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Experimental Study on the Effects of Circular Holes on Vibration and Noise in Spur Gear

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Abstract

Propose- the effect of circular-section holes on vibration and noise is calculated through a practical experiment

Methodology- comparing the number of holes 3, 5 and 7 within gears of identical dimensions regardless of lubrication and dimeter of holes is 19 mm and distance 50mm from center of rotation.

Finding- The current study conducted a comprehensive study on gears and how to increase their effectiveness by controlling their weight using circular holes and taking the effect of these holes on the machine in general and on performance in particular.

Originality- In order to verify the validity of the results, it was applied in a practical way, and the results were compared with the theoretical values. It is noteworthy that multiple cases and multiple speeds were taken for a stable load.

Keywords: Natural frequency, Modal and Spur gear and Rim holes

Paper Type- Research paper

1. Introduction

In light of the development in the field of technology, modern equipment requires less weight, and therefore reducing the weight of gears has a great benefit in reducing the weight of any engine, as well as reducing the noise generated through the process of interlocking between gears is one of the main requirements

Yokota, Taguchi and Mitsuo G [1998] We tackled the problem of optimal weight design of gears and solved it using improved genetic algorithmic techniques with nonlinear design constraints [1]. Özek, F [2007] investigated the effect of different geometric hole types on the loading of spur gears to reduce their weight. They found that the model with circular hole and step wheel was the most suitable model in terms of stress value [2]. Nidal H. and Mohammad A [2014] They explored the effect of different knot parameters to make holes in the gear body and face profile using the first model and the second model, respectively. In this case, the hardness of the gear was greatly affected [3]. Satoshi O. et al [1986], They was found that when the rim thickness of the internal gear is thick, the maximum root stress is equal regardless of the wall thickness or the angle of the wall sector [4]. Jianxing Z. and Sun W [2014], He then studied the temporal course of node dynamics and the gearbox noise spectrum, then he gave an example of the effect and finally the changes in the gearbox's radiate noise with the load using simulations and cut formulas [8].

Symbol	Parameter	Unite
I_P	Moment of Inertia for pully	$kg.m^2$
I_{GP}	Moment of Inertia for pinon	$kg.m^2$
I_f	Moment of Inertia for flywheel	$kg.m^2$
I_{GG}	Moment of Inertia for Gear	$kg.m^2$
$\acute{I}_G$	Equivalent Mass Moment of Inertia for Gears Meshing	$kg.m^2$
l_1	Length of small shaft	m
l_2	Length of big shaft	m
K_1	Stiffness of small shaft	N/m

K_2	Stiffness of big shaft	N/m
$\acute{K}_2$	Equivalent Stiffness of big shaft	N/m
ω_n	Natural Frequency	Hz
ω_{mesh}	Rotation Frequency of the Gear Shaft	Hz
n_G	The Rotational Speed of Gear	rpm
n_P	The Rotational Speed of Pinion	rpm
N2	Number of the Gear Teeth	-
N1	Number of the Pinion Teeth	-

Table (1): Parameters and Symbol

2. Defining an optimization problem in gears using circular holes

As mentioned earlier, this study examined the effects of vibration and noise due to pinion holes. The aim of this study is to

1- Reduce the weight of the stimulator as much as possible without increasing vibrations or noise.

2- To test the possibility of lowering the "net weight" of the unit (gearwheel) and then saving costs.

3- Reduction of raw material inventory through standardization, taking into account economies of scale.

Modeling vibrations and other elastic structures of the gearbox housing walls are important for the noise and vibrations emitted by the surrounding system. The pattern activity of the home walls is directly related to the structure and intensity of the noise emitted from the gearbox to the surroundings. Therefore, sample activity research is common.

Importance for modeling the noise and vibration generation process in mechanical systems. The noise and vibrations emitted by the gearbox to the surroundings are mainly the result of natural oscillations of the casing.

3- Model of Exiting Gear

Parameter	**Pinon**	**Gear**
Thickness	7.4 mm	6.6 mm
Width	40 mm	40 mm
Root radius	1.4 mm	1.4 mm
Pitch Circle radius	87.5 mm	175 mm
Module	3.5	3.5
No. of teeth	25	50

Table (2): Gears parameters

1. Material Used: CK35
2. Type Spur Gear
3. Application for case Study: laboratory apparatus

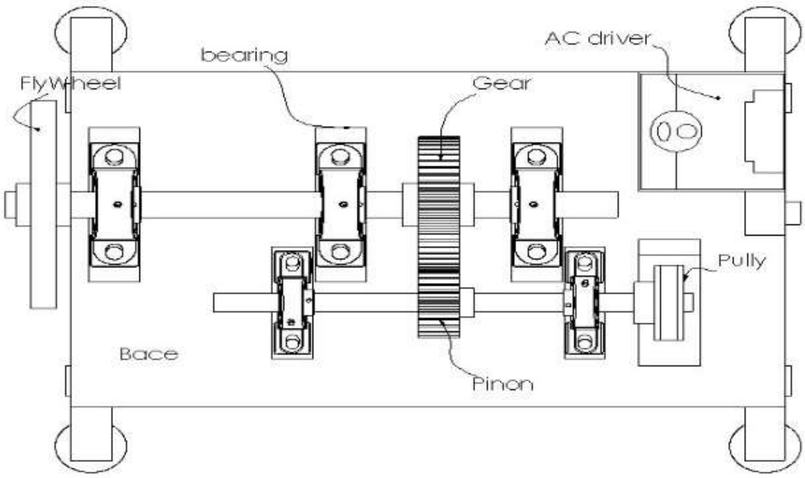

Figure (1): System

Mechanical Part of system
1-Base
2-Bearing
3-flywheel
4-Shafts
5-Gear and Pinon

4- Analytical Method

Analytical modeling is based on the following assumptions: In addition to the 1-teeth, the gear was assumed to be rigid. 2-axes are designed as springs in the direction of rotation and turning.
The ideal model taken for an under-investigated gear drive system is shown in Fig. (2)

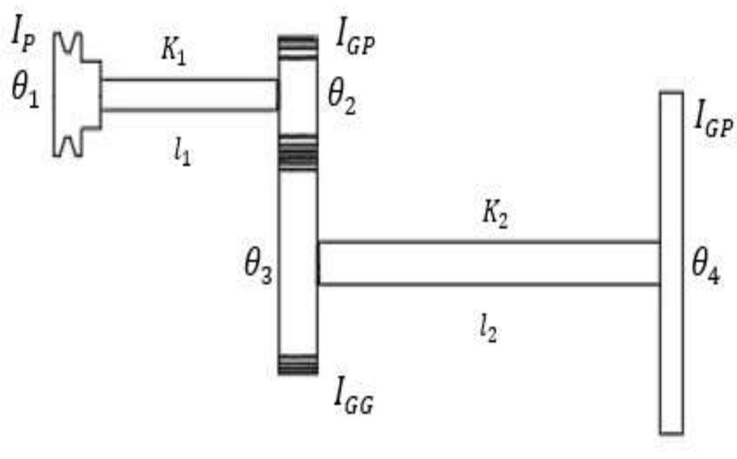

Figure (2): Gear System Model

The rigid machine model, the "steel body system" which provides "stable speed" loading, is the simplest model of dynamic loading of the drive system. To represent the static moment Figure (2), "input torque" produces an angle of velocity that can be calculated using the "low moment of inertia" [6]:

$$I_{read} = I_P + I_{GP} + \left[(I_{GG} + I_F) \left(\frac{1}{\mathrm{i}}\right)^{2} \right]$$

Then the angular accelerations are:

$\ddot{\theta}_1 = \ddot{\theta}_2 = \frac{M_T}{I_{read}}$, $\ddot{\theta}_4 = \ddot{\theta}_3 = \ddot{\theta}_1(\frac{1}{i})$

For torsional moments, the two shafts of the gear mechanism show different values, and these values are measured by the free body diagram:

$M_1 = (I_{read} - I_P)\ddot{\theta}_1$

$M_4 = \ddot{\theta}_4 I_4$

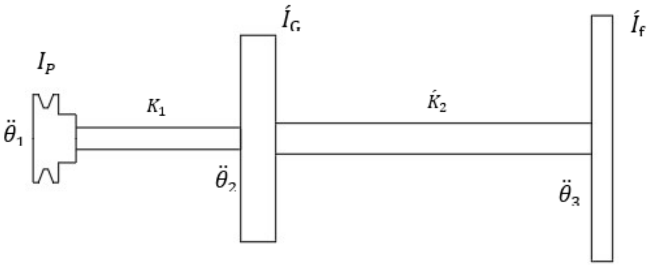

Figure (3): Equivalent Gear Model

Figure (3), shows the reducing oscillator chain of a drive gear system with an unbranched standard model compared with figure (2).

$I_P\ddot{\theta}_1 + K_1\theta_1 - K_1\theta_2 = 0$
$\acute{I}_G\ddot{\theta}_2 - K_1\theta_1 + (K_1 + \acute{K}_2)\theta_2 - \acute{K}_2\theta_3 = 0$
$\acute{I}_f\ddot{\theta}_3 - \acute{K}_2\theta_2 + \acute{K}_2\theta_3 = 0$ …..(EQM)

Matrix will be:

$$\begin{bmatrix} I_P & 0 & 0 \\ 0 & \acute{I}_G & 0 \\ 0 & 0 & \acute{I}_f \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} K_1 & -K_1 & 0 \\ -K_1 & (K_1 + \acute{K}_2) & -\acute{K}_2 \\ 0 & -\acute{K}_2 & \acute{K}_2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = 0$$

To find natural frequency by solve (EQM)
Natural frequency determined from this equation

$$-\omega_n^2 \left[\omega_n^4 - \omega_n^2 \left(K_1 * \frac{I_P + \acute{I}_G}{I_P\, \acute{I}_G} + \acute{K}_2 \frac{\acute{I}_G + \acute{I}_f}{\acute{I}_G\, \acute{I}_f}\right) + K_1\, \acute{K}_2 \frac{I_P + \acute{I}_G + \acute{I}_f}{I_P\, \acute{I}_G\, \acute{I}_f}\right] = 0$$

Applied dynamic Absorber will be

$$\omega_{n\,2,3}^2 = \frac{1}{2}\left(K_1 * \frac{I_P + \acute{I}_G}{I_P\, \acute{I}_G} + \acute{K}_2 \frac{\acute{I}_G + \acute{I}_f}{\acute{I}_G\, \acute{I}_f}\right) \mp \sqrt{\frac{1}{4}\left(K_1 * \frac{I_P + \acute{I}_G}{I_P\, \acute{I}_G} + \acute{K}_2 \frac{\acute{I}_G + \acute{I}_f}{\acute{I}_G\, \acute{I}_f}\right)^2 - K_1\, \acute{K}_2 \frac{I_P + \acute{I}_G + \acute{I}_f}{I_P\, \acute{I}_G\, \acute{I}_f}}$$

From the above equation, it is possible to find the first, second and third natural frequency of the system in each case, where the table (4) shows the frequency values
Since the natural frequency depends largely on the moment of inertia, these values must be calculated after calculating their mass in advance show in table (3) [8].

$I_{GG} = I_{GG} - I_{hole}$

$I_{hole} = \frac{m\, r^2}{2} + m\, d^2$

$\acute{I}_G$	0.02607780	$kg.m^2$
$\acute{I}_G3$	0.025563893	$kg.m^2$
$\acute{I}_G5$	0.025221289	$kg.m^2$
$\acute{I}_G7$	0.024878684	$kg.m^2$

Table (3): the moment of inertia

Name	Natural Frequency Hz		
	ω_{n1}	ω_{n2}	ω_{n3}
R0-D0-N0	0	163.679	545.742
R43-D19-N3	0	164.460	546.425
R43-D19-N5	0	164.990	546.893
R43-D19-N7	0	165.527	547.371

Table (4): Theoretical natural frequency

In order to compare the practical results with the theoretical results, the gear frequency must be calculated, which represents the practical frequency for all cases, which is constant because it depends on the speed of rotation and the number of teeth of the gear.

Each gear produces frequencies (gear mesh frequencies) associated with the gear and gear teeth running speed [7].

$\omega_{mesh} = \frac{nZ}{60}$

As the frequency was calculated for the different speeds, which are 500, 1000, 1500 and 2000 as shown in the table (5)

Speed rpm	Frequency Hz
500	33.1740
1000	66.3481
1500	99.5222
2000	132.6963

Table (5): Gear Mesh Frequency

5- Practical Vibration Result

As the operation frequency was calculated for the different speeds, which are 500, 1000, 1500 and 2000 at room temperature with the use of dampers through the (acceleration) sensor, then the data was processed by the sigveiw program using the FFT transformation to find the frequencies as shown in the tables below

500			
Name	Practical Frequency Hz		
	Near Gears		
	X	Y	Z
R0-D0-N0	30.542	23.95	31.934
R50-D19-N3	31.787	30.103	33.252
R50-D19-N5	18.896	33.325	37.32
R50-D19-N7	22.852	26.66	29.297

Table (6): Practical Frequency for 500 rpm

1000			
Name	Practical Frequency Hz		
	Near Gears		
	X	Y	Z
R0-D0-N0	57.593	54.348	60.669
R50-D19-N3	58.105	64.087	71.265
R50-D19-N5	75.537	63.403	36.914
R50-D19-N7	76.733	62.72	62.549

Table (7): Practical Frequency for 1000 rpm

Name	1500		
	Practical Frequency Hz		
	Near Gears		
	X	Y	Z
R0-D0-N0	55.908	101.32	113.77
R50-D19-N3	70.068	76.904	83.645
R50-D19-N5	77.881	69.58	87.891
R50-D19-N7	87.891	69.336	110.35

Table (8): Practical Frequency for 1500 rpm

Name	2000		
	Practical Frequency Hz		
	Near Gears		
	X	Y	Z
R0-D0-N0	142.190	99.375	47.188
R50-D19-N3	133.13	80.313	106.88
R50-D19-N5	80	118.44	111.25
R50-D19-N7	122.81	118.13	122.19

Table (9): Practical Frequency for 2000 rpm

To clarify how the values were extracted from the LabVIEW program and analyzed by the sigveiw program

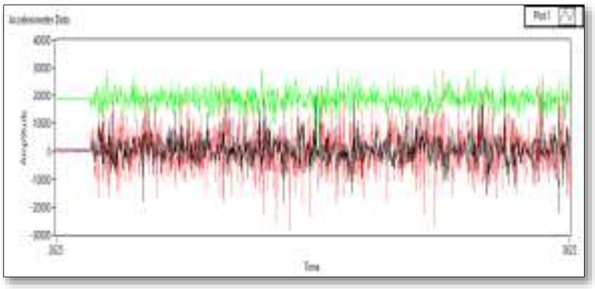

Figure (4): LabVIEW Program

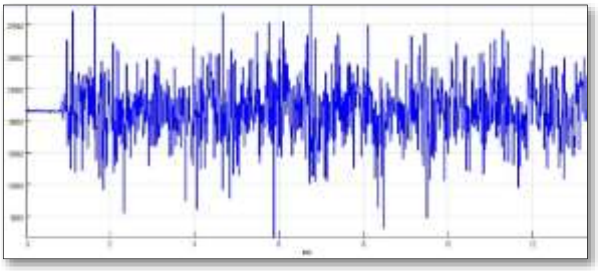

Figure (5): Time Domain (Sigview)

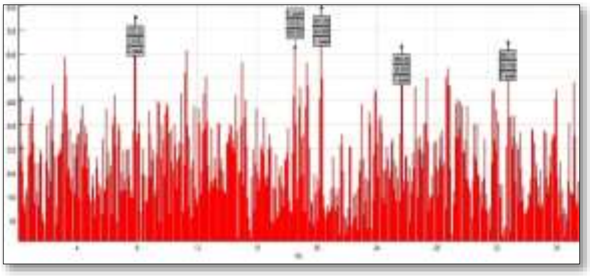

Figure (6): Frequency Domain (Sigview)

When applying the practical case with the theoretical case using the Ansys program, it was found that the value of the natural frequency is higher than the hypothetical case, due to the presence of dampers and bearings, where the value of the frequency is 229 Hz, while the default value is 164 Hz Figure (7) shows the mod shape 1 of the system and the vibration centering point.

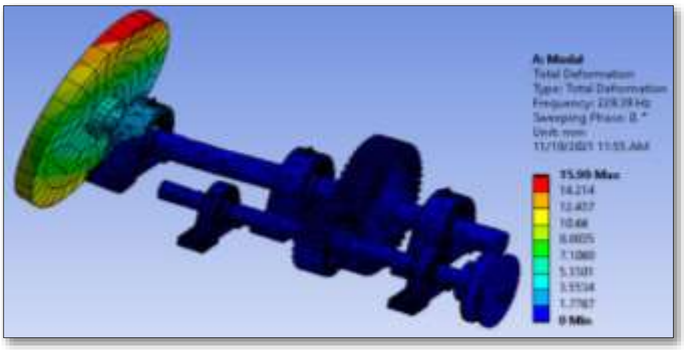

Figure (7): Mode Shape 1

The representation of the practical values extracted on the graph is promised as shown in Figure (8).

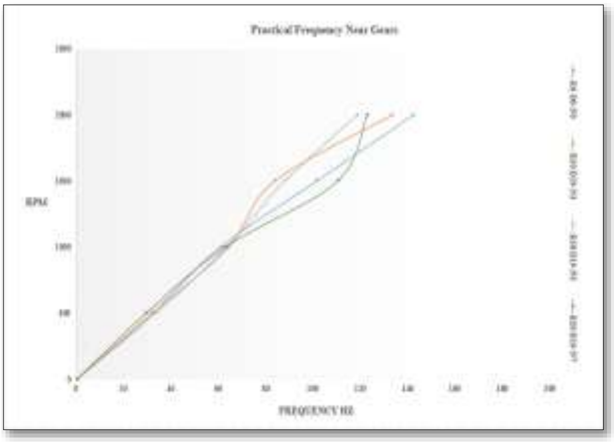

Figure (8): Speed & Frequency chart

From the graph, where it shows the relationship between speed and frequency according to the number of holes, as it becomes clear that in the case of holes in the gears, the relationship is closer to a linear relationship when the number of the gear is 5 in the gear, and using the graph program, an equation can be found for the curve R50-D19-N3 and it is as follows

$$speed(rpm) = 10.3733 * x^{1.1049}$$

x: is value of frequency
where the value of x represents the practical frequency of the system that contains the gear with 5 holes.

It was found that if there are 5 holes in the gear, the frequency is somewhat stable and better than the gears that contain 3 and 7 respectively.

6- Practical Noise Result

The practical values of noise were extracted for a period of 15 seconds, as the tables below show these values at different speeds (500, 1000, 1500 and 2000).

#	R0-D0-N0	R50-D19-N3	R50-D19-N5	R50-D19-N7
1	89	94	95.8	94
2	88.9	94.4	95.4	93.3
3	89.6	89.6	94.9	94.4
4	88.8	93.3	93.8	93.1
5	90	88.6	92.3	88.6
6	90.1	94.5	92.7	94.1
7	91.3	92.4	93.1	92.7
8	90.8	90.6	94.8	94.4
9	92.6	95.2	93.1	93.9
10	91.2	93.4	94.3	89.3
11	90.5	95.1	94.5	85.5
12	91.2	92.1	94.6	87
13	93.7	94.6	94.8	94.4
14	91.6	92.4	94.3	94.5
15	91.8	92.3	92.8	90.4
16	93.2	94.6	94.8	85.3
17	94.6	94.3	93.3	94.3
18	92.8	94.8	93.9	89.7
19	91.9	94.1	94.2	94.9
20	92	93.8	94.1	93.6
Min	88.8	88.6	92.3	85.3
Max	94.6	95.2	95.8	94.9
Av.	91.3	93.2	94.1	91.9

Table (10): Noise Measurement – Gear R50-D19 500 rpm

#	R0-D0-N0	R50-D19-N3	R50-D19-N5	R50-D19-N7
1	94.9	98.5	99.8	98.2
2	93.9	99	100.7	100.2
3	95.2	98.9	101.6	98.3
4	95	99.7	100.2	98.1
5	95	100.3	100.8	98.2
6	96.8	97.1	101.6	99.6
7	96	98.6	101	99.2
8	95.5	98.7	100.8	96.3
9	95.3	99	100.6	98.4
10	96.5	98.1	101.5	99.3
11	96.2	98.6	101.4	96.9
12	96	99.6	101.7	100.1
13	95.8	99.4	101	98.9
14	94.4	100.1	100.1	96.9
15	95	99.2	101.6	99.1
16	95.8	99	100.3	99.8
17	96.9	98.4	101	99.4
18	96.1	99.3	99.7	98.1
19	95.9	98.8	100.7	97.3
20	96	98.6	100	98.9
Min	93.9	97.1	99.7	96.3
Max	96.9	100.3	101.7	100.2
Av.	95.6	98.9	100.8	98.6

Table (11): Noise Measurement – Gear R50-D19 1000 rpm

#	R0-D0-N0	R50-D19-N3	R50-D19-N5	R50-D19-N7
1	96.6	104.5	104.1	101.5
2	96.7	104.8	104.7	103.3
3	97.2	105	105.2	100.7
4	97	103.5	105	100.4
5	97.8	104.7	106.1	101.2
6	97.1	105.2	104.4	101.9
7	97.9	104.6	104.7	104.3
8	96.8	103.7	104.6	102.7
9	98	104.3	105.4	100.9
10	97.9	104.9	104	100.6
11	98.2	104.6	103.2	101.4
12	98	104.9	104.7	102.2
13	98.3	105.1	105.3	104.3
14	98.1	104.3	103.5	104.9
15	99.2	105.6	105.4	102
16	99	104.7	105.8	100.8
17	99.6	104.5	104.5	102
18	99.1	104.8	103.5	101.3
19	99.3	105.5	103.8	104.8
20	99.5	106.1	104.9	102.7
Min	96.6	103.5	103.2	100.4
Max	99.6	106.1	106.1	104.9
Av.	98.1	104.8	104.6	102.2

Table (12): Noise Measurement – Gear R50-D19 1500 rpm

#	R0-D0-N0	R50-D19-N3	R50-D19-N5	R50-D19-N7
1	99.6	107.7	106.4	106.2
2	98.8	108.2	106.7	104.7
3	99.9	108.5	105.9	104.5
4	98.6	108.7	106.9	106.2
5	100.2	108	106.6	105.8
6	100.8	109.5	106.2	104.2
7	101	107.8	107.3	107.1
8	100.7	109.1	106.9	104.7
9	101.2	107.9	107.3	104
10	101.3	108.2	106	106.6
11	101.5	108.9	106.1	106.3
12	101.4	107.3	107.1	105.4
13	101.9	108.7	108	106.6
14	101.7	107.8	106.7	104.6
15	102.3	108	106.8	105.1
16	102.1	107.6	106.3	106.2
17	102.5	106.8	106.5	106.5
18	101.7	107.4	106.7	106.1
19	101.6	107.6	107.4	104.9
20	101.9	108	107.6	107.6
Min	98.6	106.8	105.9	104.0
Max	102.5	109.5	108.0	107.6
Av.	101.0	108.1	106.8	105.7

Table (13): Noise Measurement – Gear R50-D19 2000 rpm

The above tables represent the noise values resulting from the collision of the gear teeth with each other with an applied load without lubrication.

When these values are represented in graphs between time and noise for different holes as shown in the figures (9), (10), (11) and (12):

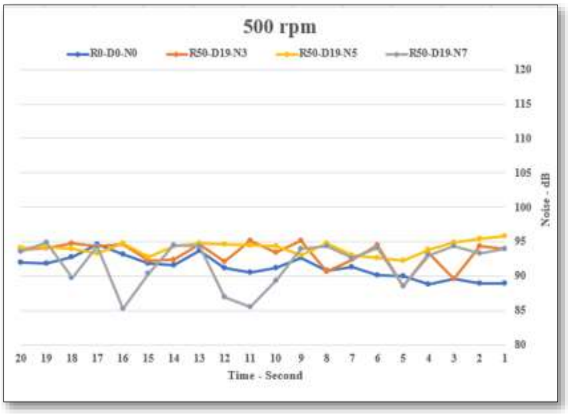

Figure (9): Noise Measurement 500 rpm

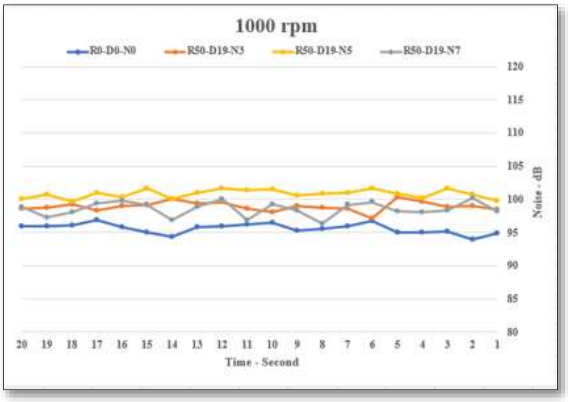

Figure (10): Noise Measurement 1000 rpm

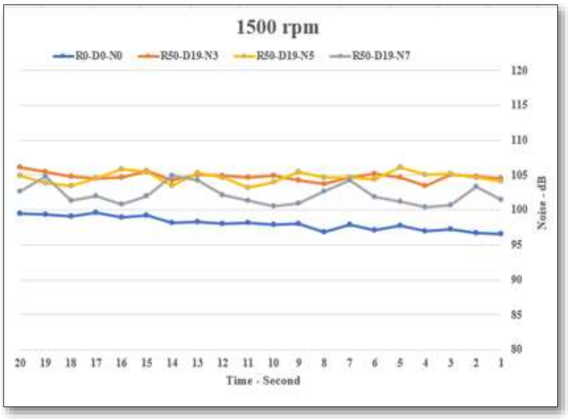

Figure (11): Noise Measurement 1500 rpm

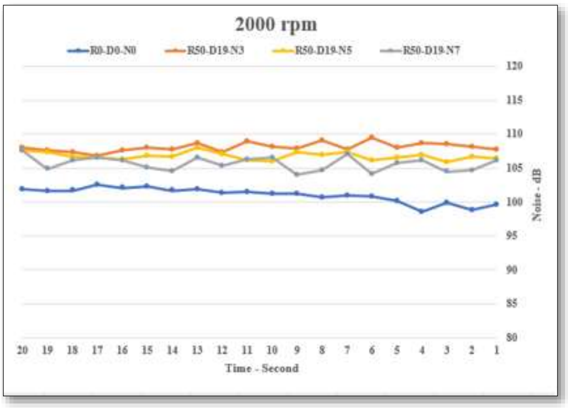

Figure (11): Noise Measurement 2000 rpm

After practically measuring the noise at speeds of 500, 1000, 1500 and 2000, it turns out that the gear that does not contain holes has little noise, but it has a large weight. In the case of holes, the noise increases as in the previous figures, where the higher the speed, the higher the noise generated.
When reviewing the data, it was found that in the case of 5 holes in the gear, the noise is stable, i.e., its duration ranges from 95 to 105 dB.

7- Experimental Set Up

4 gears were used, the first gear does not contain a hole, and the rest contains 3, 5 and 7 holes, respectively.

Figure (13): Gears

A 2-horsepower motor was used in the experiment, in isolation from the external environment

Figure (14): System

8- Discussion

Using the approximate truck power and dungeon method, and the determinants (dynamic observers), we calculated the natural frequencies of some of the model systems according to some of the determinants described in Chapter 3. ANSYS software uses SolidWorks software to detect the first seven natural frequencies of the imported system model. The closest calculation that can be compared to the natural frequency modeling model is the irregular series type. Especially in positions 5 and 6, there are around 4-5 locking modes with results slightly similar to the theoretical calculation. This represents a good confluence of results. The theoretical excitation frequency depends on the speed of rotation and the number of teeth. When comparing the excitation frequencies with the calculated actual frequencies, a good approximation provides them as a basis for comparison. The frequency behavior associated with an increase in the number of holes is accompanied by an increase in the calculated frequency in almost all the calculated cases. The general conclusion about the operating conditions associated with the model used in the experiment is that it reaches resonance at high speeds of 1438 rpm, which negatively affects the life of the gear. Experimental experience has shown that increasing the number of holes increases the vibration of the system. This is useful for applications where the required weight is low. I echo frequently

Following all the results, it is evident that the noise level decreases at different speeds, and as the rotation speed increases, the effect of the noise begins to increase with different effects, the lower the noise produced in the gears. Undrilled gears with noise level below 85 decibels. All other case readings are considered to have relatively high noise levels with most being above 90 dB, which is considered dangerous when the object is open for closed or extended periods of time. When studying only the noise results (without studying the vibration analysis), the noise increases with the number of holes. And it reached a good position when the gear had 5 holes, at a distance of 50 from the center of rotation, with a diameter of 19mm, where the noise was constant at different speeds with the range (92-106) dB

9- Conclusion

The current study conducted a comprehensive study on gears and how to increase their effectiveness by controlling their weight using circular holes and taking the effect of these holes on the machine in general and on performance in particular. The ideal case for the system containing perforated gears with circular holes was found by calculating vibration and noise values as the two factors The two principals in the performance of the system, as it was found that gears with 5 holes are better than gears with 3 and 7 holes in terms of frequency, noise and weight.

9-Acknowledgment

Thanks go to the family, especially to my father, may God bless him in his mercy, and also thanks to everyone for the moral support and I thank the University of Technology.

10- References

1- Yokota, T.; Taguchi, T.; Mitsuo, G. A "Solution Method for Optimal Weight Design Problem of the Gear Using Genetic" Algorithms ,1998

2- Özek, F. "Optimum Ağırlıklı Düz Dişli Çark Tasarımı ve Gerilme Analizi", 2007

3- N. H. Abu-Hamdeh and M. A. Alharthy "A Study on the Influence of using Stress Relieving Feature on Reducing the Root Fillet Stress in Spur Gear", ResearchGate, ISSN 2227-4588, Interlaken, Switzerland, February, 2014.

4- S. Oda, K. Miyachika, and T. Sayama, "Effects Of Rim and Web Thicknesses on Bending Fatigue Strength of Internal Gear", Bulletin of JSME, volume 29, pages 586–592, 1986.

5- Journal of Sound and vibration (2000) 234(2), 311}329

6- H. Dresig and F. Holzweißig, "Dynamics of Machinery - Theory and Applications", pp. 225, Springer-Verlag Berlin Heidelberg, 2010.

7- J. D. Smith, "Gear Noise and Vibration", 2nd edition - Revised and Expanded, pp. 8, Marcel Dekker, Inc, 2003.

8- J. Zhou and S. Wenlei, "Vibration and Noise Radiation Characteristics of Gear Transmission System", Xinjiang University, 830047, China, volume 33, pages 485–502, 2014.

9- S. Shweiki, A. Palermo, and D. Mundo, "A Study on the Dynamic Behaviour of Lightweight Gears", Hindawi Publishing Corporation, Article ID 7982170, 2017.

10- J. Hiremagalur and B. Ravani, "Effect of Backup Ratio on Root Stresses in Spur Gear Design", University of California, Mechanical and Aeronautical Engineering, one shields Avenue, Davis, CA 95616, 2004.

11- G. D. Bibel, S. K. Reddy, M. Savage, and R. F. Handschuh, "Effects of Rim Thickness on Spur Gear Bending Stress", University of Akron & U. S. Army Aviation Systems Command, NASA Lewis Research Center, Cleveland, Ohio, Technical Memorandum104388, 1991.

12- F. Karpat and B. Engin, "Effects Of Rim Thickness on Tooth Root Stress and Mesh Stiffness of Internal Gears", Department of Mechanical Engineering, Uludag University, Bursa, 16059, Turkey, November, 2014.

13- S. Oda, K. Miyachika, and T. Sayama, "Effects Of Rim and Web Thicknesses on Bending Fatigue Strength of Internal Gear", Bulletin of JSME, volume 29, pages 586–592, 1986.

14- S. Mahendran, K. M. Eazhil, and L. Senthil Kumar, "Design and Analysis of Composit Spur Gear", Research and Scientific Innovation Society RSIS International, ISSN 2321-2705, November, 2014.

15- A. R. Hassan, G. Thanigaiyarasu, and V. Ramamurti, "Effect of Natural Frequency and Rotational Speed on Dynamic Stress in Spur Gear", World Academy of Science, Engineering and Technology, volume 2, 2008.

F. L. Litvin and A. Fuentes, *Gear Geometry and Applied Theory*. Cambridge, Cambridge University Press, 2004.

16- J. R. Colbourne, the Geometry of Involute Gears. Newyork, Springer Verlag, 2012.

17- E. Buckingham, Analytical Mechanics of Gears. Courier Corporation, 1988.

18- S. P. Radzevich and W. D. Darle, Handbook of Practical Gear Design. CRC press, 1994.

19- Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth. ANSI/AGMA 2001-B88, 1999.

20- Calculation of Load Capacity of Spur and Helical Gears, ISO 6336, 2006.

21- S. Oda et al., “Stress analysis of thin rim spur gears by finite element method,” Bulletin of JSME, vol. 24, no. 193, pp. 1273-1280, 1981.

22- N. Arai et al., “Research on bending strength properties of spur gears with a thin rim,” Bulletin of JSME, vol. 24, no. 195, pp. 1642-1650, 1981.

23- D. G. Lewicki and R. Ballarini, “Effect of rim thickness on gear crack propagation path,” Journal of Mechanical Design, vol. 119, no.1, pp. 89-95, 1997.

24- J. Hiremagalur and B. Ravani, “Effect of backup ratio on root stresses in spur gear design,” Mechanics Based Design of Structures and Machines, vol. 32, no. 4, pp. 423-440, 2004.

25- J. Kramberger et al., “Numerical calculation of bending fatigue life of thin-rim spur gears,” Engineering Fracture Mechanics, vol. 71, no. 4, pp. 647-656, 2004.