Study of Multiple Stenosed Artery with Hall Current Impact on MHD Pulsatile Blood Fluid Through Porous Channel Unsteady Wall Suction/Injection

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Abstract
A mathematical model has been developed for pulsatile blood fluid through on multi-stenosis artery. The blood fluid channel is bounded by porous in unsteady with time included suction and injection at the boundary rate of fluid wall. The effect of external magnetic field with Hall current is consider in the blood flow of horizontal channel. The system of governing differential equations solved analytically by Perturbation technique velocity field, volumetric flow rate and shear strain at the wall are calculated. The essential results are discussed for variation of characteristics parameters displayed plots.

Keywords: Hall current, MHD, Multi-Stenosis, Porous medium, Suction/Injection.

1 Introduction
Characteristics of blood flow on arteries are significantly altered between arterial disease, such as stenosis and aneurysm. They are closely connected to the nature of blood flow development in dynamical behaviour of blood fluid vessels. The cardiovascular diseases is abnormal when carry blood from the heart to all the organs and tissues of the body particularly kidney, brain, heart, gut and muscles itself.

Many authors studying pulsatile blood flow mild stenosed vessels have been investigated Mishra and Chakravorty[4], Nirmala Rathagar and Vijayakumar. The mathematical models are developed through overlapping stenosis time dependent and blood is carreau fluid analyzed by Sankar[5]. A mathematical model is inclined of the porous channel and flow resistance to mild with multiple stenosis analyzed by Maruthi Prasad and Prabhakar[8]. Rekha Bali and Usha Awasthi[7] conducted the blood fluid flow multi stenosed in Cassonfluid with impact of magnetic induction.


Many researchers studied the injection/suction on two parallel porous plates in[2][3]. The micropolar fluid is unsteady flow moving in the channel of suction and injection under the impact of magnetic induction analysed by Venkataswamy et.al., [9].

The present paper is study the pulsate fluid in blood flow moving in x way in u(y) multi stenosis cardiovascular system. Porous channel is considered the dominatated to unsteady based on suction and injection at boundary of the walls. The horizontal channel with Hall current effect of on uniform magnetic field B0 and permeable walls slip velocity of the blood fluid. The solution are analytically solved by perturbation method. In the following numerical values for the different dimensionless parameters are plotted and discussed.

2 Mathematical formulation
We consider the pulsatile flow of multi stenosis blood fluid between two parallel walls from y∗ = 0 to y∗ = R. We assume the model is unsteady, incompressible, and viscous electrically conducting blood fluid in resistance of constant Hall current effect on external magnetic field B0 deputy perpendicular to the channel. The channel is fill up to the porous medium.
The governing equations can be written as follows:

\[ \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} = - \frac{1}{\rho} \frac{\partial p^*}{\partial x} + \Theta \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) - \frac{\partial u^*}{\partial t} - \frac{\sigma B_0^2 u^*}{\rho(1+m^2)} \]  \hspace{1cm} (2)

\[ \frac{\partial v^*}{\partial t} + u^* \frac{\partial v^*}{\partial x} + v^* \frac{\partial v^*}{\partial y} = - \frac{1}{\rho} \frac{\partial p^*}{\partial y} + \Theta \left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} \right) \]  \hspace{1cm} (3)

where \( u^*, v^* \) is velocity of the blood fluid, \( \rho \) is density, \( p^* \) is pressure, \( k \) is permeability, \( t^* \) is time, \( \sigma \) is electrically conducting fluid, \( m \) is the effect of Hall current, \( B_0 \) is the external magnetic field.

We consider the blood fluid is sucked and injected off through the medium of the channel with time depending the velocity field \( V \) is specified as,

\[ V = V_0(1 + \epsilon e^{i \omega t}) \]  \hspace{1cm} (4)

where \( V_0 \) be the uniform transpiration velocity(injection/suction \( V_0 > 0 / V_0 < 0 \) ), \( \epsilon \) is the small amplitude of oscillation and \( \epsilon < 1 \). Equation (3) substituting in (4) becomes,

\[ -\frac{\partial p^*}{\partial y} = \rho \nu_0(i \omega e^{i \omega t}) + \frac{\mu_0 \nu_0}{\rho k} (1 + \epsilon e^{i \omega t}) \]  \hspace{1cm} (5)

From equation (2) substituting in(4) becomes,

\[ \frac{\partial u^*}{\partial t} + V_0(1 + \epsilon e^{i \omega t}) \frac{\partial u^*}{\partial y} = - \frac{1}{\rho} \frac{\partial p^*}{\partial x} + \Theta \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) - \frac{\partial u^*}{\partial t} - \frac{\sigma B_0^2 u^*}{\rho(1+m^2)} \]  \hspace{1cm} (6)

Assuming that the blood flow is symmetric then centerline of the channel is \( y^* = 0 \) ie, flow in the region \( 0 \leq y^* \leq R^*(x) \). The appropriate boundary condition for the model are given as,

\[ \frac{\partial u^*(y, t)}{\partial y} = 0 \hspace{1cm} \text{at} \hspace{1cm} y^* = 0 \]  \hspace{1cm} (7)

\[ u^*(y, t) = \beta \frac{\partial u^*(y, t)}{\partial y^*} \hspace{1cm} \text{at} \hspace{1cm} y^* = R^*(x) \]  \hspace{1cm} (8)
blood fluid is handle to pulsatile, action of the heart which makes an unsteady pressure gradient approximated as, Ogulu[1993]

\[-\frac{\partial p^*}{\partial x^*} = P_s + \epsilon P_0 \cos(\omega t), t > 0\]  \hspace{1cm} (9)

where, \(P_s + \epsilon P_0\) the amplitude of the throbbing component giving rise to blood vessel or arterial pressure and heartbeat pressure \(w = 2\pi f\) with \(f\), the heart burst frequency.

The geometry of the multi-stenosis in the arterial lumen is described mathematically as,

\[R(x^*) = \begin{cases} R_0 & 0 \leq x^* \leq d_1^* \\ R_0 - \frac{\delta_i^*}{2} \left( 1 + \cos \frac{2\pi}{l_i} \left( x^* - d_i^* - \frac{l_i^*}{2} \right) \right) & d_i^* \leq x^* \leq d_i^* + l_i^* \\ R_0 - \frac{\delta_i^*}{2} \left( 1 + \cos \frac{2\pi}{l_i} \left( x^* - d_i^* - \frac{l_i^*}{2} \right) \right) & d_i^* + l_i^* \leq x^* \leq d_{i+1}^* \\ R_0 - \frac{\delta_i^*}{2} \left( 1 + \cos \frac{2\pi}{l_i} \left( x^* - d_i^* + \frac{l_i^*}{2} \right) \right) & d_i^* \leq x^* \leq d_i^* + l_i^* \\ R_0 & d_i^* + l_i^* \leq x^* \leq l^* \end{cases} \]

where, \(R(x^*)\) is the radius of the artery, \(R\) is normal of the radial artery, \(l\) and \(\delta_i\), \((i = 1, 2, 3)\) are the length and maximum thickness of three stenosis \((\delta << R_0)\), \(l\) is length of the cardivacular artery and interval(distance) \(d\) is equispaced point.

Introducing the following non dimensional quantities,

\[\alpha = R_0 \sqrt{\frac{\rho \omega}{\mu}}, \lambda = \frac{k}{\kappa_0}, \frac{\beta^*}{\kappa_0}, \frac{\beta^*}{\kappa_0} U_0, l = \frac{l^*}{\kappa_0}, \delta = \frac{\delta^*}{\kappa_0}, R = \frac{R^*}{\kappa_0} \]  \hspace{1cm} (10)

where \(\alpha\) is Womersley parameter, \(Re\) is Reynolds number, \(\lambda\) is permeability parameter and \(Kn\) is Knudsen number. By using above non dimensional quantities, then the equations (6) in (7), (8) and neguleciting the stars, we get,

\[\frac{\alpha^2 Re}{2\pi} \frac{\partial u}{\partial t} + (1 + \epsilon e^{i \omega t}) \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - \left( \frac{1}{\lambda + \frac{M}{1 + m^2}} \right) u \]  \hspace{1cm} (11)

\[\frac{\partial u(y,t)}{\partial y} = 0 \hspace{1cm} \text{at} \hspace{1cm} y = 0 \]  \hspace{1cm} (12)

\[u(y,t) = Kn \frac{\partial u(y,t)}{\partial y} \hspace{1cm} \text{at} \hspace{1cm} y = R(x) \]  \hspace{1cm} (13)
The dimensionless form is given by,

\[
R(x) = \begin{cases} 
1 & 0 \leq x \leq d_1 \\
1 - \frac{\delta_1}{2} \left(1 + \cos \frac{2\pi}{l_1} (x - d_1 - \frac{l_1}{2})\right) & d_1 \leq x \leq d_1 + l_1 \\
1 - \frac{\delta_2}{2} \left(1 + \cos \frac{2\pi}{l_2} (x - d_2 - \frac{l_2}{2})\right) & d_1 + l_1 \leq x \leq d_2 \\
1 - \frac{\delta_3}{2} \left(1 + \cos \frac{2\pi}{l_3} (x - d_3 - \frac{l_3}{2})\right) & d_2 \leq x \leq d_2 + l_2 \\
1 & d_2 + l_2 \leq x \leq d_3 \\
1 - \frac{\delta_3}{2} \left(1 + \cos \frac{2\pi}{l_3} (x - d_3 - \frac{l_3}{2})\right) & d_3 \leq x \leq d_3 + l_3 \\
1 & d_3 + l_3 \leq x \leq l
\end{cases}
\]

3 Method of solution

The differential equation of the form in perturbation techniques we get,

\[
u(y, t) = u_0 + \varepsilon \ u_1(y) e^{2\pi i t} \tag{14}
\]

where, small amplitude of oscillation is \( \varepsilon \), \( u_0 \) is state steady and \( u_1 \) is the state transient.

from equation (14) substitute (11) using the boundary conditions (12) and (13) we get,

(i) Steady state(Zero order):

\[
\frac{1}{Re} \frac{d^2 u_0}{dy^2} - \frac{du_0}{dy} - \left(\frac{1}{\lambda} + \frac{M}{1+m^2}\right) u_0 = -P_s \tag{15}
\]

with boundary conditions,

\[
\frac{du_0}{dy} = 0 \quad \text{at} \quad y = 0 \tag{16}
\]

\[
u_0 = Kn \frac{d u_0}{dy} \quad \text{at} \quad y = R(x) \tag{17}
\]

Solving (15) we get the general solution of the form

\[
u_0 = a_1 e^{n_1 y} + a_2 e^{n_2 y} + \frac{P_s}{Re \left(\frac{1}{\lambda} + \frac{M}{1+m^2}\right)} \tag{18}
\]

(ii) Transient state (first order):

\[
\frac{1}{Re} \frac{d^2 u_1}{dy^2} - \left(1 + \varepsilon e^{2\pi i t}\right) \frac{du_1}{dy} - \left(i a^2 Re + \frac{1}{\lambda} + \frac{M}{1+m^2}\right) u_1 = \frac{d u_0}{dy} - P_0 \cos(2\pi t) e^{-2\pi i t} \tag{19}
\]

Subject to the boundary condition

\[
\frac{du_1}{dy} = 0 \quad \text{at} \quad y = 0 \tag{20}
\]
\[ u_1 = Kn \frac{du_1}{dy} \text{ at } y = R(x) \quad (21) \]

Solving (19) we get the general solution of the form

\[ u_1 = a_3 e^{3y} + a_4 e^{4y} + a_5 e^{y} + a_6 e^{2y} + a_7 \quad (22) \]

substituting steady state and transient state from equation (18) and (22) into (14), we obtain the velocity \( u(y,t) \) expression as,

\[ u(y,t) = a_1 e^{n_1 y} + a_2 e^{n_2 y} + \frac{P_s}{Re \left( \frac{1}{x} + \frac{M}{1 + m^2} \right)} + a_3 e^{n_3 y} + a_4 e^{n_4 y} + a_5 e^{n_5 y} + a_7 \quad (23) \]

**Flow rate**

The volumetric of the rate of fluid flow \( Q \) we get,

\[ Q = \int_0^{R(x)} u(y,t) dy \quad (24) \]

Applying above equation (23) and (24) we get,

\[ Q = a_1 e^{n_1 R(x) - 1} + a_2 e^{n_2 R(x) - 1} + \frac{P_s R(x)}{Re \left( \frac{1}{x} + \frac{M}{1 + m^2} \right)} + e^{2\pi i t \alpha_1} e^{n_3 R(x) - 1} = a_4 e^{n_4 R(x) - 1} + a_5 e^{n_5 R(x) - 1} + a_7 R(x) \quad (25) \]

**Shear stress at the wall**

\[ \tau_w = \left[ \frac{\partial u}{\partial y} \right] \quad (26) \]

Substituting from equation (23) in equation (26) and calculating the shear stress of the wall can be written as,

\[ \tau_w = a_1 n_1 e^{n_1 R(x)} + a_2 n_2 e^{n_2 R(x)} + e^{2\pi i t \alpha_1} e^{n_3 R(x) + \alpha_5 n_1 e^{n_1 R(x)} + \alpha_6 e^{n_2 R(x)}} \quad (27) \]

4 Results and Discussion

The physical insight of the problem, velocity, rate of flow, shear stress and pressure gradient have been discussed in stenosed region by assigning numerical values to various parameter obtained in mathematical formulation of the problem and the results are shown graphically.

Assumed the multi-stenosis values \( l_1 = l_2 = l_3 = 0.2; \ P_0 = 7; \ \alpha = 0.5; \ \varphi = 0.01; \ t = 1.0 ; \ Re = 0.1 \), distance between the multi stenosis is equal to hight of the three stenosis when the blockage of the vessels are 10 , 30 , 20 (\( \delta_1 = 0.1, \delta_2 = 0.3, \delta_3 = 0.2 \)).

The velocity versus non-dimensional transverse y-axis at the multi-stenosis with various parameters are represent figure (2)-(12).

Figure 2, described parabolic of the velocity blood has maximal. It is clearly velocity of blood decreased while increasing Kn . (ie) Axial direction of blood decreases moving slowly.

Figure 3, shows that the velocity profile is decreased while increasing the \( M \). It is noted velocity field in blood is increases can be release by acceptable magnetic induction.

Figure 4, displays a effect of periodic increases, as (\( \lambda \)) also increases. Figure 5.It is shows that periodic velocity field decreases while increases the Reynolds number. It is defined by the strong injection it is noted the velocity account to rising with decreasing curvature as Reynolds number increases.

The different flow of volumetric blood rate along with (x-direction) of the channel for various quantities are result in Figure 6 to 8. It is described figure 6, the rate of flow is maximal concerning the end of multi- stenosis while have the low range at the throat.

Figure 6 and 8 that the rate of flow decreased while increasing the height of the multi stenosis (\( \delta_1, \delta_2, \delta_3 \)) the magnetic parameter and Reynolds number. Rate of fluid flow increased while increasing the effect of Hall current \( m \) shown in Figure 7.
The shear stress of the wall is consequence part in cardiovascular diseases. The study of dimensionless in physical quantities various of the shear stress.

Figure 9, represents the different non dimensional of the magnetic parameter

It is decreases with increasing of the wall shear stress. Figure 10, depicts the various values of the Reynolds number $Re$ is decreases with increasing the value of Reynolds number. Figure 11, indicates the shear strain increases while increased Darcian linear drag parameter. Figure 12, gives wall shear stress is variation for different values Hall current decreases with increase $m$.

Figure 2: Plots of velocity while distance axis for various values "Kn"

Figure 3: Plots of velocity while distance axis for various values "M"

Figure 4: Plots of velocity while distance axis for various values "$\lambda$"

Figure 5: Plots of velocity while distance axis for various values "Re"
Figure 6: Plots of flow rate while distance axis for various values "M"

Figure 7: Plots of flow rate while distance axis for various values "m"

Figure 8: Plots of flow rate while distance axis for various values "m"

Figure 9: Plots of shear stress while distance axis for various values "M"

Figure 10: Plots of shear stress while distance axis for various values "Re"
5 Conclusion

The mathematical model deals with the effect of Hall current pulsatile blood fluid through on porous of the human body with time included sucked and injected walls of the multi-stenosis artery. In our present paper is slip velocity blood fluid through an cardiovascular diseases control those regions, velocity is low and shear stress of the wall is low. The results are useful to analyzing the bearing pressure gradient interior artery during heat exhaustion and surgical process. In these view of the arguments present study is useful to blood pressure control of the blood flow in diseased state of affairs.

References

Appendix:

\[
\begin{align*}
\frac{Re + \sqrt{Re^2 + 4Re\left(\frac{1}{\lambda} + \frac{M}{1 + m^2}\right)}}{2},
\end{align*}
\]

\[
\begin{align*}
\frac{Re - \sqrt{Re^2 + 4Re\left(\frac{1}{\lambda} + \frac{M}{1 + m^2}\right)}}{2},
\end{align*}
\]

\[
\begin{align*}
a_1 &= \frac{n_2}{(n_1 n_2 Kn (-e^{n_1 R(x)} - e^{n_2 R(x)}) + n_1 e^{n_2 R(x)} - n_2 e^{n_1 R(x)})};
\end{align*}
\]

\[
\begin{align*}
a_2 &= \frac{n_1}{(n_1 n_2 Kn (-e^{n_1 R(x)} + e^{n_2 R(x)}) - n_1 e^{n_2 R(x)} + n_2 e^{n_1 R(x)})};
\end{align*}
\]

\[
\begin{align*}
n_3 &= \frac{Re(1 + e^{2\pi i t}) + \sqrt{Re(1 + e^{2\pi i t}) + 4Re\left(i \alpha^2 + \frac{1}{\lambda} + \frac{M}{1 + m^2}\right)}}{2};
\end{align*}
\]

\[
\begin{align*}
n_4 &= \frac{Re(1 + e^{2\pi i t}) - \sqrt{Re(1 + e^{2\pi i t}) + 4Re\left(i \alpha^2 + \frac{1}{\lambda} + \frac{M}{1 + m^2}\right)}}{2};
\end{align*}
\]

\[
\begin{align*}
a_3 &= \frac{a_7 n_4 + a_{31} + a_{32}}{n_4 e^{n_4 R(x)} - n_4 e^{n_3 R(x)} + n_3 n_4 Kn n_1 n_4 (e^{n_4 R(x)} - e^{n_3 R(x)})};
\end{align*}
\]

\[
\begin{align*}
a_{31} &= a_5 \left(n_4 e^{n_4 R(x)} - n_1 e^{n_1 R(x)} + Kn n_1 n_4 (e^{n_4 R(x)} - e^{n_1 R(x)})\right);
\end{align*}
\]

\[
\begin{align*}
a_{32} &= a_6 \left(n_4 e^{n_4 R(x)} - n_2 e^{n_2 R(x)} + Kn n_2 n_4 (e^{n_4 R(x)} - e^{n_2 R(x)})\right);
\end{align*}
\]

\[
\begin{align*}
a_4 &= \frac{a_7 n_3 + a_{41} + a_{42}}{n_4 e^{n_3 R(x)} - n_3 e^{n_4 R(x)} + n_3 n_4 Kn n_3 n_4 (e^{n_4 R(x)} - e^{n_3 R(x)})};
\end{align*}
\]

\[
\begin{align*}
a_{41} &= a_5 \left(n_3 e^{n_1 R(x)} - n_1 e^{n_3 R(x)} + Kn n_1 n_3 (e^{n_3 R(x)} - e^{n_1 R(x)})\right);
\end{align*}
\]

\[
\begin{align*}
a_{42} &= a_6 \left(n_3 e^{n_2 R(x)} - n_2 e^{n_3 R(x)} + Kn n_2 n_3 (e^{n_3 R(x)} - e^{n_2 R(x)})\right);
\end{align*}
\]

\[
\begin{align*}
a_5 &= \frac{Re a_1 n_4 e^{n_4 R(x)}}{n_1 Re n_4 Re\left(i \alpha^2 + \frac{1}{\lambda} + \frac{M}{1 + m^2}\right)};
\end{align*}
\]
\[ a_6 = \frac{\text{Re} a_2 n_2 e^{n_2 y}}{n_2^2 \text{Re} n_2 - \text{Re}(i \alpha^2 + \frac{\pi}{\lambda} + \epsilon \gamma)}; \]

\[ a_7 = \frac{P_0 \cos(2\pi \alpha t)e^{-2\pi \alpha t}}{i \alpha^2 + \frac{1}{\lambda} + \frac{M}{1 + \mu^2}}; \]