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Optimization of Stiffened Beam with Various Loading Conditions

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Abstract:
The objective of this paper is to obtain an optimal cross section parameters of a uniform cross-section hollow rectangular cantilever beam with transverse stiffeners subjected to contracted load and uniform distributed load (UDL) considering weight minimization. Finite element method in conjunction with optimization algorithm is used to analyses the effect cross sectional geometrical parameters such as (cross section thickness, height of the beam) and state variables such as (total equivalent stress) on the beam weight. Commercial finite element software (Solidworks) is used to simulate the cantilever hollow rectangular beam subjected to a contracted load and uniform distributed load (UDL) at its end and top face of beam respectively, then perform a series of optimization iteration in order to obtain the optimal design parameters for a selected objective function (beam weight). In this study, the objective function is the minimum beam weight. The beam is made of AS1163 grade C450L0. The goal of the simulation and optimization process is to optimize the cross-section parameters to withstand the exerted load yet with minimum material keeping the total equivalent stress just below the maximum yield stress.

Keywords. Finite element method, Cross-section thickness, height of the beam, material, mass, optimization.

1. Introduction and Background

Hollow rectangular cross-section beam used in different applications for construction of base structure of machine, bridge structure, off-shore structures, aerospace and mining structures. Design engineers are facing several challenges for the design of product or system with low weight and cost without compromise in the strength of the product or system. Depending on the kind of design optimization problem, different parameters are affecting on the optimum design link geometry, material, and manufacturing process in order to meet design requirements and maximize its performance with consideration of minimization of its cost and weight **[1, 2].** Numerical methods for shape optimization problems have been used for a long time. The first challenge for shape optimization was carried out by Galileo in1938, who found the minimum weight of a cantilever parabolic beam by developing a mathematical formulation for shape optimization [3, 4]. Structural optimization is a substantial technique for designers to tailor a structure for a specific required performance level [5, 6]. In the design optimization problem, the objective function depends on the kind of problem. In time line, many researchers have made efforts and developed new methods for the optimization.

There were different optimization methods like Particle Swam Optimization (PSO) [3, 7-9], Numerical Optimization [5, 12-13, 17, 20, 21, 33, 35-36], Genetic Algorithm (GA) [10-11], Topology Optimization [13, 14, 18, 19, 22-26, 28, 31-32], Superposition Method [15], Johnson's method [19], Simulated Annealing algorithm [27], Experimental Approach [29] and Adaptive Single-Objective Method [34] etc. have been applied by different researchers for their optimization problem to fulfill the design requirements.

Different beam cross sections, such as hollow circular [10, 16], hollow rectangular [12, 14, 16], dual rectangular [14], hollow square [12, 16, 21], solid rectangular [13], solid square [21], I–section [14, 16], U-section [14], L-section [14], polygonal hollow structural section (PHSS) [17], and so on, are used in various applications reported by various researchers in their research.

Different types of beams, structures and mechanical components like cantilever [5, 10-14, 23], continuous [15], simply supported [17, 31], ten truss bar system [27], box girder [32], bus body frame [34], rotor blade [18], taper turbine disc [23] etc. were optimized by various researchers according to their problem formulation and to meet the optimum design requirements using different optimization approaches with specific objective functions. There were different objective functions considered by different researchers for their optimization problem like minimization of inertia weight [7], outer and inner radii [10], mass [11], deflection [15, 20], shear deformation [16], geometrical/shape parameter [18, 23, 24, 27, 31-32], cost [19, 28], material [29-30], manufacturing process [29-30], stress [22, 32], volume [13, 21], weight [5, 14, 21, 33-34] etc. and maximization of stiffness for a given amount of material [22, 25-26].

The commercial finite element software (e.g., Solidworks software) is used to simulate the hollow rectangle cantilever beam. The cantilever beam is supported from one of its ends and subjected to a combined load of 15000N (Concentrated Load) at free end and 7.5 N/mm (UDL) at top face of beam. A series of optimization iterations were carried out using design optimization algorithm to improve the design by changing the design variables, in order to meet the optimal design dimensions, for a particular objective function which is in this case minimum weight.

2. Finite Element Analysis of Rectangle Hollow Beam

Several optimization techniques have been developed to integrate geometry design and material selection, consequently reducing time and cost. Solidworks simulator which is finite element modelling software was used to simulate a hollow cantilever rectangle beam with 2000 mm length made of AS1163 grade C450L0. It is subjected to a combined load of 15000N (Concentrated Load) at free end and 7.5 N/mm (UDL) at top face of beam. Figure 1 shows the initial geometry of the hollow rectangular beam.

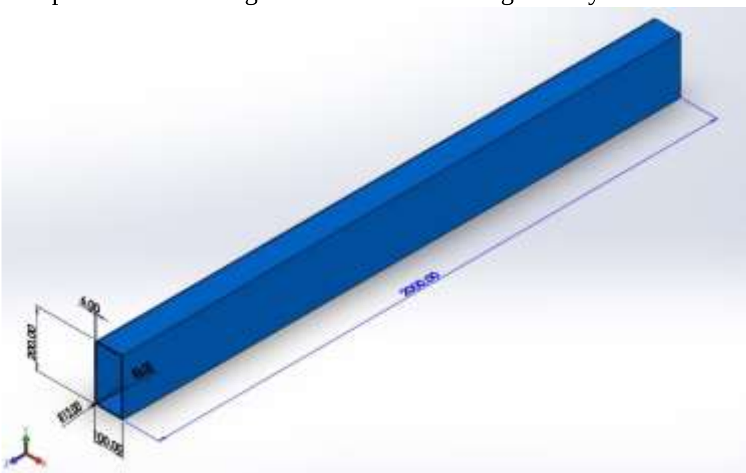


Figure 1. Initial geometry of the hollow rectangular beam

2.1. Analytical Analysis

Equation 1 is represented deflection of beam under concentrated load and uniform distributed load (UDL) which was derived by using Mohr's Moment Area Method (MMAM). Deflection at the free end for constant cross section is presented by the following way [35].

$$\delta = \frac{PL^3}{3IE}\left\{\begin{array}{c} \begin{pmatrix} 1+\sum_{i=1}^{n}(1-\varepsilon-(i-1)\phi-(i-1)\alpha)^3\left(\frac{1}{\beta}-1\right)+ \\ +\sum_{i=1}^{n}(1-\varepsilon-i\phi-(i-1)\alpha)^3\left(1-\frac{1}{\beta}\right)+(6\phi^3)\left(\frac{1}{\beta}-1\right) \end{pmatrix}+ \\ +\frac{3WL}{8P}\left\{\begin{array}{c} \begin{pmatrix} \varepsilon^2+(n+2)\gamma\phi^2+(n-1)\alpha^2+\eta^2+2\gamma\phi\Big((n+2)\varepsilon+P_{n+1}\phi+\big(P_{n-1}+2(n-1)\big)\alpha+2\eta\Big)+ \\ +2\alpha\big((n-1)\varepsilon+P_{n-1}\phi+P_{n-2}\alpha\big)+2\eta(\varepsilon+n\phi+(n-1)\alpha) \end{pmatrix} \\ +\sum_{i=1}^{n}\left[\begin{pmatrix} (n+3-i)\gamma\phi^2+(n-i)\alpha^2+\eta^2+ \\ +2\gamma\phi\big(P_{n+2-i}\phi+\big(P_{n-i}+2(n-i)\big)\alpha+2\eta\big)+ \\ +2\alpha(P_{n-i}\phi+P_{n-1-i}\alpha)+2\eta(n\phi+(n-i)\alpha) \end{pmatrix}(1-\varepsilon-(i-1)\phi-(i-1)\alpha)^2\left(\frac{1}{\beta}-1\right)\right] \\ +\sum_{i=1}^{n}\left[\begin{pmatrix} (n+2-i)\gamma\phi^2+(n-i)\alpha^2+\eta^2+ \\ +2\gamma\phi\big(P_{n+1-i}\phi+\big(P_{n-i}+2(n-i)\big)\alpha+2\eta\big)+ \\ +2\alpha(P_{n-1-i}\phi+P_{n-1-i}\alpha)+2\eta\big((n-i)\phi+(n-i)\alpha\big) \end{pmatrix}(1-\varepsilon-i\phi-(i-1)\alpha)^2\left(1-\frac{1}{\beta}\right)\right] \\ +(16\gamma\phi^4)\left(\frac{1}{\beta}-1\right) \end{array}\right\} \end{array}\right\} \quad (1)$$

$$\delta = 11.194 \text{ mm}$$

Equation 2 is representing deflection of beam under concentrated load and uniform distributed load (UDL) which was derived by using Classical Beam Theory (CBT) and detail description given in [36]. Stress at the support end for constant cross section is presented by following way.

Von mises criterion Stress (σ_v) is relate with bending stress (σ_b) and Shear stress (τ_{xy}) according to [36].

$$\sigma_v = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \quad (2)$$

Where:

σ_x = Bending stress in x direction, σ_y = Bending stress in y direction, τ_{xy} = Shear stress in xy plane, Yield strength, σ_y = 450 N/mm^2, Ultimate strength, σ_u = 500 N/mm^2.

From Equation 3, the obtained value of maximum Von mises stress is 172.19 MPa for problem taken under study with consideration of $\sigma_1 = (\sigma_b)_{max}$ and $\sigma_2 = 0$.

$$\sigma_v = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \quad (3)$$

2.2. Structural Analysis of Rectangular Hollow Beam

Standard practice of Finite Element Analysis has been adopted for the structural optimization of hollow rectangular cross-section cantilever beam.

Step-1 Pre-processing

Figure 2 shows the isometric view of prepared 3D model of a beam. Table 1 shows the material and material properties considered for the beam.

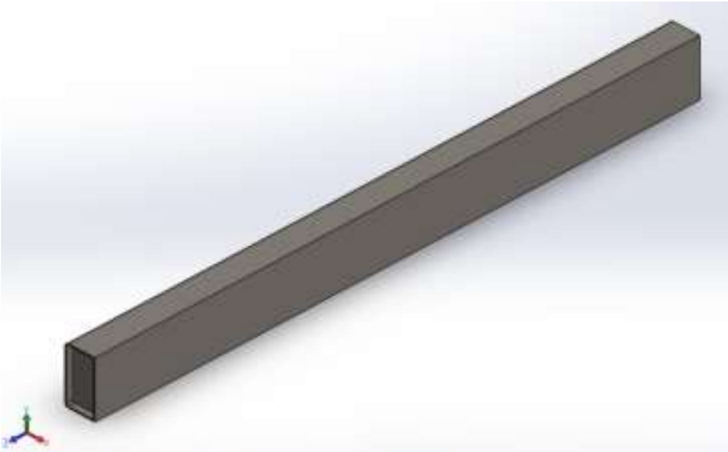


Figure 2. Solid model of the hollow rectangular beam

Table 1. Material and material properties of the beam (Initial beam)

Structure	**Material used**	**Young Modulus (GPa)**	**Yield Strength (MPa)**	**Poisson's Ratio**	**Density (Kg/m^3)**
Rectangular Hollow Beam	AS1163 grade C450L0	200	450	0.27	7850

Step-2 Processing

Programmed gendered fine mesh was used for meshing the beam and used the element size of 37.6358 mm, number of nodes of 11968 and number of elements of 6107. Figure 3 shows the meshed beam.

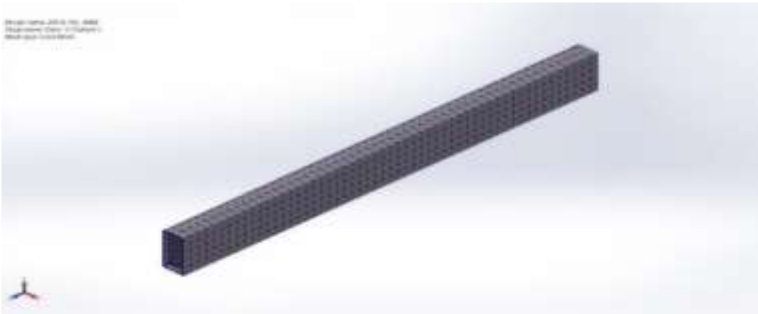


Figure 3. Meshing of rectangular hollow beam

Defined boundary conditions:

- Beam is fixed (constrained all degree of freedom) at left hand side end.
- The point load and UDL are applied at end and top side of beam 15000N and 7.5 N/mm respectively.

Figure 4 shows the boundary (All DOF restricted at fixed end of the beam) and loading conditions (concentrated load applied at free end of the beam and UDL on the upper surface of the beam throughout its length) to the beam.

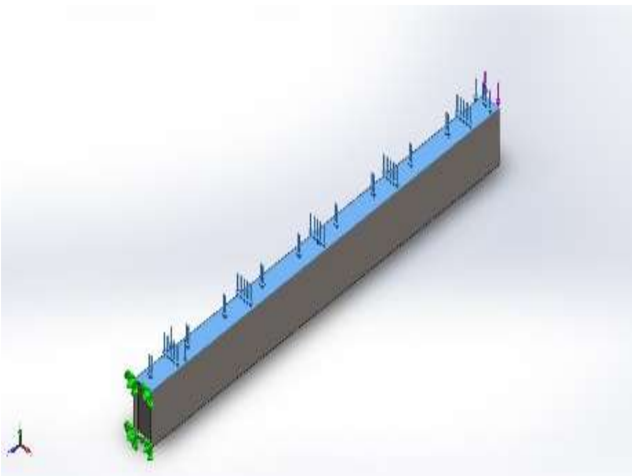


Figure 4. Boundary and loading conditions

Step-3 Post Processing

The post processing performed to identify the equivalent stress and deflection.

Table 2. Equivalent Von mises stress for static analysis (Initial beam)

Name	Type	Min.	Max.
Stress	Von mises stress (Equivalent stress)	0.260 MPa Node: 6992	109.755 MPa Node: 672

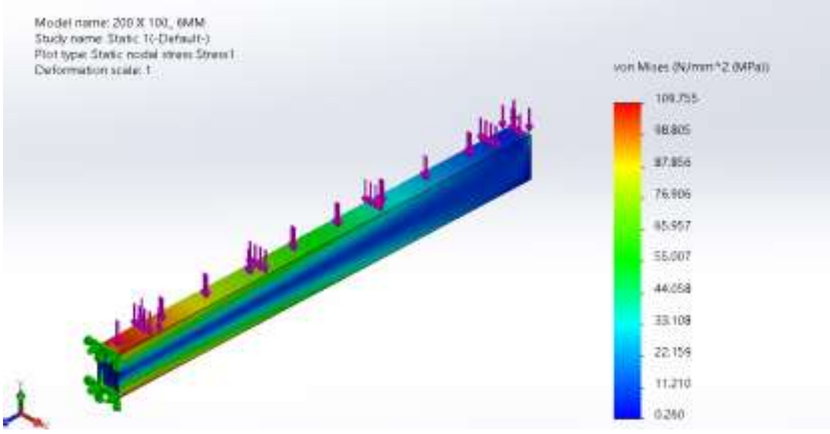


Figure 5. Equivalent Von misses stress (Initial beam)

Figure 5 shows the finite element result of the beam deflection. The maximum value is at the free end and its value is equal to 2.7941 mm.

Table 3. Deflection for static analysis (Initial beam)

Name	Type	Min.	Max.
Deflection	Total deflection	0.0 mm Node: 1	7.429 mm Node: 61

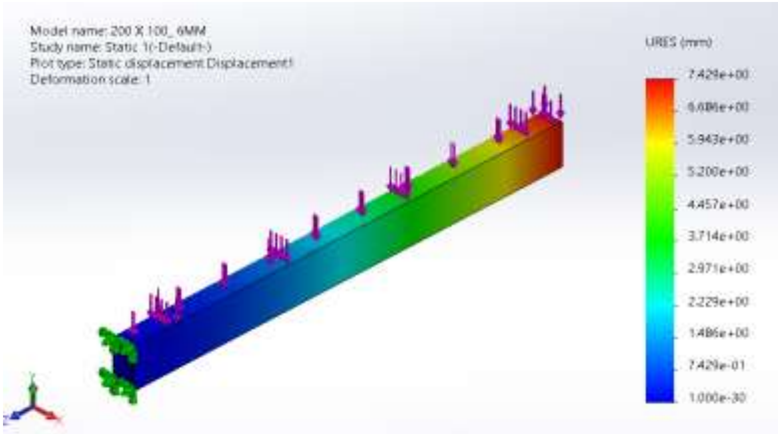


Figure 6. Deflection of beam (Initial beam)

Numerical model of the cantilever beam has been built and solved using Solidworks simulation static structural tool. The results of the maximum value of Von mises stress and the maximum deflection values were compared with the analytical solution. Figure 5 shows the finite element result of Von mises equivalent stress distribution through the beam length for the geometry of initial beam. The maximum value of Von mises stress located at the support is σ = 169.9 MPa, which is lower than the yield strength of the material of the beam (AS1163 grade C450L0) (450MPa) as well as lower than the safe stress (225 MPa) with consideration of factor of safety as 2. It shows that there is a scope of optimization. Figure 6 shows the finite element result of the beam deflection, with maximum value of 10.87 mm at the free end of the cantilever beam. Analytical and finite element results are listed in Table 4, where these results are compatible with each other.

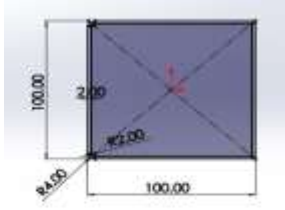
Table 4. Comparison between numerical and analytical results of the initial geometry of the beam

	Analytical results	FEA results	%Error
Max equivalent stress (MPa)	172.19	169.9	1.33
Maximum Deflection (mm)	11.194	10.87	2.89

3. Design Variables

The design variables are some geometrical parameters that the designers want to change to fulfill the optimization goal, which in this case, is the weight minimization. The geometrical parameters that need to be changed are thickness of cross section and the height of the beam, as shown in Table 5. The optimization algorithm will search for the minimum objective function (minimum weight) within the design space specified by the upper and the lower limits of the design variables and constraints.

Table 5. Initial set and the lower and upper limit of design variables

Parameters	Initial set	Lower limit	Upper limit	
Thickness of cross section (mm)	2	2	10	
Height of the beam (mm)	100	100	200	

In the optimization procedure, Solidworks optimization toolbox performs a series of analysis evaluation-modification cycles. That is, an analysis of the initial design is performed, the results are evaluated against specified design parameters, and the design is modified as necessary. The process is repeated until all specified criteria are met.

4. Constraints

In order to utilize the material to it extent, the maximum Von mises stress must be kept just at the maximum allowable stress. Therefore, it is very important to specify the allowable stress as a constraint for optimization algorithm to avoid critical situation and be safe. The yield stress of AS1163 grade C450L0 is 450 MPa and the allowable stress is equal to 450⁄2 = 225 MPa.

5. Objective Function

The main goal of the design engineer is lowering the production cost, and one of these costs is the material used in the production, so one of the objectives is minimization of the material. The goal of the simulation is to find out the shape of the beam that leads to a minimum weight. The mass calculated by Solidworks modeler is defined as the objective function to the optimization toolbox. The optimization toolbox keeps changing the beam cross section parameters until the minimum weight is obtained.

6. Results and Discussion

Table 6. Evaluation of parameters for beam cross section size 100 mm x 100 mm

Cross section size	100 mm x 100 mm	**No. of stiffeners**	5
Intermediate stiffener spacing	480 mm	Max.	
Intermediate stiffener thickness	5	mm	
Thickness of cross section	**First stiffener**	**Stress (MPa)**	**Deflection (mm)**
2	20	1224	160.3
2	100	1284	160.4
2	200	1260	160.5
2	300	1258	160.6
2	500	1295	161.1
4	20	651.8	86.38
4	100	690.1	86.24
4	200	668.8	86.26
4	300	668.5	86.29
4	500	671.4	86.36
6	20	467.8	61.84
6	100	481.9	61.86
6	200	497.3	61.89
6	300	485.8	61.89
6	500	498.3	61.94
8	20	386.8	49.92
8	100	389.2	49.9
8	200	390.2	49.94
8	300	387.5	49.92

8	500	391.7	49.96
10	20	308.9	41.79
10	100	322.8	41.80
10	200	326.4	41.81
10	300	325	41.81
10	500	328.9	41.82

Table 7. Evaluation of parameters for beam cross section size 155 mm x 100 mm

Cross section size	155 mm x 100 mm	**No. of stiffeners**	5
Intermediate stiffener spacing	480 mm	Max.	
Intermediate stiffener thickness	5	mm	
Thickness of cross section	**First stiffener**	**Stress (MPa)**	**Deflection (mm)**
2	20	678.8	57.93
2	100	712.7	57.97
2	200	714.3	57.99
2	300	714.2	58.02
2	500	714.3	58.13
4	20	361.8	30.58
4	100	373.4	30.59
4	200	368.5	30.6
4	300	368.3	30.62
4	500	368.1	30.64
6	20	254.5	21.52
6	100	282.7	21.54
6	200	264.5	21.54
6	300	263.7	21.53
6	500	278.5	21.56
8	20	204	17.04
8	100	210.4	17.05
8	200	212	17.05
8	300	211.1	17.04
8	500	211.3	17.06
10	20	163.8	14.07
10	100	161.3	14.06
10	200	170.7	14.06
10	300	168.6	14.07
10	500	170.8	14.07

Table 8. Evaluation of parameters for beam cross section size 200 mm x 100 mm

Cross section size	200 mm x 100 mm	**No. of stiffeners**	5
Intermediate stiffener spacing	480 mm	Max.	
Intermediate stiffener thickness	5	mm	
Thickness of cross section	**First stiffener**	**Stress (MPa)**	**Deflection (mm)**
2	20	495.2	31.56
2	100	506.6	31.59
2	200	507.9	31.6
2	300	506.9	31.61
2	500	506.9	31.66
4	20	254.1	16.53
4	100	261.5	16.54
4	200	259.9	16.54
4	300	259.7	16.55
4	500	259.6	16.56
6	20	169.9	10.87
6	100	189.6	11.54
6	200	194.8	11.55
6	300	186.5	11.54
6	500	195.1	11.55
8	20	140.9	9.05

8	100	145.9	9.05
8	200	145.4	9.05
8	300	145.2	9.05
8	500	144.9	9.06
10	20	109.7	7.43
10	100	110.8	7.42
10	200	115.9	7.42
10	300	114.3	7.43
10	500	115.9	7.43

The geometrical parameters that need to be changed are thickness of cross section and the height of the beam, as shown in Table 6 to 8 and found values of von mises stress and deflection for respective set of condition.

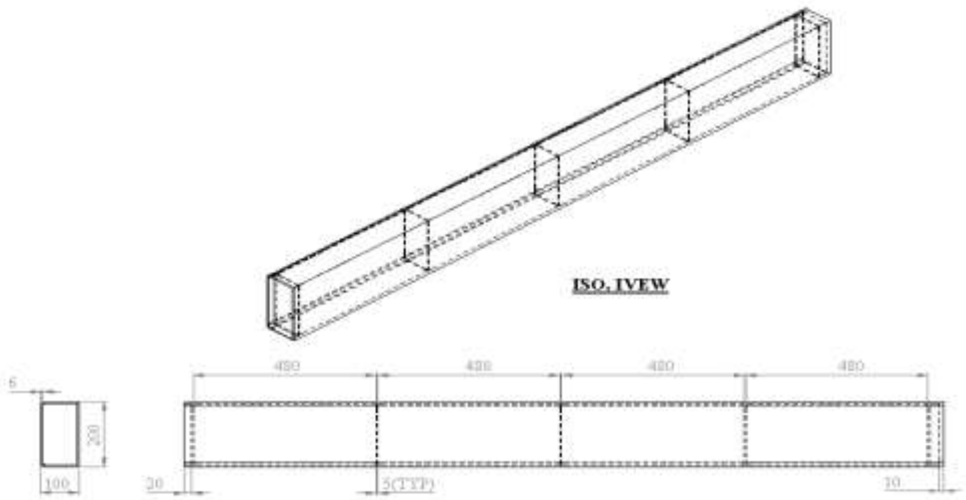


Figure 7. Detail drawing of optimum stiffener location w.r.t. maximum equivalent stress and deflection

Table 9. Optimum value of n = 5, t_s = 5 mm, first stiffener position (w.r.t fixed end) = 20 mm (ε = 0.01) and distance between intermediate stiffeners = 480 mm (α = 0.24)

Iteration	Cross-section thickness (mm)	Beam height (mm)	Max. equivalent stress (MPa)	Beam deflection (mm)	Geometry mass (kg)
1	2	100	1206	160.3	14.76
2	2	155	686	57.93	19.66
3	2	200	487.7	31.57	23.63
4	4	100	661.8	86.36	26.12
5	4	155	374.6	30.62	34.41
6	4	200	256.8	16.53	41.2
7	6	100	470.4	61.87	36.81
8	6	155	254.8	21.52	48.51
9	6	200	169.9	10.87	58.07
10	8	100	381.8	49.9	46.86
11	8	155	204	17.04	61.94
12	8	200	141	9.05	74.29
13	10	100	307.6	41.79	57.55
14	10	155	163.7	14.07	76.03
15	10	200	109.7	7.43	91.14

Figure 7 shows the detail drawing of optimum stiffener location w.r.t. maximum equivalent stress and deflection The finite element analysis and optimization results of some design sets are presented in Figure 8 and Figure 9. Figure 8 shows the optimization iterations (design sets) versus maximum equivalent stress as primary axis and beam mass as secondary axis. Design sets with equivalent stress higher than 225 MPa are called infeasible design sets (1, 2, 3, 4, 5, 6, 7, 8, 10, 13) (Refer Table 9), and they are out of the design space. The rest are within the design space and called feasible design sets (9, 11, 12) (Refer Table 9). Among this, only one is the optimum design set (9) which has the deflection (10.87 mm which under limit as per Eurocode3) and maximum equivalent stress value of 169.9MPa.

Figure 9 shows the optimization iterations (design sets) versus beam deflection as primary axis and beam height as secondary axis. The optimum design sets have maximum deflection of 10.87 mm, and the optimum beam cross section height of 200 mm simultaneously. Several iteration optimization design sets were carried out by using Solidworks package simulation software. Table 9 shows the finite element analysis and optimization results of the best candidate design sets (Iteration number 9). It also reveals that the optimum design set with maximum equivalent stress, mass geometry and maximum deflection are 169.9 MPa, 58.07 Kg and 10.87 mm, respectively.

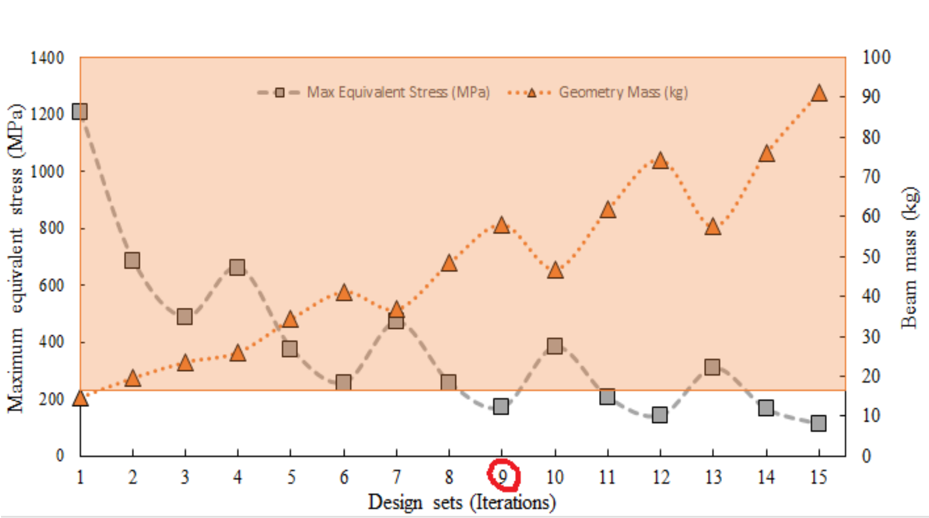


Figure 8. Maximum equivalent stress and beam mass versus optimization iteration number

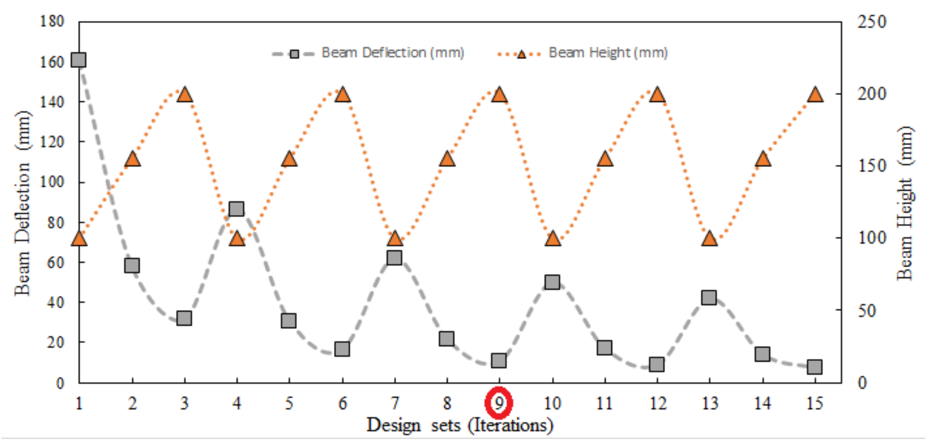


Figure 9. Beam deflection and height versus optimization iteration number

Table 10. Equivalent Von mises stress obtained from static analysis (Optimized beam)

Name	Type	Min.	Max.
Stress	VON: von Mises stress (Equivalent stress)	0.230 MPa Node: 5145	169.9 MPa Node: 115

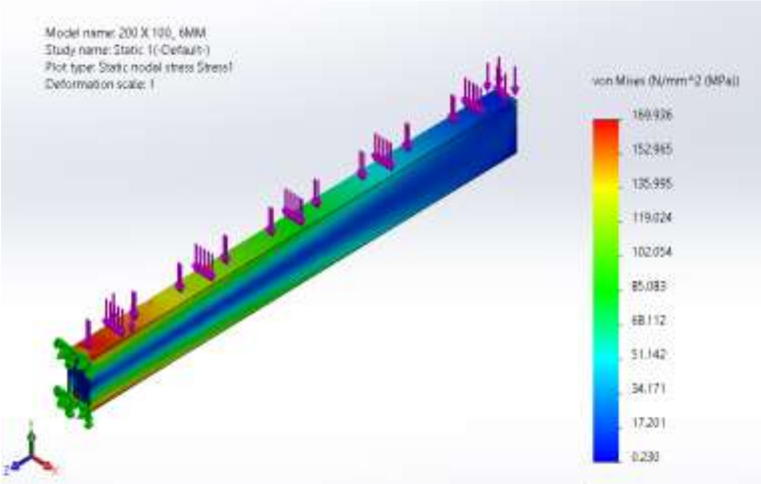


Figure 10. Equivalent Von misses stress (Optimized beam)

Table 11. Deflection of beam obtained from static analysis (Optimized beam)

Name	Type	Min.	Max.
Deflection	Total deflection	0.00 mm Node: 1	10.87 mm Node: 62

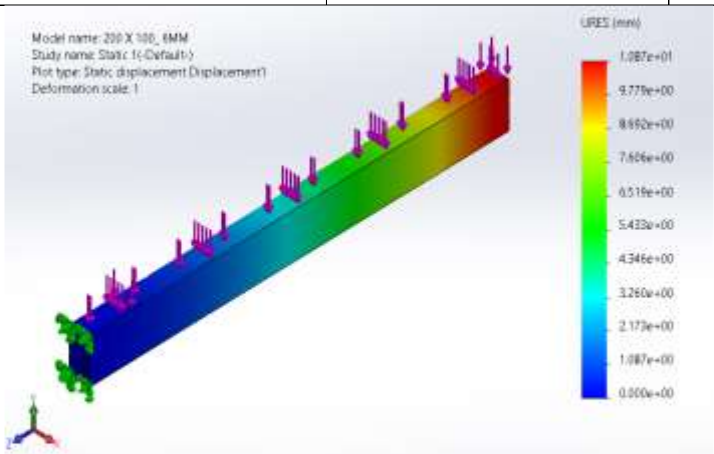


Figure 11. Deflection of beam (Optimized beam)

Table 12. Finite element analysis and optimization results of the best candidate design sets

Candidates No.	Cross-section thickness (mm)	Beam height (mm)	Max equivalent stress (MPa)	Beam deflection (mm)	Geometry mass (kg)
Best Candidate No. 9	6	200	169.9	10.87	58.07

Table 13. Comparison of finite element results for the initial and the optimum geometry

Parameter	Initial geometry results	Optimum geometry results	% Optimized
Max. equivalent stress (MPa)	109.7	169.9	35.43%
Geometry mass (kg)	89.84	58.07	35.36%
Total deflection (mm)	7.43	10.87	31.64%

Table 10 shows the equivalent Von mises stress of optimized beam obtained from static analysis. Table 11. Shows the deflection of beam obtained from static analysis for optimized beam. Table 12 indicates the comparison of finite element results for the initial and the optimum geometry. The maximum equivalent stress, geometry mass and total deflection are improved in about, 35.43% (increased), 35.36% (decreased), and 31.64% (increased) respectively (Ref. Table 13).

7. Conclusions

Several iteration optimization design sets were carried out by using Solidworks package simulation software. Table 9 shows the finite element analysis and optimization results of the best candidate design sets (Iteration number 9). It also reveals that the optimum design set with maximum equivalent stress, mass geometry and maximum deflection, are 169.9 MPa, 58.07 Kg and 10.87 mm, respectively. The comparison of FEA results for the initial and the optimum geometry of beam shows that the maximum equivalent stress increased by 35.43%, geometry mass reduced by 35.36% and total deflection increased by 31.64%. The maximum equivalent stress is within the stress limit (225 MPa), the deflection is below the limiting value of deflection as per EUROCODE3.

References

1. Ruoxu Jia, Junling Hu, Xingguo Xiong, and Linfeng Zhang (2014), “Shape Optimization Design and Material Selection for a Fitness Equipment”, ASEE 2014 Zone I Conference, April 3-5, 2014, University of Bridgeport, Bridgeport, CT, USA.
2. K.L. Edwards (2003), Designing of engineering components for optimal materials and manufacturing process utilization, Materials and Design, 24, 2003, pp. 355- 366.
3. Alhadi A. Abosbaia and Mohammed Jolgaf (1995), “Particle Swarm Optimization”, IEEE, 0-7803-2768-3/95, pp. 1942-1948.
4. M.F. Ashby, Y.J.M. Brechet, D. Cebon, and L. Salvo (2004), “Selection strategies for materials and processes”, Materials and Design, 25, 2004, 51-67.
5. Alhadi A. Abosbaia and Mohammed Jolgaf (2019), “Shape Optimization of a Hollow Cantilever Beam for Weight Minimization Using Finite Element Method”, International Science and Technology Journal, Vol. 17, pp. 1-16.
6. Sangeeta Yadav, K. K. Pathak, Rajesh Shrivastava (2010), “Shape Optimization of Cantilever Beams Using Neural Network”, Applied Mathematical Sciences, Vol. 4, 2010, no. 32, 1563 – 1572.
7. Russell C. Eberhart and Yuhui Shi (2001) “Particle Swarm Optimization: Development, Applications and Resources”, IEEE, Vol. 3 (01), pp. 81-86.
8. A. Rezaee Jordehi (2015), “A review on constraint handling strategies in particle swarm optimisation”, Natural Computing and Applications, Springer, DOI 10.1007/s00521-014-1808-5.
9. Mahamad Nabab Alam (2016), “Particle Swarm Optimization: Algorithm and its Codes in MATLAB, DOI:10.13140/RG.2.1.4985.3206, https://www.researchgate.net/profile/Mahamad_Alam.
10. K R Indu and Airin M G (2015), “Optimization of Tapered Cantilever Beam Using Genetic Algorithm: Interfacing MATLAB and ANSYS”, International Journal of Innovative Research in Science, Engineering and Technology, International Journal of Innovative Research in Science, Engineering and Technology, Vol. 4 (10),pp. 10145-10150. DOI:10.15680/IJIRSET.2015.0410062.
11. V.Muthukumaran, R. Rajmurugan, and V.K.Ram kumar (2014), “Cantilever Beam and Torsion Rod Abstract: Design Optimization Using Genetic Algorithm”, International Journal of Innovative Research in Science, Engineering and Technology, Volume 3, Special Issue 3, pp. 2682-2690.
12. Timo Björk, Antti Ahola and Tuomas Skriko (2020), “On the distortion and warping of cantilever beams with hollow section”, Welding in the World, Vol. 64, pp. 1269–1278. https://doi.org/10.1007/s40194-020-00911-5.
13. Behzad Ahmed Zai, M.K. Park, Seungchul Lim, Joongwon Lee, and Rashid Ali Sindhu (2011), “Structural Optimization of Cantilever Beam in Conjunction with Dynamic Analysis”, Journal of the Korean Institute of Gas, Vol. 15(5), pp. 31-36, DOI: 10.7842/kigas.2011.15.5.031.
14. Jiantao Bai and Wenjie Zuo (2019), “Hollow structural design in topology optimization via moving morphable component method”, Structural and Multidisciplinary Optimization https://doi.org/10.1007/s00158-019-02353-0.
15. Abdulamir Atalla Almayah (2018), “Simplified Analysis of Continuous Beams”, International Journal of Applied Engineering Research, Volume 13 (2), pp. 922-928. www.ripublication.com.
16. Reza Moazed, Mohammad Amir Khozeimeh and Reza Fotouhi (2021), “Simplified Approach for Parameter Selection and Analysis of Carbon and Glass Fiber Reinforced Composite Beams”, Journal of Composites Science,Vol. 5, pp. 220-241. https://doi.org/10.3390/jcs5080220.
17. John Samuel Kabanda and Colin MacDougall (2016), “Optimization of a Polygonal Hollow Structural Steel Section in the Elastic Region”, Resilient Infrastructure, pp. 959-967.
18. Zhijun Wang, Akke S.J. Suiker, H`erm Hofmeyer, Twan van Hooff, and Bert Blocken (2021), “Optimization of thin-walled beam structures: Monolithic versus staggered solution schemes”, Thin-Walled Structures, Vol. 159, pp. 107182-107199.http://www.elsevier.com/locate/tws.
19. S Darshan, Akshay Varik, Anirudh N Katti, Amit Kumar Singh, and Rahul R Kamath (2013), “Size and Topological Optimization of Cantilever Beam”, International Journal of Engineering Trends and Technology, Vol. 4 (5), pp. 2077-2082.
20. Long Yanze, Zhang Ke, Shi Huaitao , Li Songhua, and Zhang Xiaochen, (2021), “Theoretical and Experimental Analysis of Thin-Walled Curved Rectangular Box Beam under In-Plane Bending”, Hindawi, Vol. 2021, pp. 1-19.https://doi.org/10.1155/2021/8867142.
21. Chander Sen and Jitendra Prasad (2020), “Introducing Design Space-Independence in Beam Cross-Section Optimization”, Engineering Engrxiv, pp. 1-12. DOI: 10.31224/osf.io/ntjf2
22. Dulyachot Cholaseu (2006), “A Stress-Based Material Distribution Method for Optimum Shape Design of Mechanical Parts”, Thammasat International Journal of Sc. Tech., Vol. 11 (3), pp. 17-21.
23. Jeffrey M. Brown, “Three-Dimensional Shape Optimization with Probabilistic Constraints using Parametric Session Files”, Air Force Research Laboratory, pp. 1-15.
24. M. P. Bendsùe, and O. Sigmund (1999), “Material interpolation schemes in topology optimization”, Archive of Applied Mechanics, Springer-Verlag, Vol. 69, pp. 635-654. DOI: 10.1007/s004190050248.
25. Taemin Earmme (2009), “Evolutionary Structural Optimization with Multiple Performance Constraints by Large Admissible Perturbations”, Mechanical Engineering and Naval Architecture and Marine Engineering, The University of Michigan, Ph. D. Thesis.
26. Q. Li, G.P. Steven, and Y.M. Xie (1999), “On equivalence between stress criterion and stiffness criterion in evolutionary structural optimization”, Structural Optimization, Springer-Verlag, Vol. 18, pp. 67-73.
27. P. Starry, E. Dupinet and M. Mekhilef (1993), “A new way to optimize mechanical systems using simulated annealing”, Transactions on the Built Environment, WIT Press, vol. 2, pp. 569-583.www.witpress.com.

28. Ruoxu Jia, Junling Hu, Xingguo Xiong, and Linfeng Zhang (2014), “Shape Optimization Design and Material Selection for a Fitness Equipment”, ASEE 2014 Zone I Conference, April 3-5, 2014, University of Bridgeport, Bridgeport, CT, USA. https://www.researchgate.net/publication/283570404.
29. K.L. Edwards (2003), “Designing of engineering components for optimal materials and manufacturing process utilisation”, Materials & Design, Vol. 24 (5), pp. 355-366.https://doi.org/10.1016/S0261-3069(03)00041-4.
30. M.F. Ashby, Y.J.M. Bréchet, D. Cebon, and L. Salvo (2004), “Selection strategies for materials and processes”, Materials & Design, Vol. 25 (1), pp. 51-67.
31. Sangeeta Yadav, K. K. Pathak and Rajesh Shrivastava (2010), “Shape Optimization of Cantilever Beams Using Neural Network”, Applied Mathematical Sciences, Vol. 4, 2010, no. 32, 1563 - 1572.
32. Qian Wang, Wen-liang Qiu, and Sheng-li Xu (2015), “Structural Optimization of Steel Cantilever Used in Concrete Box Girder Bridge Widening”, Hindawi Publishing Corporation Mathematical Problems in Engineering, Volume 2015, Article ID 105024, 14 pages, http://dx.doi.org/10.1155/2015/105024.
33. Mohamed Jolgaf (2017), “Optimization of Cantilever Beam for Minimum Weight Using Finite Element Analysis”, Proceedings of FLCMCPT2017 First Libyan Conference on Metal Casting and Processing Technologies September 12-13, 2017, Sidi Al-Sayeh, Tripoli, Libya.
34. Ismoyo Haryanto, Fuad Arief Raharjo, Ojo Kurdi, Gunawan Dwi Haryadi, Sigit Puji Santosa, Leonardo Gunawan (2018), “Optimization of Bus Body Frame Structure for Weight Minimizing with Constraint of Natural Frequency using Adaptive Single-Objective Method”, International Journal of Sustainable Transportation Technology Vol. 1, No. 1, April 2018, 9-14.
35. Patel BP, and Patel DP (2021) “Deflection analysis of uniform cross sectional hollow rectangular cantilever beam with transverse stiffeners under UDL and concentrated load”, Journal of Engineering Research Express, Institute of Physics Publishing (IOP) Publication, 3: 035018-41 DOI: 10.1088/2631-8695/ac1847.
36. Patel DP, and Patel BP (2021) “Stress analysis of uniform a cross-section hollow rectangular cantilever beam with transverse stiffeners under combined load of concentrated and uniformly distributed load”, Design Enginering, ISSN: 0011-9342,Issue: 7,Pages: 15556- 15586.