Evaluation the Reliability of ARPA Network By Using Delta-Star and Reduction Methods

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Abstract. This paper provides expressions of star delta transformation to simplify complex reliability block diagrams for Advanced Research Projects Agency Network (ARPANET). We also use reduction technique to evaluate the reliability of the system. In this approach, parallel and series subsystems present in a complex system architecture must always be identified and parallel and series cuts must be applied

Keywords: ARPA network, Reduction methods, Delta-Star

1. Introduction

In engineering systems planning, design and operation, reliability assessment is an essential and integral aspect. The term “reliability” is commonly used to refer to the reliability of a system in order to continue performing its intended role [1, 2]. Several techniques for evaluating reliability have been developed but their effectiveness and benefits may vary widely. Some of these methods are pivotal decomposition ,generalized decomposition method , generation of minimal paths and minimal cuts , path tracing method , minimal cut method and others. This paper presents two methods to evaluate the reliability of Advanced Research Projects Agency Network (ARPANET) which is known as one of the first broadband networks to operate packet switching, in addition to the distributed control feature The Advanced Research Projects Agency Network (ARPANET) was and one of the first networks to implement the TCP/IP protocol suite[13]. Both technologies became the technical foundation of the Internet, The first method is to convert a star delta into a similar reliability device. In simplifying complex networks it is very useful. Some examples are: (1) Only by series-parallel technique can Wheatstone Bridge be resolved. It can easily be solved by transforming some delta into stars or vice versa. (2) Kelvin Double-ratio Arm Bridge, after transforming delta into star, decreases to traditional Wheatstone Bridge. (3) Anderson Bridge becomes the classic Maxwell Bridge after transforming the resistance star into an identical delta [2, 4, 5]. The correct relationship between the reliability of the three components of the delta configuration and the identical star configuration was defined all by Gupta & Sharma. The second method is the Reduction method where reduction in series and parallel reduction are essential for simplifying complex networks. It is possible to substitute links in series or in parallel with one link with equivalent reliability [1, 5].

2. A Delta-Star transformation approach for reliability evaluation

This is the best and most practical approach to the reliability of bridge network testing. Delta Star technology transforms the bridge network into parabolic and parallel chain form. The grid reduction method can be extended to achieve network stability [4,6]. However, the Star Delta strategy can effectively deal with networking with more than one bridge configuration. Furthermore, bridge networks consisting of devices with two mutually exclusive failure modes can be introduced. Use this approach to calculate the reliability of the bridge network shown in Figure.1 to display delta-star conversion to an equivalent reliability diagram, it is assumed that the delta configuration is made up of three units of a system with components $R_1$, $R_2$ and $R_3$, and its star equivalent configuration units (components) are $R_{12}$, $R_{13}$, and $R_{23}$. We're assuming it,

$$R_{12} = \frac{\alpha \beta}{\gamma}$$  \hspace{1cm} (1)  \\
$$R_{13} = \frac{\gamma \alpha}{\beta}$$  \hspace{1cm} (2)  \\
$$R_{23} = \frac{\gamma}{\beta}$$  \hspace{1cm} (3)
Where,

\[ \alpha = R_1 + (1 - R_1)R_2R_3 = R_1 + R_2R_3 - R_1R_2R_3, \quad \beta = R_3 + (1 - R_3)R_1R_2 = R_3 + R_1R_2 - R_1R_2R_3, \]

\[ \gamma = R_2R_3 + R_2R_3 + R_2R_3 - 2R_1R_2R_3 \]

Simplify equation (1), (2) and (3), we obtain

\[ R_{12} = \frac{(R_1 + R_2R_3 - R_2R_3)(R_3 + R_2R_3)}{R_1R_2 + R_2R_3 - 2R_1R_2R_3} \]

\[ R_{13} = \frac{R_1R_2 + R_2R_3 + R_2R_3 - 2R_1R_2R_3}{R_1 + R_2R_3} \]

\[ R_{23} = \frac{R_1R_2 + R_2R_3 + R_2R_3 - 2R_1R_2R_3}{R_3 + R_1R_2R_3} \]

There is also Three units of the system comprising \( R_7, R_8 \) and \( R_9 \) components are delta configuration and equivalent stellar units (components) are \( R_{79}, R_{78} \) and \( R_{78} \). We're presuming that [6, 7],

\[ R_{89} = \frac{\alpha^* \beta^*}{\gamma^*} \quad (4) \]

\[ R_{79} = \frac{\gamma^*}{\beta^*} \quad (5) \]

\[ R_{78} = \frac{\gamma^*}{\alpha^*} \quad (6) \]

where,

\[ \alpha^* = R_8 + (1 - R_8)R_7R_9 = R_8 + R_7R_9 - R_7R_8R_9, \quad \beta^* = R_7 + (1 - R_7)R_8R_9 = R_7 + R_8R_9 - R_7R_8R_9, \]

\[ \gamma^* = R_2R_3 + R_2R_3 + R_2R_3 - 2R_1R_2R_3 \]

Simplify equation (4), (5) and (6), we obtain

\[ R_{89} = \frac{(R_8 + R_7R_9 - R_7R_8R_9)(R_7 + R_8R_9 - R_7R_8R_9)}{R_7R_9 + R_7R_9 + R_7R_9 - 2R_7R_8R_9} \]

\[ R_{79} = \frac{R_7R_8 + R_7R_9 + R_7R_9 - 2R_7R_8R_9}{R_7 + R_8R_9 - R_7R_8R_9} \]

\[ R_{78} = \frac{R_7R_8 + R_7R_9 + R_7R_9 - 2R_7R_8R_9}{R_8 + R_7R_9 - R_8R_9} \]
Now, we will rewrite the symbols as follows:

\[ R_a = R_{12}, \quad R_b = R_{13}, \quad R_c = R_{23}, \quad R_d = R_{78}, \quad R_e = R_{79}, \quad R_f = R_{89} \]

**Figure 2.** Bridge system after delta-star transformation.

In order to complete the idea of simplifying the complex network, we must use another method of reduction, as explained in the next section of this paper.

### 3. Reduction Method (RM)

To estimate the reliability of series, parallel, parallel-series, series-parallel and k-out-of-n systems, the RM is simple and efficient. This approach is to successively transform a system into a simple system, with each system described by an analogous reliability block diagram. Two instances will be analyzed in this approach [6,8,9]:

**case 1:** In series reduction, the n-component series subsystem is replaced by a supercomponent whose reliability is equal to that of the reliability of the subsystem components.

**case 2:** In parallel reduction, a parallel subsystem with n components is replaced with a super component whose unreliabilities of the components in the subsystem [9,10].

We apply the method to Figure 2, considering a device whose block diagram of reliability is given in Figure 3a. There are nine components to the system. The reliability and unreliability of component \( i \) are given and denoted by \( P_i = R_i \) and \( q_i = 1 - R_i \), respectively (\( i = 1, 2, \ldots, 9 \)). Use series and parallel reductions for system reliability evaluation. From the system structure given in Figure 3a [11,12].

**Step 1:** we see that components 1 and 4 form a series subsystem, which can be represented by a “supercomponent” denoted by \( 1–4 \). The reliability of supercomponent \( 1–4 \) is equal to the product of the reliabilities of components 1 and 4:

\[ R_{1–4} = R_1 R_4 \]  \hspace{1cm} (7)
Step 2: Components 5, 6, 9 also form a series subsystem, which can be represented by a supercomponent denoted by 5–6–9. The reliability of supercomponent 5–6–9 is equal to the product of the reliabilities of components 5, 6, 9:

\[ R_{5-6-9} = R_5 R_6 R_9 \]  \hspace{1cm} (8)

Step 3: Components 6, 9 also form a series subsystem, which can be represented by a supercomponent denoted by 6–9. The reliability of supercomponent 6–9 is equal to the product of the reliabilities of components 6, 9 [5, 8, 13]:

\[ R_{6-9} = R_6 R_9 \]  \hspace{1cm} (9)

Step 4: Components 5, 8 also form a series subsystem, which can be represented by a supercomponent denoted by 5–8. The reliability of supercomponent 5–8 is equal to the product of the reliabilities of components 5, 8:

\[ R_{5-8} = R_5 R_8 \]  \hspace{1cm} (10)

Step 5: After these series reductions, the reliability block diagram in Figure 3a is transformed into that in Figure 3b. Examination of Figure 3b reveals that component 8 and component 5–6–9 form a parallel subsystem denoted by supercomponent 5–6–9, whose unreliability is the product of the unreliabilities of component 8 and supercomponent 5–6–9:

\[ R_8 R_{5-6-9} = 1 - \left( \left[1 - R_8 \right] \left[1 - R_5 R_6 R_9 \right] \right) \]

\[ = R_8 + R_3 R_6 R_9 - R_3 R_6 R_8 R_9 \]  \hspace{1cm} (11)

Step 6: Components 6–9 and component 5–8 form a parallel subsystem denoted by supercomponent 5–8, whose unreliability is the product of the unreliabilities of component 6–9 and supercomponent 5–8:

\[ R_{6-9} R_{5-8} = 1 - \left( \left[1 - R_6 R_9 \right] \left[1 - R_5 R_8 \right] \right) \]

\[ = R_5 R_8 + R_6 R_9 - R_5 R_6 R_8 R_9 \]  \hspace{1cm} (12)

Figure 3b.

Step 7: After this parallel reduction, the reliability block diagram in Figure 3b is transformed into that in Figure 3c. Figure 3c shows that component 1–4 and supercomponent 2 form parallel subsystem denoted by 1–2–4, whose reliability is

\[ R_2 R_{1-4} = 1 - \left( \left[1 - R_2 \right] \left[1 - R_1 R_4 \right] \right) \]
\[ R = R_2 + R_4 - R_2R_4 \]  \hspace{1cm} (13)

**Figure 3d.**

**Step 8:** Component 1-2-4 and supercomponent 5-6-8-9 form a series subsystem and a series reduction produces a supercomponent denoted by 1-2-4-5-6-8-9, we will rewrite the symbol as follow:

\[ R_{1,2,4,5,6,8,9} = R_b \]

whose reliability is

\[ R_{1,2,4,5,6,8,9} = R_1R_2R_4R_5R_6R_8R_9 \]  \hspace{1cm} (14)

**Figure 3e.**

**Step 9:** The system reliability is equal to the reliability of the equivalent series system as shown in Figure 3e and the reliability is:

\[ R_s = R_a \times R_b \times R_f \]  \hspace{1cm} (15)

4. Conclusion

In this paper two techniques have been shown as Delta star and reduction in order to simplify the ARPA network to the series system and this technique allow to us to corporate with given network as easiest way to studying several thinks which related the work properly or optimize the system such as reliability allocation, reliability importance, redundancy, ..., etc.

5. References


