t) + z(t).$$

$$y'(t) = J\left(t, x(t), y(t), z(t)\right) = -y(t) - z(t).$$

$$z'(t) = \omega\left(t, x(t), y(t), z(t)\right) = 2x(t) + y(t) + z(t).$$

Under initial fuzzy conditions:

$$x(0,\varsigma) = (\varsigma, 2-\varsigma), \qquad y(0,\varsigma) = (\varsigma-1, 1-\varsigma), \qquad z(0,\varsigma) = (\varsigma-3, -2\varsigma).$$

Apply general formula in sections 3 with Theorem (2.3) such as following:

Case (1): If $x(t), z(t)$ and $y(t)$ are (i)-differentiable, then

$$\begin{cases} s\hat{A}[x(t)] \ominus \dfrac{1}{s}x(0) = \hat{A}[x(t) + y(t) + z(t)], \\ s\hat{A}[y(t)] \ominus \dfrac{1}{s}y(0) = \hat{A}[-y(t) - z(t)]. \\ s\hat{A}[z(t)] \ominus \dfrac{1}{s}z(0) = \hat{A}[2x(t) + y(t) + z(t)]. \end{cases}$$

After substitution initial condition.

$$\begin{cases} (s-1)A\left[\underline{x}(t,\varsigma)\right] - A\left[\underline{y}(t,\varsigma)\right] - A[\underline{z}(t,\varsigma)] = \dfrac{\varsigma}{s}, \\ (s-1)A[\overline{x}(t,\varsigma)] - A[\overline{y}(t,\varsigma)] - A[\overline{z}(t,\varsigma)] = \dfrac{2-\varsigma}{s}, \\ sA\left[\underline{y}(t,\varsigma)\right] + A[\overline{y}(t,\varsigma)] + A[\overline{z}(t,\varsigma)] = \dfrac{\varsigma-1}{s}, \\ sA[\overline{y}(t,\varsigma)] + A\left[\underline{y}(t,\varsigma)\right] + A\left[\underline{z}(t,\varsigma)\right] = \dfrac{1-\varsigma}{s}, \\ (s-1)A\left[\underline{z}(t,\varsigma)\right] - 2A\left[\underline{x}(t,\varsigma)\right] - A\left[\underline{y}(t,\varsigma)\right] = \dfrac{\varsigma-3}{s}, \\ (s-1)A[\overline{z}(t,\varsigma)] - 2A[\overline{x}(t,\varsigma)] - A[\overline{y}(t,\varsigma)] = \dfrac{-2\varsigma}{s}. \end{cases}$$

With simple calculation:

$$A[\underline{x}(t,\varsigma)] = \frac{-\varsigma + \varsigma s^3 - \varsigma s^2 - \varsigma s - 4s^2 + 7s + 1}{s^5 - 4s^4 + s^3 + 6s^2}.$$

$$A[\overline{x}(t,\varsigma)] = \frac{\varsigma - \varsigma s^3 + \varsigma s^2 + 4\varsigma s + 2s^3 - 5s^2 + 2s - 1}{s^5 - 4s^4 + s^3 + 6s^2}.$$

$$A[\underline{y}(t,\varsigma)] = \frac{\varsigma + \varsigma s^4 - \varsigma s^3 - 4\varsigma s^2 + 3\varsigma s - s^4 + 3s^3 - 6s^2 + 9s - 1}{s^6 - 4s^5 + s^4 + 6s^3}.$$

$$A[\overline{y}(t,\varsigma)] = \frac{-\varsigma - \varsigma s^4 + 2\varsigma s^3 + s^4 + 10s^2 + 12s + 1}{s^6 - 4s^5 + s^4 + 6s^3}.$$

$$A[\underline{z}(t,\varsigma)] = \frac{-\varsigma + \varsigma s^3 - \varsigma s^2 - \varsigma s - 3s^3 + 11s^2 - 11s + 1}{s^5 - 5s^4 + 6s^3}.$$

$$A[\overline{z}(t,\varsigma)] = \frac{\varsigma - 2\varsigma s^3 + 5\varsigma s^2 - 2\varsigma s + 5s^2 - 10s - 1}{s^5 - 5s^4 + 6s^3}.$$

Using the inverse Aboodh transform obtained the solution of case (1)

$$\underline{x}(t,\varsigma) = \varsigma\left(\frac{-1}{6} + \frac{1}{6}\, e^{-t} - \frac{1}{6}\, e^{2t} + \frac{7}{6}\, e^{3t}\right) + \frac{1}{6} + \frac{5}{6}\, e^{-t} + \frac{1}{6}\, e^{2t} - \frac{7}{6}\, e^{3t}.$$

$$\overline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6} + \frac{1}{12}\, e^{-t} - \frac{5}{6}\, e^{2t} - \frac{5}{12}\, e^{3t}\right) - \frac{1}{6} + \frac{5}{6}\, e^{-t} + \frac{1}{6}\, e^{2t} + \frac{7}{6}\, e^{3t}.$$

$$\underline{y}(t,\varsigma) = \varsigma\left(\frac{17}{36} + \frac{1}{6}\, t - \frac{1}{3}\, e^{-t} + \frac{1}{12}\, e^{2t} + \frac{7}{9}\, e^{3t}\right) + \frac{55}{36} - \frac{1}{6} t - \frac{5}{3}\, e^{-t} - \frac{1}{12}\, e^{2t} - \frac{7}{9}\, e^{3t}.$$

$$\overline{y}(t,\varsigma) = \varsigma\left(\frac{1}{36} - \frac{1}{6}\, t - \frac{1}{3}\, e^{-t} + \frac{1}{12}\, e^{2t} - \frac{7}{9}\, e^{3t}\right) + \frac{71}{36} + \frac{1}{6} t - \frac{5}{3}\, e^{-t} - \frac{1}{12}\, e^{2t} + \frac{7}{9}\, e^{3t}.$$

$$\underline{z}(t,\varsigma) = \varsigma\left(\frac{-11}{36} - \frac{1}{6}\, t - \frac{1}{4}\, e^{2t} + \frac{14}{9} e^{3t}\right)\frac{-61}{36} + \frac{1}{6}\, t + \frac{1}{4}\, e^{2t} - \frac{14}{9} e^{3t}.$$

$$\overline{z}(t,\varsigma) = \varsigma\left(\frac{-7}{36} + \frac{1}{6}\, t - \frac{1}{4}\, e^{2t} - \frac{14}{9} e^{3t}\right)\frac{-65}{36} - \frac{1}{6}\, t + \frac{1}{4}\, e^{2t} + \frac{14}{9} e^{3t}.$$

Case (2): If $x(t)$ and $z(t)$ are (i)-differentiable but $y(t)$ is (ii)-differentiable, then similar with case (1), by taking Aboodh transform and substitution, initial condition, yields:

$$\begin{cases} (s-1)A\left[\underline{x}(t,\varsigma)\right] - A\left[\underline{y}(t,\varsigma)\right] - A[\underline{z}(t,\varsigma)] = \frac{\varsigma}{s}, \\ (s-1)A[\overline{x}(t,\varsigma)] - A[\overline{y}(t,\varsigma)] - A[\overline{z}(t,\varsigma)] = \frac{2-\varsigma}{s}, \\ (s+1)A\left[\underline{y}(t,\varsigma)\right] + A\left[[\underline{z}(t,\varsigma)\right] = \frac{\varsigma - 1}{s}, \\ (s+1)A[\overline{y}(t,\varsigma)] + A[\overline{z}\,(t,\varsigma)] = \frac{1-\varsigma}{s}, \\ (s-1)A\left[\underline{z}(t,\varsigma)\right] - 2A\left[\underline{x}(t,\varsigma)\right] - A\left[\underline{y}(t,\varsigma)\right] = \frac{\varsigma - 3}{s}, \\ (s-1)A[\overline{z}(t,\varsigma)] - 2A[\overline{x}(t,\varsigma)] - A[\overline{y}(t,\varsigma)] = \frac{-2\varsigma}{s}. \end{cases}$$

Therefore

$$A[\underline{x}(t,\varsigma)] = \frac{\varsigma s + 2\varsigma + 4}{s^3 - s^2 - 2s}.$$

$$A[\overline{x}(t,\varsigma)] = \frac{-\varsigma s - 3\varsigma + 1 + 2s}{s^3 - s^2 - 2s}.$$

$$A[\underline{y}(t,\varsigma)] = \frac{-2\varsigma + \varsigma s^2 - 3\varsigma s - 2 + 5s - s^2}{s^4 - s^3 - 2s^2}.$$

$$A[\overline{y}(t,\varsigma)] = \frac{\varsigma - \varsigma s^2 + 4\varsigma s - 5 - 2s + s^2}{s^4 - s^3 - 2s^2}.$$

$$A[\underline{z}(t,\varsigma)] = \frac{2\varsigma + \varsigma s + 2 - 3s}{s^3 - 2s^2}.$$

$$A[\overline{z}(t,\varsigma)] = \frac{-\varsigma - 2\varsigma s + 5}{s^3 - 2s^2}.$$

The inverse Aboodh transform obtained the solution of case (2)

$$\underline{x}(t,\varsigma) = \varsigma\left(-\frac{1}{3}\, e^{-t} + \frac{4}{3}\, e^{2t}\right) - \frac{4}{3}\, e^{-t} + \frac{4}{3}\, e^{2t}.$$

$$\overline{x}(t,\varsigma) = \varsigma\left(\frac{2}{3}\,e^{-t} - \frac{5}{3}\,e^{2t}\right) + \frac{1}{3}\,e^{-t} + \frac{5}{3}\,e^{2t}.$$

$$\underline{y}(t,\varsigma) = \varsigma\left(1 + \frac{2}{3}e^{-t} - \frac{2}{3}\,e^{2t}\right) + 1 - \frac{8}{3}e^{-t} + \frac{2}{3}\,e^{2t}.$$

$$\overline{y}(t,\varsigma) = \varsigma\left(-\frac{1}{2} - \frac{4}{3}e^{-t} + \frac{5}{6}\,e^{2t}\right) + \frac{5}{2} - \frac{2}{3}e^{-t} - \frac{5}{6}\,e^{2t}.$$

$$\underline{z}(t,\varsigma) = \varsigma(-1 - 2e^{2t}) + 1 - 2e^{2t}.$$

$$\overline{z}(t,\varsigma) = \varsigma\left(\frac{1}{2} - \frac{5}{2}e^{2t}\right) - \frac{5}{2} + \frac{5}{2}e^{2t}.$$

Case (3): If $x(t)$ and $y(t)$ are (i)-differentiable but $z(t)$ is (ii)-differentiable, then

$$\begin{cases} s\widehat{A}[x(t)] \ominus \frac{1}{s}x(0) = \widehat{A}\,[x(t) + y(t) + z(t)], \\ s\widehat{A}[y(t)] \ominus \frac{1}{s}y(0) = \widehat{A}[-y(t) - z(t)], \\ -\frac{1}{s}z(0) \ominus -s\widehat{A}[z(t)] = \widehat{A}[2x(t) + y(t) + z(t)]. \end{cases}$$

After substitution, initial condition and simple calculation

$$A[\,\underline{x}(t,\varsigma)] = \frac{-6\varsigma + \varsigma s^3 + \varsigma s^2 - 4\varsigma s - 4s^2 + 8s + 2}{s^5 - 2s^4 + s^3 - 4s}.$$

$$A[\,\overline{x}(t,\varsigma)] = \frac{4\varsigma - \varsigma s^3 - 2\varsigma s^2 + 5\varsigma s + 2s^3 - s^2 - s - 8}{s^5 - 2s^4 + s^3 - 4s}.$$

$$A[\,\underline{y}(t,\varsigma)] = \frac{4\varsigma + \varsigma s^4 + \varsigma s^3 - 7\varsigma s^2 + \varsigma s - 4s^4 + s^3 + s^2 + 7s - 12}{s^6 - 2s^5 + s^4 - 4s^2}.$$

$$A[\,\overline{y}(t,\varsigma)] = \frac{-6\varsigma - \varsigma s^4 + 5\varsigma s^2 + 2\varsigma s + 4s^4 + 2s^3 - 11s^2 + 6s - 2}{s^6 - 2s^5 + s^4 - 4s^2}.$$

$$A[\,\underline{z}(t,\varsigma)] = \frac{-4\varsigma + \varsigma s^3 - 8\varsigma s^2 + 13\varsigma s - 3s^3 + 14s^2 - 21s + 12}{s^5 - 3s^4 + 4s^3 - 4s^2}.$$

$$A[\,\overline{z}(t,\varsigma)] = \frac{6\varsigma - 2\varsigma s^3 + 10\varsigma s^2 - 16\varsigma s - 4s^2 + 8s + 2}{s^5 - 3s^4 + 4s^3 - 4s^2}.$$

Using the inverse Aboodh transform to obtain the solution of case (3)

$$\underline{x}(t,\varsigma) = \varsigma\left(\frac{1}{2}e^{-t} - \frac{1}{2}e^{2t} + e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{6}{7}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) + \frac{5}{6}e^{-t} + \frac{1}{6}e^{2t} - e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{6}{7}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

$$\overline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6}e^{-t} - \frac{1}{6}e^{2t} - e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{6}{\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) + \frac{5}{6}e^{-t} + \frac{1}{6}e^{2t} + e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{6}{\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

$$\underline{y}(t,\varsigma) = \varsigma\left(-1 - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t} + \frac{9}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{11}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) + 3 - \frac{5}{3}e^{-t} - \frac{1}{12}e^{2t} - \frac{9}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{11}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

$$\overline{y}(t,\varsigma) = \varsigma\left(\frac{3}{2} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t} - \frac{9}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{11}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) + \frac{1}{2} - \frac{5}{3}e^{-t} - \frac{1}{12}e^{2t} + \frac{9}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{11}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

$$\underline{z}(t,\varsigma) = \varsigma\left(1 - \frac{1}{4}e^{2t} + \frac{1}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{37}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) - 6 + \frac{7}{4}e^{2t} + \frac{5}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{\sqrt{7}}{4}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

$$\overline{z}(t,\varsigma) = \varsigma\left(\frac{-3}{2} - \frac{1}{4}e^{2t} - \frac{1}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{37}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) - \frac{1}{2} + \frac{1}{4}e^{2t} + \frac{1}{4}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{37}{4\sqrt{7}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

Case (4): If $y(t)$ and $z(t)$ are (i)-differentiable but $x(t)$ is (ii)-differentiable, then

$$\begin{cases} -\frac{1}{s}\underline{x}(0) \ominus -s\hat{A}[x(t)] = \hat{A}\,[x(t) + y(t) + z(t)], \\ \quad s\hat{A}[y(t)] \ominus \frac{1}{s}y(0) = \hat{A}[-y(t) - z(t)], \\ s\hat{A}[z(t)] \ominus \frac{1}{s}z(0) = \hat{A}[2x(t) + y(t) + z(t)]. \end{cases}$$

With steps similar to the pervious cases, it is possible to obtain the solutions following cases, such as :

$$A[\underline{x}(t,\varsigma)] = \frac{\varsigma + \varsigma s^3 - 2\varsigma s^2 - 2s^2 + 7}{s^5 - 2s^4 - 5s^3 + 6s^2 + 8s}.$$

$$A[\overline{x}(t,\varsigma)] = \frac{3\varsigma - \varsigma s^3 + 3\varsigma s + 2s^3 - 3s^2 - 5s + 5}{s^5 - 2s^4 - 5s^3 + 6s^2 + 8s}.$$

$$A[\underline{y}(t,\varsigma)] = \frac{\varsigma + \varsigma s^4 + \varsigma s^3 - 4\varsigma s^2 - 5\varsigma s - s^4 + s^3 - 2s^2 + s + 15}{s^6 - 2s^5 - 5s^4 + 6s^3 + 8s^2}.$$

$$A[\overline{y}(t,\varsigma)] = \frac{3\varsigma - \varsigma s^4 + 2\varsigma s^2 - 2\varsigma s + s^4 + 2s^3 - 8s^2 - 6s + 13}{s^6 - 2s^5 - 5s^4 + 6s^3 + 8s^2}.$$

$$A[\underline{z}(t,\varsigma)] = \frac{-\varsigma + \varsigma s^3 + \varsigma s^2 - 5\varsigma s - 3s^3 + 5s^2 + 9s - 15}{s^5 - 3s^4 - 2s^3 + 8s^2}.$$

$$A[\overline{z}(t,\varsigma)] = \frac{-3\varsigma - 2\varsigma s^3 + \varsigma s^2 + 8\varsigma s + 5s^2 - 4s - 13}{s^5 - 3s^4 - 2s^3 + 8s^2}.$$

By the inverse Aboodh transform obtained the solution of case (4)

$$\underline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6}e^{-t} - \frac{1}{6}e^{2t} + e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) + \frac{2}{\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right)\right) + \frac{5}{6}e^{-t} + \frac{1}{6}e^{2t} - e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) - \frac{2}{\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right).$$

$$\overline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6}e^{-t} - \frac{1}{6}e^{2t} - e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) - \frac{2}{\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right)\right) + \frac{5}{6}e^{-t} + \frac{1}{6}e^{2t} + e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) + \frac{2}{\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right).$$

$$\underline{y}(t,\varsigma) = \varsigma\left(\frac{1}{8} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t} + \frac{9}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) + \frac{31}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right)\right) + \frac{15}{8} - \frac{5}{3}e^{-t} - \frac{1}{12}e^{2t} - \frac{9}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) - \frac{31}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right).$$

$$\overline{y}(t,\varsigma) = \varsigma\left(\frac{3}{8} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t} - \frac{9}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) - \frac{31}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right)\right) + \frac{13}{8} - \frac{5}{3}e^{-t} - \frac{1}{12}e^{2t} + \frac{9}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) + \frac{31}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right).$$

$$\underline{z}(t,\varsigma) = \varsigma\left(-\frac{1}{8} - \frac{1}{4}e^{2t} + \frac{11}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) + \frac{61}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right)\right) - \frac{15}{8} + \frac{1}{4}e^{2t} - \frac{11}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) - \frac{61}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right).$$

$$\overline{z}(t,\varsigma) = \varsigma\left(-\frac{3}{8} - \frac{1}{4}e^{2t} - \frac{11}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) - \frac{61}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right)\right) - \frac{13}{8} + \frac{1}{4}e^{2t} + \frac{11}{8}e^{\frac{t}{2}}\cos\left(\frac{\sqrt{17}t}{2}\right) + \frac{61}{8\sqrt{17}}e^{\frac{t}{2}}\sin\left(\frac{\sqrt{17}t}{2}\right).$$

Case (5): If $x(t), z(t)$ and $y(t)$ are (ii)-differentiable, then

$$A[\underline{x}(t,\varsigma)] = \frac{\varsigma + \varsigma s^3 - 2\varsigma s^2 - 3\varsigma s + 3s^2 - 3s - 1}{s^5 + 2s^4 - 5s^3 - 6s^2}.$$

$$A[\overline{x}(t,\varsigma)] = \frac{-\varsigma - \varsigma s^3 + \varsigma s^2 + 2s^3 - 6s + 1}{s^5 + 2s^4 - 5s^3 - 6s^2}.$$

$$A[\underline{y}(t,\varsigma)] = \frac{\varsigma + \varsigma s^4 - 2\varsigma s^2 - 2\varsigma s - s^4 + 2s^3 + 4s^2 - 10s - 1}{s^6 + 2s^5 - 5s^4 - 6s^3}.$$

$$A[\overline{y}(t,\varsigma)] = \frac{-\varsigma - \varsigma s^4 + \varsigma s^3 + 4\varsigma s^2 - \varsigma s + s^4 + s^3 - 2s^2 - 11s + 1}{s^6 + 2s^5 - 5s^4 - 6s^3}.$$

$$A[\underline{\mathfrak{z}}(t,\varsigma)] = \frac{-\varsigma + \varsigma s^3 - 4\varsigma s^2 + 2\varsigma s - 3s^3 + 2s^2 + 10s + 1}{s^5 + s^4 - 6s^3}.$$

$$A[\overline{\mathfrak{z}}(t,\varsigma)] = \frac{\varsigma - 2\varsigma s^3 + 2\varsigma s^2 + \varsigma s - 4s^2 + 11s - 1}{s^5 + s^4 - 6s^3}.$$

Using the inverse Aboodh transform obtained the solution of case (5)

$$\underline{x}(t,\varsigma) = \varsigma\left(-\frac{1}{6} + \frac{1}{6}e^{-t} - \frac{1}{6}e^{2t} + \frac{7}{6}e^{-3t}\right) + \frac{1}{6} + \frac{5}{6}e^{-t} + \frac{1}{2}e^{2t} - \frac{7}{6}e^{-3t}.$$

$$\overline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6} + \frac{1}{6}e^{-t} - \frac{1}{6}e^{2t} - \frac{7}{6}e^{-3t}\right) - \frac{1}{6} + \frac{5}{6}e^{-t} + \frac{1}{2}e^{2t} + \frac{7}{6}e^{-3t}.$$

$$\underline{y}(t,\varsigma) = \varsigma\left(\frac{17}{36} - \frac{1}{6}t - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t} + \frac{7}{9}e^{-3t}\right) + \frac{55}{36} + \frac{1}{6}t - \frac{5}{3}e^{-t} - \frac{1}{12}e^{2t} - \frac{7}{9}e^{-3t}.$$

$$\overline{y}(t,\varsigma) = \varsigma\left(\frac{1}{36} + \frac{1}{6}t - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t} - \frac{7}{9}e^{-3t}\right) + \frac{71}{36} - \frac{1}{6}t - \frac{5}{3}e^{-t} - \frac{1}{12}e^{2t} + \frac{7}{9}e^{-3t}.$$

$$\underline{z}(t,\varsigma) = \varsigma\left(\frac{-11}{36} + \frac{1}{6}t - \frac{1}{4}e^{2t} + \frac{14}{9}e^{-3t}\right) + \frac{-16}{36} - \frac{1}{6}t + \frac{1}{4}e^{2t} - \frac{14}{9}e^{-3t}.$$

$$\overline{z}(t,\varsigma) = \varsigma\left(\frac{-7}{36} - \frac{1}{6}t - \frac{1}{4}e^{2t} - \frac{14}{9}e^{-3t}\right) + \frac{-65}{36} + \frac{1}{6}t + \frac{1}{4}e^{2t} + \frac{14}{9}e^{-3t}.$$

Case (6): If $x(t)$ and $z(t)$ are (ii)-differentiable but $y(t)$ is (i)-differentiable, then

$$A[\underline{x}(t,\varsigma)] = \frac{6\varsigma + s^3\varsigma - 4\varsigma s^2 + 3s^2 - 4s - 2}{s^5 - 5s^3 + 4s}.$$

$$A[\overline{x}(t,\varsigma)] = \frac{-4\varsigma - \varsigma s^3 + 3\varsigma s^2 - \varsigma s + 2s^3 - 4s^2 - 3s + 8}{s^5 - 5s^3 + 4s}.$$

$$A[\underline{y}(t,\varsigma)] = \frac{4\varsigma + \varsigma s^4 + 3\varsigma s^3 - 7\varsigma s^2 - 7\varsigma s - s^4 - s^3 + 5s^2 - s + 4}{s^6 - 5s^4 + 4s^2}.$$

$$A[\overline{y}(t,\varsigma)] = \frac{-2\varsigma - \varsigma s^4 - 2\varsigma s^3 + 7\varsigma s^2 + 4\varsigma s + s^4 + 4s^3 - 9s^2 - 12s + 10}{s^6 - 5s^4 + 4s^2}.$$

$$A[\underline{z}(t,\varsigma)] = \frac{4\varsigma + \varsigma s^2 - 5\varsigma s - 3s^2 + 5s + 4}{s^4 - 4s^2}.$$

$$A[\overline{z}(t,\varsigma)] = \frac{-2\varsigma - 2\varsigma s^2 + 4\varsigma s - 4s + 10}{s^4 - 4s^2}.$$

Using the inverse Aboodh transform obtained the solution of case (6)

$$\underline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6}e^{-t} - \frac{1}{2}e^{t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{2t}\right) - \frac{1}{6}e^{-t} + \frac{1}{2}e^{t} + \frac{3}{2}e^{-2t} + \frac{7}{6}e^{2t}.$$

$$\overline{x}(t,\varsigma) = \varsigma\left(\frac{1}{6}e^{-t} + \frac{1}{2}e^{t} - \frac{3}{2}e^{-2t} - \frac{1}{6}e^{2t}\right) + \frac{13}{6}e^{-t} - \frac{11}{2}e^{t} + \frac{5}{6}e^{-2t} + \frac{5}{6}e^{2t}.$$

$$\underline{y}(t,\varsigma) = \varsigma\left(1 - \frac{1}{3}e^{-t} + e^{t} - \frac{3}{4}e^{-2t} + \frac{1}{12}e^{2t}\right) + 1 - \frac{5}{3}e^{-t} - e^{t} + \frac{3}{4}e^{-2t} - \frac{1}{12}e^{2t}.$$

$$\overline{y}(t,\varsigma) = \varsigma\left(-\frac{1}{2} - \frac{1}{3}e^{-t} - e^{t} + \frac{3}{4}e^{-2t} + \frac{1}{12}e^{2t}\right) + \frac{5}{2} - \frac{5}{3}e^{-t} + e^{t} - \frac{3}{4}e^{-2t} - \frac{1}{12}e^{2t}.$$

$$\underline{z}(t,\varsigma) = \varsigma\left(-1 + \frac{9}{4}e^{-2t} - \frac{1}{4}e^{2t}\right) + 1 - \frac{9}{4}e^{-2t} + \frac{1}{4}e^{2t}.$$

$$\overline{z}(t,\varsigma) = \varsigma\left(\frac{1}{2} - \frac{9}{4}e^{-2t} - \frac{1}{4}e^{2t}\right) - \frac{5}{2} + \frac{9}{4}e^{-2t} + \frac{1}{4}e^{2t}.$$

Case (7): If $x(t)$ and $y(t)$ are (ii)-differentiable but $z(t)$ is (i)-differentiable, then

$$A[\underline{x}(t,\varsigma)] = \frac{4\varsigma + s^3\varsigma - 4\varsigma s^2 + 3s^2 - 8}{s^5 - s^3 - 4s^2 - 4s}.$$

$$A[\overline{x}(t,\varsigma)] = \frac{-6\varsigma - s^3\varsigma + 3\varsigma s^2 - \varsigma s + 2s^3 - 4s^2 + s + 2}{s^5 - s^3 - 4s^2 - 4s}.$$

$$A[\underline{y}(t,\varsigma)] = \frac{4\varsigma + \varsigma s^4 - 2\varsigma s^3 - 3\varsigma s^2 + 6\varsigma s - s^4 + 4s^3 + s^2 - 6s - 12}{s^6 - s^4 - 4s^3 - 4s^2}.$$

$$A[\overline{y}(t,\varsigma)] = \frac{-6\varsigma - \varsigma s^4 + 3\varsigma s^3 + 7\varsigma s^2 - 5\varsigma s + s^4 + 4s^3 + s^2 - 6s - 12}{s^6 - s^4 - 4s^3 - 4s^2}.$$

$$A[\underline{z}(t,\varsigma)] = \frac{-4\varsigma + \varsigma s^3 + 3\varsigma s^2 - 10\varsigma s - 3s^3 - s^2 + 10s + 12}{s^5 - s^4 - 4s^2}.$$

$$A[\overline{z}(t,\varsigma)] = \frac{6\varsigma - 2\varsigma s^3 - 3\varsigma s^2 + 9\varsigma s + 5s^2 - 9s + 2}{s^5 - s^4 - 4s^2}.$$

Using the inverse Aboodh transform obtained the solution of case (7)

$$\underline{x}(t,\varsigma) = \varsigma\left(e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{6}{\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) + \frac{1}{6}\,e^{-t} - \frac{1}{6}\,e^{2t}\right) + \frac{1}{2}e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{3}{2\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) + \frac{11}{6}\,e^{-t} + \frac{2}{3}\,e^{2t}.$$

$$\overline{x}(t,\varsigma) = \varsigma\left(-e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{6}{\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) + \frac{1}{6}\,e^{-t} - \frac{1}{6}\,e^{2t}\right) + e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{6}{\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) + \frac{5}{6}\,e^{-t} + \frac{1}{6}\,e^{2t}.$$

$$\underline{y}(t,\varsigma) = \varsigma\left(-1 + \frac{9}{4}\,e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{11}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) - \frac{1}{3}\,e^{-t} + \frac{1}{12}\,e^{2t}\right) + 3 - \frac{9}{4}e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{11}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) - \frac{5}{3}\,e^{-t} - \frac{1}{12}\,e^{2t}.$$

$$\overline{y}(t,\varsigma) = \varsigma\left(\frac{3}{2} - \frac{9}{4}\,e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{11}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) - \frac{1}{3}\,e^{-t} + \frac{1}{12}\,e^{2t}\right) + \frac{1}{2} + \frac{9}{4}e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{11}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right) - \frac{5}{3}\,e^{-t} - \frac{1}{12}\,e^{2t}.$$

$$\underline{z}(t,\varsigma) = \varsigma\left(1 - \frac{1}{4}\,e^{2t} + \frac{1}{4}\,e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{37}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) - 3 + \frac{1}{4}\,e^{2t} - \frac{1}{4}e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{37}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

$$\overline{z}(t,\varsigma) = \varsigma\left(-\frac{3}{2} - \frac{1}{4}\,e^{2t} - \frac{1}{4}\,e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) - \frac{37}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right)\right) - \frac{1}{2} + \frac{1}{4}\,e^{2t} + \frac{1}{4}e^{\frac{-t}{2}}\cos\left(\frac{\sqrt{7}t}{2}\right) + \frac{37}{4\sqrt{7}}e^{\frac{-t}{2}}\sin\left(\frac{\sqrt{7}t}{2}\right).$$

Case (8): Finally, if $y(t)$ and $z(t)$ are (ii)-differentiable but $x(t)$ is (i)-differentiable, then solution set for the last case takes the following form :

$$\underline{x}(t,\varsigma) = \varsigma\left(\frac{9}{2} - \frac{10}{3}e^{-t} - \frac{1}{6}e^{2t}\right) - \frac{9}{2} + \frac{13}{3}e^{-t} + \frac{1}{6}e^{2t}.$$

$$\overline{x}(t,\varsigma) = \varsigma\left(-\frac{9}{2} + \frac{11}{3}e^{-t} - \frac{1}{6}e^{2t}\right) + \frac{9}{2} - \frac{8}{3}e^{-t} + \frac{1}{6}e^{2t}.$$

$$\underline{y}(t,\varsigma) = \varsigma\left(\frac{-23}{4} + \frac{9}{2}t + \frac{20}{3}\,e^{-t} + \frac{1}{12}e^{2t}\right) + \frac{31}{4} - \frac{9}{2}t - \frac{36}{3}\,e^{-t} - \frac{1}{12}e^{2t}.$$

$$\overline{y}(t,\varsigma) = \varsigma\left(\frac{25}{4} - \frac{9}{2}t - \frac{22}{3}\,e^{-t} + \frac{1}{12}e^{2t}\right) - \frac{17}{4} + \frac{9}{2}t + \frac{16}{3}\,e^{-t} - \frac{1}{12}e^{2t}.$$

$$\underline{z}(t,\varsigma) = \varsigma\left(\frac{5}{4} - \frac{9}{2}t - \frac{1}{4}\,e^{2t}\right) - \frac{13}{4} + \frac{9}{2}t + \frac{1}{4}e^{2t}.$$

$$\overline{z}(t,\varsigma) = \varsigma\left(\frac{-7}{4} + \frac{9}{2}t - \frac{1}{4}\,e^{2t}\right) - \frac{1}{4} - \frac{9}{2}t + \frac{1}{4}e^{2t}.$$

Conclusion: The fuzzy Aboodh transform is used to find solutions to a system of n-dimensional fuzzy linear first order differential equations. To show the method's efficiency and quality, we provided a practical example.

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