Neutrosophic Bipolar Soft Topological Compact Spaces

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***Abstract* - The present manuscript we introduce a new idea about neutrosophic bipolar soft (NBS) compact space. This is related to topological space and neutrosophic bipolar soft set. Next we establishedfew basic definitions with respect to NBS compact set with respect to topological space. In additional we obtain the results based on NBS sets with respect to compact space.**

INTRODUCTION

By Smarandache, Neutrosophic theory is well established and many more applications are there on it. After that many authors are developed this theory in all fields of different branches. The base of this topic is Intuitionistic fuzzy theory. Here we arranged in sequential order to how to develop from Intuitionistic fuzzy to Neutrosophic theory [1, 18].
Bayramov and Gunduz [2] discussed about intuitionistic fuzzy topological soft spaces, in this the authors are given the basic definitions and some of the results on topological soft spaces. Soft neutrosophic topological spaces are explained by Bera and Mahapatra [3].The Neutrosophic sets launch by Smarandache [16] are a great exact implement for the situation uncertainty in the real world. The compactness on soft neutrosophic spaces with metric has been studied byBera and Mahapatra[4]. The topological spaces with respect to fuzzy theorystudied by Chang [6].
The representation of the neutrosophic sets are truth, indeterminacy and falsity value. These T, I, F values belongs to standard or non-standard unit interval denoted by $]^{-}0, 1^{+}[$ [14,15].Cagman etal.,[5] introducesthe concept of soft topology and its properties and importance together with some results on soft topology.On intuitionistic points and intuitionisticsets,relations on soft sets with respect to neutrosophic theory and some of its properties, different operations on soft neutrosophic sets and soft neutrosophic topological spaceseliberate by many authors [6-11].These uncertainty idea comes from the theories of fuzzy sets [7], intuitionistic fuzzy sets [4, 6] and interval valued intuitionistic fuzzy sets [5].Ozturk T and Shabir M[12, 17] are successfully established a new approach to operate on neutrosophic soft sets to neutrosophic soft topological spaces. In the present study, we are discussing more on soft topological compact space with respect to bipolar neutrosophic theory, and also continuous the work on soft topological space with separation axioms[13].
Venkateswara Rao etal., introduces pre-open sets and pre-closed sets in neutrosophic topology and extended this study complex neutrosophic graphs with Broumi[27, 28]. Upender Reddy etal., extend the neutrosophic theory to bipolar single valued theory on graphs as well as bipolar topological neutrosophic set[19-22]. Siva Nageswara Raoetal., collaborated work on bipolar neutrosophic weekly closed sets and interior and boundary vertices on bipolar neutrosophic graphs[23-25]. Broumi et al., demonstrate a new trend in neutrosophic theory in probability, decision making problems, graph theory, topological space, soft sets relations and some properties[29-33].

2. PREPARATORY

Here we introduce some notations (Short cuts) which are using further simplification purpose. Further we explained some basic definitions on neutrosophic bipolar soft set and we deduce some results based on neutrosophic bipolar soft set, and compact set.
Abbreviations:
1. Neutrosophic Set(NS)
2. Neutrosophic Soft Sets(NSS)
3. Neutrosophic Bipolar Soft (NBS)
4. Neutrosophic Bipolar Soft Sets(NBSS)

5. Neutrosophic Bipolar Soft Topology(NBST)
6. Neutrosophic Bipolar Soft Topological Space(NBSTS)
7. Neutrosophic Bipolar Soft Open(NBSO)
8. Neutrosophic Bipolar Soft Open Sets(NBSOS)
9. Neutrosophic Bipolar Soft Closed Set (NBSCS)
10. Neutrosophic Bipolar Soft Interior (NBSI)
11. Neutrosophic Bipolar Soft Compact (NBSC)
12. Neutrosophic Bipolar Soft Compact Set(NBSCS)

Explanation 2.1. Let the domain be Υ along with acollection of variables η. Consider $N(\Upsilon)$ designate thecollection of every NS of domain. A NSS λ_η over Υ is establish by awell-defined collection appraisemapping λ act fora mapping λ from η to $N(\Upsilon)$ is a multivalued mapping then λ_η over Υ is said to be NSS, here λ_η is approximated by λ. In other words, λ_η is a restricted householdelementsoft $N(\Upsilon)$ and symbolically written as $\lambda_\eta = \{(\mathrm{l}, \langle \Upsilon_1, \lambda_{\Phi(\mathrm{l})}(\Upsilon_1), \lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)\rangle : \Upsilon_1 \in \Upsilon) : \mathrm{l} \in \eta\}$, where $0 \le \lambda_{\Phi(\mathrm{l})}(\Upsilon_1), \lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1) \le 1$ said to be the membership about truth, neutral and falsitymapping of $\lambda(\mathrm{l})$, respectively. As the least upper bound of all Φ, Θ, Λ is unity, so the values of $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1), \lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)$ lies between zero to three.

All round this exertion, $BN_{S\hat{S}}(\Upsilon_\eta)$ mention the variety of each and every NBSSover Υ.

Explanation 2.2. Consider $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. The notation λ_η^{C} indicates the complement of λ_η and isexplained by $\lambda_\eta^{C} = \{(\mathrm{l}, \langle \Upsilon_1, \lambda_{\Phi(\mathrm{l})}(\Upsilon_1), 1-\lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)\rangle : \Upsilon_1 \in \Upsilon) : \mathrm{l} \in \eta\}$. Obvious that, $(\lambda_\eta^{C})^{C} = \lambda_\eta$.

Explanation 2.3. Consider $\lambda_\eta, \bar{\phi}_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. Then λ_η is a subset of $\bar{\phi}_\eta$, denoted by $\lambda_\eta \subseteq \bar{\phi}_\eta$. If for $\forall \eta_1 \in \eta, \forall \Upsilon_1 \in \Upsilon$;

1. $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1) \le \bar{\phi}_{\Phi(\mathrm{l})}(\Upsilon_1)$
2. $\lambda_{\Theta(\mathrm{l})}(\Upsilon_1) \le \bar{\phi}_{\Theta(\mathrm{l})}(\Upsilon_1)$
3. $\lambda_{\Lambda(\mathrm{l})}(\Upsilon_1) \le \bar{\phi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$.

λ_η equals to $\bar{\phi}_\eta$ if $\lambda_\eta \subseteq \bar{\phi}_\eta$ and $\bar{\phi}_\eta \subseteq \lambda_\eta$. It is denoted by $\lambda_\eta = \bar{\phi}_\eta$.

Explanation 2.4. Let $\lambda_\eta, \bar{\phi}_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. The union of λ_η and $\bar{\phi}_\eta$ is equal to $\bar{\psi}_\eta$ and is interpretedas $\bar{\psi}_\eta = \{(\mathrm{l}, \langle \Upsilon_1, \lambda_{\Phi(\mathrm{l})}(\Upsilon_1), \lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)\rangle : \Upsilon_1 \in \Upsilon) : \mathrm{l} \in \eta\}$, where $\bar{\psi}_{\Phi(\mathrm{l})}(\Upsilon_1)$ is the maximum of $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Phi(\mathrm{l})}(\Upsilon_1)$, $\bar{\psi}_{\Theta(\mathrm{l})}(\Upsilon_1)$ is maximum of $\lambda_{\Theta(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Theta(\mathrm{l})}(\Upsilon_1)$ and $\bar{\psi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$ is minimum of $\lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$.

Explanation 2.5. Let $\lambda_\eta, \bar{\phi}_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. The intersection of λ_η and $\bar{\phi}_\eta$ is equal to $\bar{\psi}_\eta$ and is described as $\bar{\psi}_\eta = \{(\mathrm{l}, \langle \Upsilon_1, \lambda_{\Phi(\mathrm{l})}(\Upsilon_1), \lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)\rangle : \Upsilon_1 \in \Upsilon) : \mathrm{l} \in \eta\}$, where $\bar{\psi}_{\Phi(\mathrm{l})}(\Upsilon_1)$ is the minimum of $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Phi(\mathrm{l})}(\Upsilon_1)$, $\bar{\psi}_{\Theta(\mathrm{l})}(\Upsilon_1)$ is minimum of $\lambda_{\Theta(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Theta(\mathrm{l})}(\Upsilon_1)$ and $\bar{\psi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$ is maximum of $\lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$.

Explanation2.6. Let $\lambda_\eta, \bar{\phi}_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. Then, λ_η difference $\bar{\phi}_\eta$ operation on them is indicated as $\lambda_\eta \setminus \bar{\phi}_\eta = \bar{\psi}_\eta$ and is explained by $\bar{\psi}_\eta = \lambda_\eta \cap \bar{\phi}_\eta^{C}$ as follows: $\bar{\psi}_\eta = \{(\mathrm{l}, \langle \Upsilon_1, \lambda_{\Phi(\mathrm{l})}(\Upsilon_1), \lambda_{\Theta(\mathrm{l})}(\Upsilon_1), \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)\rangle : \Upsilon_1 \in \Upsilon) : \mathrm{l} \in \eta\}$, where $\bar{\psi}_{\Phi(\mathrm{l})}(\Upsilon_1)$ is the minimum of $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Phi(\mathrm{l})}(\Upsilon_1)$, $\bar{\psi}_{\Theta(\mathrm{l})}(\Upsilon_1)$ is minimum of $\lambda_{\Theta(\mathrm{l})}(\Upsilon_1)$ and $1-\bar{\phi}_{\Theta(\mathrm{l})}(\Upsilon_1)$ and $\bar{\psi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$ is maximum of $\lambda_{\Lambda(\mathrm{l})}(\Upsilon_1)$ and $\bar{\phi}_{\Lambda(\mathrm{l})}(\Upsilon_1)$.

Explanation2.7. $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$.Then, λ_η is called,

1. an emptyNSS if $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1) = 0, \lambda_{\Theta(\mathrm{l})}(\Upsilon_1) = 1, \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1) = 1$; $\forall \mathrm{l} \in \eta, \forall \Upsilon_1 \in \Upsilon$. It is denoted by 0_{Υ_η}.
2. complete NSS if $\lambda_{\Phi(\mathrm{l})}(\Upsilon_1) = 1, \lambda_{\Theta(\mathrm{l})}(\Upsilon_1) = 1, \lambda_{\Lambda(\mathrm{l})}(\Upsilon_1) = 0 \ \forall \mathrm{l} \in \eta, \forall \Upsilon_1 \in \Upsilon$. It is denoted by 1_{Υ_η}.

Clearly, $0^{C}{}_{\Upsilon_\eta} = 1_{\Upsilon_\eta}$ and $1^{C}{}_{\Upsilon_\eta} = 0_{\Upsilon_\eta}$.

Result 2.8. Let $\lambda_\eta, \bar{\phi}_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. Then the

1. accompaniment of union of λ_η & $\bar{\phi}_\eta$ is same as common part of complement of λ_η & $\bar{\phi}_\eta$
2. complement ofintersection of λ_η & $\bar{\phi}_\eta$ is equal to union of complement of λ_η & $\bar{\phi}_\eta$

Explanation 2.9. Let $\overset{BN_{SS}}{*\tau}$ be a subset $BN_{S\hat{S}}(\Upsilon_\eta)$. Then $\overset{BN_{SS}}{*\tau}$ is called a NBST on Υ if,

1. $0_{\Upsilon_\eta}, I_{\Upsilon_\eta} \in \overset{BN_{SS}}{*\tau}$
2. $\lambda_\eta, \bar{\phi}_\eta \in \overset{BN_{SS}}{*\tau}, \lambda_\eta \cap \bar{\phi}_\eta \in \overset{BN_{SS}}{*\tau}$
3. $(\lambda_\eta)_i, \forall i \in I$,the arbitrary union of $(\lambda_\eta)_i$ is also an element of $\overset{BN_{SS}}{*\tau}$. The triplet $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is known as(NBSTS). The elements of $\overset{BN_{SS}}{*\tau}$ are said to be NBSOin Υ.

Explanation 2.10. Consider $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is aNBSTS over Υ and $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$.

(a) Then, λ_η is known as NBSCSif λ_η^C is a NBSOS.

(b) Then, the NSI of λ_η, indicated by $(\lambda_\eta)^o$, is explained byNBSjoining of every NBSO subsets of λ_η. $(\lambda_\eta)^o = \cup\{\bar{\phi}_\eta \in BN_{S\hat{S}}(\Upsilon_\eta) : \bar{\phi}_\eta \text{ is a BNSOS and } \bar{\phi}_\eta \subset \lambda_\eta\}$

Clearly, $(\lambda_\eta)^o$ is the maximum of NBSOSbelongs to λ_η.

(c) Then, the NBS closure of λ_η, indicated by $,\overline{(\lambda_\eta)^c}$, is explained by the NBScommon of every NBSCcontains λ_η. $\overline{(\lambda_\eta)^c} = \bar{\cap}\{\chi_\eta \in BN_{S\hat{S}}(\Upsilon_\eta) : \chi_\eta \text{ is a BNSCS and } \lambda_\eta \subset \chi_\eta\}$

Clearly, $\overline{(\lambda_\eta)^c}$ is the minimum of NBSCScontained in λ_η.

Explanation2.11. The NBSS $\Upsilon_1{}^1{}_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})}$ is called a NBS point,for every $\forall j \in \Upsilon$, $-1 \le \Im^{*-}, \Delta^{*-}, h^{*-} \le 0$, $0 \le \Im^{*+}, \Delta^{*+}, h^{*+} \le 1$ for every $1 \in \eta$, and is describe by: $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})}(\wp)(h) = \begin{cases}(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+}), & if\ \wp' = \wp \text{ and } h = j, \\ (0,0,-1;0,0,1), & if\ \wp' = \wp \text{ or } h \ne j\end{cases}$

Explanation 2.12. Consider λ_η is aNBS set on top of Υ, said to be $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})} \in \lambda_\eta$,if $\Im^{*-} > B^{\Phi_{\lambda(1)}}(j)$, $\Delta^{*-} > B^{\Theta_{\lambda(1)}}(j)$ and $h^{*-} < B^{\Lambda_{\lambda(1)}}(j)$

$\Im^{*+} < B^{\Phi_{\lambda(1)}}(j)$, $\Delta^{*+} < B^{\Theta_{\lambda(1)}}(j)$ and $h^{*+} > B^{\Lambda_{\lambda(1)}}(j)$.

Explanation 2.13. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ and $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. Then, λ_η is called a NBS neighborhood of the NBS $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})} \in \lambda_\eta$, if $\exists$ a NBS open set $\bar{\phi}_\eta$ such $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})} \in \lambda_\eta \in \bar{\phi}_\eta \subset \lambda_\eta$.

Result 2.14. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS and $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. Then, λ_η is a NBS open set $\Leftrightarrow$ λ_η is a NBSneighborhood of itsNBS points.

Explanation 2.15. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ and $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})}$ and $h^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})}$ are distinct NBS points. If $\exists$ NBSopen sets λ_η and $\bar{\phi}_\eta$ such that, $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})} \in \lambda_\eta$, $j^1_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})} \cap \bar{\phi}_\eta = 0_{\Upsilon_\eta}$ and $y^{1'}_{(\Im^{*-\prime},\Delta^{*\prime},h^{*-\prime},\Im^{*+\prime},\Delta^{*+\prime},h^{*+\prime})} \in \bar{\phi}_\eta$ $y^{1'}_{(\Im^{*-\prime},\Delta^{*\prime},h^{*-\prime},\Im^{*+\prime},\Delta^{*+\prime},h^{*+\prime})} \cap \lambda_\eta = 0_{\Upsilon_\eta}$

Then, $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is called a NBS T_1-space.

Explanation 2.16. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ and $j^{1}_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})}$, and $h^{1}_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})}$ are distinct NBS points. If $\exists$ NBSopen sets λ_η and $\bar{\phi}_\eta$ such that, $j^{1}_{(\Im^{*-},\Delta^{*-},h^{*-},\Im^{*+},\Delta^{*+},h^{*+})} \in \lambda_\eta$, $h^{1'}_{(\Im^{*-'},\Delta^{*'},h^{*-'},\Im^{*+'},\Delta^{*+'},h^{*+'})} \in \bar{\phi}_\eta$ and $\lambda_\eta \cap \bar{\phi}_\eta = 0_{\Upsilon_\eta}$.

Then, $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is called a NBS T_2-space (NBS Hausdorff space).

Explanation 2.17. Ais calleda NBS normal spaceover Υ *is indicated by* $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ (NBSTS) and is explained as for any two different NBSCS λ^1_η, λ^2_η, $\exists$ disjoint NBSopen $\bar{\phi}^1_\eta, \bar{\phi}^2_\eta$ such that $\lambda^1_\eta \subset \bar{\phi}^1_\eta$ and $\lambda^2_\eta \subset \bar{\phi}^2_\eta$. $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is aNBS T_4-space it satisfies both NBSnormal and neutrosophic soft T_1-space.

Explanation 2.18. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ and λ_η be an arbitrary NBSS. Then $\tau_{\lambda_\eta} = \{(\lambda_\eta \cap \bar{\psi}_\eta) : \bar{\psi}_\eta \in \tau\}$ is said to beNBST on λ_η and $(\lambda_\eta, \overset{BN_{SS}}{*\tau_{\lambda_\eta}}, \eta)$ is called a NBST subspace of $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$.

SOFT COMPACT SPACES WITH RESPECT TO BIPOLAR NEUTROSOPHIC

In the present division, we discuss basic terminology of NBSCspace on a NBSS. Also we derived some prepositions on it.

Explanation 3.1. (i) ConsiderJis anyset $\Omega = \{(\lambda_\eta)_j : j \in J\}$ a collection of subset of $BN_{S\hat{S}}(\lambda_\eta)$. If $\lambda_\eta \subset \cup_{j\in J}(\lambda_\eta)_j$, then Ω is cover of the NBS subset J_{Υ_η}. Ω is NBS coverof J_{Υ_η} if it satisfies $\lambda_\eta = J_{\Upsilon_\eta}$

(ii) Ω is a finite or countable NBS cover, if the cardinality of J_{Υ_η} is finite or countable.

(iii) Consider $\Omega = \{(\lambda_\eta)_j : j \in J\}$ is a NBS cover of J_{Υ_η}.If, J_{Υ_η} is the NBS subfamily of Ω and it is also a coverof J_{Υ_η}, then Ω' is NBS sub-cover of Ω.

(iv) Every countableNBS subclasses of Ω has non-empty intersection is known as centralized NBS, where Ω is a class of NBSS.

Explanation 3.2. (i) Consider $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ and Ω be a NBS cover of J_{Υ_η}. If every element of the cover Ω is a NBS open (closed) in $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$, then Ω is c a NBS open (closed)cover.

(ii) The intersection between the neighborhoodof some NBS with the cover Ω is countablefor every $j^{1}_{(\Im^{*},\Delta^{*},h^{*})} \in BN_{S\hat{S}}(\Upsilon_\eta)$, Ω is called a NBS locally finite cover.

(iii) Ω is a NBS star finite cover if for every $(\lambda_\eta)_j$ of the cover Ω meets only a countable collection of objects.

(iv) Ω isNBS point finite cover if for every $j^{1}_{(\Im^{*},\Delta^{*},h^{*})} \in BN_{S\hat{S}}(\Upsilon_\eta)$ into only a countable number of objectives of the cover Ω

Explanation 3.3. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ and $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$.

(i) $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is NBScompact space if for any NBS open-cover ofNBSTScontains a countable NBS sub-cover.

(ii) λ_η isNBScompact set if $(\lambda_\eta, \overset{BN_{SS}}{*\tau_{\lambda_\eta}}, \eta)$ is a NBS compact spacewith respect to $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$

Result 3.4. Consider $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTSover Υ, $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$. λ_η is aNBSCSifffor all NBSO cover of λ_η has a countableNBSsub cover in NBSTS.

Proof. Consider $\lambda_\eta \in BN_{S\hat{S}}(\Upsilon_\eta)$ be a NBS compact set and the family $\Omega = \{(\lambda_\eta)_j : j \in J\}$ is a NBS open cover of λ_η in $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$. Then, $\lambda_\eta \subset \cup_{j\in J}(\lambda_\eta)_j \Rightarrow \lambda_\eta = \cup_{j\in J}(\lambda_\eta \cap (\lambda_\eta)_j)$ can be written. Since each $j \in J$ is $\lambda_\eta \cap (\lambda_\eta)_j \in \overset{BN_{SS}}{*\tau_{\lambda_\eta}}$, NBS family $\{\tilde{\lambda}_\eta \cap (\tilde{\lambda}_\eta)_j\}_{j\in J}$ is a NBSopen cover of $\tilde{\lambda}_\eta$ and since $(\tilde{\lambda}_\eta, \overset{BN_{SS}}{*\tau}, \eta)$ isNBS compact, for $\exists j_1, ..., j_n.\ \tilde{\lambda}_\eta = \bigcup_{k=1}^{n}(\tilde{\lambda}_\eta \cap (\tilde{\lambda}_\eta)_{j_k}) \subset \bigcup_{k=1}^{n}(\tilde{\lambda}_\eta)_{j_k}$ is obtained, that is, family $\{(\tilde{\lambda}_\eta)_{j_k}\}_{k=1,n}$ is a NBS finitesub coverof $\tilde{\lambda}_\eta$.

Conversely, let family $\{(\phi_\eta)_j : j \in J\}$ is a NBS open cover of NBSTS. Since each $j \in J$ is $(\phi_\eta)_j \in \overset{BN_{SS}}{*\tau}_{\tilde{\lambda}_\eta}$, there is $(\tilde{\lambda}_\eta)_j \in \overset{BN_{SS}}{*\tau}_{\tilde{\lambda}_\eta}$ suchthat $(\phi_\eta)_j = \tilde{\lambda}_\eta \cap (\tilde{\lambda}_\eta)_j$. Thus, family $\{(\tilde{\lambda}_\eta)_j : j \in J\}$ is a NBS open coveringof the $\tilde{\lambda}_\eta$ in $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ and $\exists(\tilde{\lambda}_\eta)_{j_1}, ..., (\tilde{\lambda}_\eta)_{j_k} : \tilde{F}_E \subset \bigcup_{k=1}^{n}(\tilde{\lambda}_\eta)_{j_k} \Rightarrow \tilde{\lambda}_\eta = \tilde{\lambda}_\eta \cap \left(\bigcup_{k=1}^{n}(\tilde{\lambda}_\eta)_{j_k}\right) = \bigcup_{k=1}^{n}\tilde{\lambda}_\eta \cap (\tilde{\lambda}_\eta)_{j_k} = \bigcup_{k=1}^{n}(\tilde{\lambda}_\eta)_{j_k}$

Result 3.5. Consider $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTS over Υ. J_{Υ_η} isNBS compact space ifffor all classes of NBS closed setswith nullity intersection in NBSTScontains a countable subclasses with nullity intersection.

Proof. Consider $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBS compact space, also the intersection of theclass of $\Omega = \{(\tilde{\lambda}_\eta)_j : j \in J\}$ is the NBS closed sets class is nullity.

Then,the classes, $\Omega = \{(\bar{\phi}_\eta)_j = J_{\Upsilon_\eta} \setminus (\tilde{\lambda}_\eta)_j\}_{j\in J}$ is the NBS open sets family and we obtain, $\bigcup_{j\in J}(\bar{\phi}_\eta)_j = \bigcup_{j\in J}\left(J_{\Upsilon_\eta} \setminus (\tilde{\lambda}_\eta)_j\right) = J_{\Upsilon_\eta} \setminus K_{\Upsilon_\eta} = J_{\Upsilon_\eta}$

Thus, the family $\Omega = \{(\bar{\phi}_\eta)_j\}_{j\in J}$ is a NBS open covering of J_{Υ_η}. Since $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is a NBS compact space, $\exists(\bar{\phi}_\eta)_{j_1}, ..., (\bar{\phi}_\eta)_{j_n} : J_{\Upsilon_\eta} = \bigcup_{k=1}^{n}(\bar{\phi}_\eta)_{j_k}$ is derived. Next the intersection of the countable subclasses $\{(\tilde{\lambda}_\eta)_{j_k}\}_{k=1,n}$ of the NBS class of $\Omega = \{(\tilde{\lambda}_\eta)_j\}_{j\in J}$ and we derive, $\bigcap_{k=1}^{n}(\tilde{\lambda}_\eta)_{j_k} = \bigcap_{k=1}^{n}(J_{\Upsilon_\eta} \setminus (\bar{\phi}_\eta)_{j_k}) = J_{\Upsilon_\eta} \setminus (\bigcup_{j=1}^{n}(\bar{\phi}_\eta)_j = J_{\Upsilon_\eta} \setminus J_{\Upsilon_\eta} = K_{\Upsilon_\eta}$

Conversely, let family $\Omega = \{(\bar{\phi}_\eta)_j\}_{j\in J}$ be a NBS open covering of the J_{Υ_η}.

The intersection of the NBS closed sets family $\Omega = \{(\tilde{\lambda}_\eta)_j = J_{\Upsilon_\eta} \setminus (\bar{\phi}_\eta)_j\}_{j\in J}$ is empty. Really, $\bigcap_{j\in J}(\tilde{\lambda}_\eta)_j = \bigcap_{j\in J}\left(J_{\Upsilon_\eta} \setminus (\bar{\phi}_\eta)_{j_k}\right) = J_{\Upsilon_\eta} \setminus (\bigcup_{j\in J}(\bar{\phi}_\eta)_{j_k} = J_{\Upsilon_\eta} \setminus J_{\Upsilon_\eta} = K_{\Upsilon_\eta}$.

Then, from the condition of theorem $\exists(\tilde{\lambda}_\eta)_{j1}, ..., (\tilde{\lambda}_\eta)_{jn} : \bigcap_{k=1}^{n}(\tilde{F}_E)_{j_k} = K_{\Upsilon_\eta}$ can be written. Hence, $\bigcup_{k=1}^{n}(\bar{\phi}_\eta)_{j_k} = \bigcup_{k=1}^{n} J_{\Upsilon_\eta}\left(J_{\Upsilon_\eta} \setminus (\tilde{\lambda}_\eta)_{j_k}\right) = J_{\Upsilon_\eta} \setminus (\bigcap_{k=1}^{n}(\tilde{\lambda}_\eta)_{j_k} = J_{\Upsilon_\eta} \setminus K_{\Upsilon_\eta} = J_{\Upsilon_\eta}$.

Thus, the NBS finitesub covering of NBS $\Omega = \{(\tilde{\lambda}_\eta)_j : j \in J\}$ coveringof J_{Υ_η} was found. As a result, $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is NBS compactspace.

Result 3.6. Consider $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBSTSover Υ. $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is a NBS compact space if and only if the NBScommon of all members of allNBS centered closed memberclasses is disjoint fromnullity in J_{Υ_η}.

Proof. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBS compact space over Υ, and $\Omega = \{(\tilde{\lambda}_\eta)_j : j \in J\}$ be NBS centered closed sets family. Suppose it is $\bigcap_{j\in J}(\tilde{\lambda}_\eta)_j = K_{\Upsilon_\eta}$. Then,family $\Omega = \{(\bar{\phi}_\eta)_j = J_{\Upsilon_\eta} \setminus (\tilde{\lambda}_\eta)_j\}_{j\in J}$ is an open covering of J_{Υ_η}. Since $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is aNBS compact space. $\exists(\bar{\phi}_\eta)_{j_1}, ..., (\bar{\phi}_\eta)_{j_n}$ in $: J_{\Upsilon_\eta} = \bigcup_{k=1}^{n}(\bar{\phi}_\eta)_{j_k}$ written.

Particularly here $\bigcap_{j=1}^{n}(\tilde{\lambda}_\eta)_{j_k} = \bigcap_{j=1}^{n}\left(J_{\Upsilon_\eta} \setminus (\bar{\phi}_\eta)_{j_k}\right) = J_{\Upsilon_\eta} \setminus (\bigcup_{k=1}^{n}(\bar{\phi}_\eta)_{j_k}) = J_{\Upsilon_\eta} \setminus J_{\Upsilon_\eta} = K_{\Upsilon_\eta}$.

Particularly here, our presumptionisin correct, hence $\bigcap_{j\in J}(\tilde{\lambda}_\eta)_j \neq K_{\Upsilon_\eta}$.

On the other hand, $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ is not NBS compactspace, Then, there exists nocountable NBSsub-covering of $\Omega = \left\{ (\bar{\phi}_\eta)_j \right\}_{j \in J}$ NBS open covering of J_{Υ_η}. Thus, $\bigcup_{k=1}^{n} (\bar{\phi}_\eta)_{j_k} \neq J_{\Upsilon_\eta}$ is obtained for any $(\bar{\phi}_\eta)_{j_1}, ...(\bar{\phi}_\eta)_{j_n}$ infinite NBS subfamily of Ω NBS family. Consider the family of $\Omega = \left\{ (\lambda_\eta)_j = J_{\Upsilon_\eta} \setminus (\bar{\phi}_\eta)_j \right\}_{j \in J}$ NBS closed sets. Here, because $\bigcap_{k=1}^{n} (\lambda_\eta)_{j_k} = \bigcap_{k=1}^{n} (J_{\Upsilon_\eta} \setminus (\bar{\phi}_\eta)_{j_k}) = J_{\Upsilon_\eta} \setminus (\bigcup_{k=1}^{n} (\bar{\phi}_\eta)_{j_k} \neq K_{\Upsilon_\eta}$ is the Ω family is A NBS centered closed set classesalso the resultis $\bigcap_{j \in J} (\tilde{F}_E)_j \neq K_{\Upsilon_\eta}$.

Then, $J_{\Upsilon_\eta} = \bigcup_{j \in J} (\bar{\phi}_\eta)_i = \bigcup_{j \in J} \left(J_{\Upsilon_\eta} \setminus (\lambda_\eta)_{j_k} \right) = J_{\Upsilon_\eta} \setminus \left(\bigcap_{j \in J} (\lambda_\eta)_{j_k} \right) \neq J_{\Upsilon_\eta}$

So, the presumptionis in correct. Hence the result.

Result 3.7. For allNBS closed sub-set of a NBS compact topologicalspace is NBScompact.

Proof. Let $(\Upsilon, \overset{BN_{SS}}{*\tau}, \eta)$ be a NBS compact space, NBS sets family $\Omega = \left\{ (\lambda_\eta)_j : j \in J \right\}$ is an open cover of λ_η.

Particularly, $\lambda_\eta \subset \bigcup_{j \in J} (\lambda_\eta)_j$ and λ_η are NBS closed, so the $J_{\Upsilon_\eta} \setminus \lambda_\eta$ is openset. On the offensive, it is also expressed as $\exists (\lambda_\eta)_{j1}, ...(\lambda_\eta)_{jn}$ in: $(\bigcup_{k=1}^{n} (\lambda_\eta)_{j_k}) \cup (J_{\Upsilon_\eta} \setminus (\lambda_\eta)_{j_k}) = J_{\Upsilon_\eta}$.

Here, $\lambda_\eta \subset \bigcup_{k=1}^{n} (\lambda_\eta)_{j_k}$ is derived.

4. INTERPRETATIONS

At this particular context, we successfullydemonstrated the idea of neutrosophicbipolar soft compact set and their properties. Also we discussed locally bipolar soft compact space with respect to neutrosophic theory. Further we obtained the relationship between NBSC and NBS separation theorem. And also we use these results for future work.

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