ISSN: 0974-5823 Vol. 7 No. 2 February, 2022

International Journal of Mechanical Engineering

A New VAM Modification for Finding an IBFS for Transportation Problems

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Abstract:

One of the most important problems in optimization is the transportation problem (TP). The literature contains several modifications to resolve this problem. Typically, these approaches are created to find the initial basic feasible solution (IBFS) or the optimal solution (OS). In this work, we offer a new (VAM) modification for discovering an (IBFS) for virtually as good as the (OS) for the transportation problem. The proposed adjustment is supported by numerical examples that have been solved. A comparative study with the outcomes of traditional procedures was also carried out. This improved strategy produces a better solution most of the times and is extremely close to the optimal solution (and even gives the optimal solution in some cases). This procedure is so simple and effective.

Keywords: Linear Programming. Transportation Problem. Vogel's Approximation Method.

1. Introduction

The formulation and solving the (TP) has been an important and practical application of linear programming (LP). (TP) is a subcategory of (LP) concerned with our daily effectiveness [1, 2], it deals with issues of economic improvement such as cost reduction. Transportation models are primarily concerned with the most efficient technique of transferring goods from several plants or factories (supply assets) to several clients' storehouses (demand destinations) [3, 4]. The main purpose of this assignment is to determine the best shipping schedule for the commodity while still satisfying demand at each destination. In 1941 the (TP) was introduced by Hitchcock F. L. [5]. Tjalling C. Koopmans proposed his article to solve (TP) in 1947 [6]. The Two research stated are the most significant advancements in developing various strategies for solving the transportation model. (TP) can also be solved by using the simplex technique proposed by Dantzig G. B. in 1951; however (TP) has many variables and constraints, and solving them using the simplex technique takes a lot of time and effort. Many researchers have created updates to develop an IBFS that considers transportation costs. "North-West Corner method" (NWCM), "Least Cost Method" (MCM), and "Vogel's Approximation Method" (VAM) are used to obtain an (IBFS) for (TP). According to the investigations, (VAM) is the best among the three methods. The modified distribution method (MODI) and stepping stone method (SSM) are used the results of the (IBFS) to reach the (OS) to the (TP). Essentially, the differences between these approaches are primarily in terms of the (OS) from the outset versus a decent solution that would result in a lower objective value. (TP) can be classified into "balanced transportation" and "unbalanced transportation." We call it a balanced transportation problem when the number of sources equals to the number of demands. Otherwise, it is called unbalanced transportation problem [7]. Several approaches for determining (IBFS) of the transportation model have been proposed in recent years. Md. Ashraful Babu and others (2014) showed that their implied cost method (ICM) is better or equal to (VAM). In (2015), Abdul Sattar Soomro and others presented a modified (VAM). Mollah Mesbahuddin Ahmed and colleagues offered an innovative method to obtain (IBFS) for (TP) in (2017) [8]. Ravi Kumar and colleagues proposed a new approach to getting (IBFS) for (TP) in (2018) [9]. Lakhveer Kaur and colleagues also described an enhancement in the maximum difference method to identify (IBFS) for (TP) in 2018 [10]. S. C. Zelibe and C. P. Ugwuanyi proposed a new transportation strategy in (2019) [11]. The authors introduced many papers in varied fields of science such as operation research [12- 20], optimization [21- 40], but in this paper, we offer a further modification for (VAM). The result of the objective function is near to (OS) and better or equal to the solution results according to (VAM). In any case, the results of the new proposed method are much better than the results of the "North-West Corner Method" and the "Least Cost Method."

2. The Transportation Problem Mathematical Formulation:

Let m represents the number of sources.

Let n represents the number of destinations.

Let S_i represents the supply, $i = 1,2, \dots, m$.

Let D_j represents the Demand, $j = 1,2, \dots, n$.

Let c_{ij} represents the cost of transportation from the supply i to demand j.

Let x_{ij} represents the number of quantities shipped in every path than provenance i to destination j

The mathematical formula for (TP) is written as follows [41]:

(1)

$$Minimize\ \ Z\ = \sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i\ , i =\ 1,2, \dots\ \dots, m \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ , j =\ 1,2, \dots\ \dots, n \qquad (3)$$

where $\quad x_{ij} \geq 0,\ \forall\ i$ and $j \qquad (4)$

The network diagram in Fig.1 [13, 14] and the formulation schedule in Table1 [15] can both be used to describe (TP). The goal of the network diagram and formulation schedule is to find the value of x_{ij} which will reduce the overall cost of the (TP) to the lowest possible value as stated in Eq (1).

Fig 1. Network diagram of (TP)

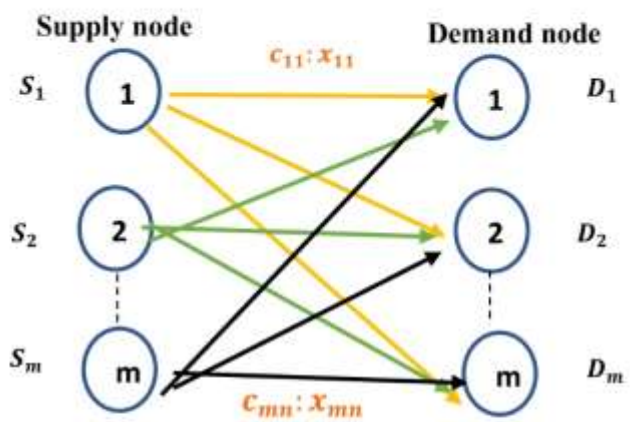

Table 1. Formulation Schedule of (TP)

Sources (i)	**Destinations (j)**				**Supply (a_i)**
	D_1	D_2	$\cdots$	D_n	
S_1	x_{11} c_{11}	x_{12} c_{12}	$\cdots$	x_{1n} c_{1n}	a_1
S_2	x_{21} c_{21}	x_{22} c_{22}	$\cdots$	x_{2n} c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	x_{ij} c_{ij}	$\vdots$	$\vdots$
S_m	x_{m1} c_{m1}	x_{m2} c_{m2}	$\cdots$	x_{mn} c_{mn}	a_m
Demand(b_j)	b_1	b_2	$\cdots$	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

3. The new Algorithm to Find IBFS

Step1: Build the transportation schedule (if it does not give) depending on the given (TP). Check whether aggregate supply equals to the aggregate demand; if it is not, then we have to balance the (TP).

Step 2: Find the "smallest cost element" in each table column and deduct that cost element from each element in that column. As a result, each column of the new table will have at least "one zero". This new table is referred to as (reduced column matrix).

This technique should be repeated for all columns in (TP).

Step 3: Determine the locations of zero cells in the original (TP) table corresponding to the cells in the reduced column matrix table.

Step 4: Allocate the uttermost possible units to the max zero cells in the chosen $x_{ij} =$ minimum (S_i, D_j).

If the maximum allocated amount meets the zeros cell, then the remainder of the assignment is added to a nonzero cell in the same row. If the specified row has no zero cells, then the allocation will start with the nonzero cell at the lowest cost (that is, for minimum (c_{ij})).

Step 5: Repeat step 4 for all rows.

Step 6: Use a balanced transportation cost matrix to compute the total transportation cost for the feasible allocations, i.e., total cost $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

4. Numerical Examples

The numerical Experiments were used to assess the efficiency of the new approach (ZT). We compare it with the three classical methods (NWCM), (MCM), and (VAM), also mention the results of (OS) to following examples of (TP).

Example 1 [42]: Consider the data of the following table:

Table 2. Data of TP

Source	Destination				Supply
	A	B	C	D	
S_1	20	22	17	4	120
S_2	24	37	9	7	70
S_3	32	37	20	15	50
Demand	60	40	30	110	240

The given transportation table is balanced since "total supply," and "demand" are equal. According to the new algorithm, we get the following tables:

Table 3. Reduced column matrix

Source	Destination			
	A	B	C	D
S_1	0	0	8	0
S_2	4	15	0	3
S_3	8	15	11	11

Table 4. Represents the solution by ZT

Source	Destination A	B	C	D	Supply
S_1	60 (20)	40 (22)	✖ (17)	20 (4)	120
S_2	✖ (24)	✖ (37)	30 (9)	40 (7)	70
S_3	✖ (32)	✖ (37)	✖ (20)	50 (15)	50
Demand	60	40	30	110	240

The total cost of the (IBFS) of (TP) is:

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}\, x_{ij}$$

$$\therefore Z = (20 \times 60) + (22 \times 40) + (4 \times 20) + (9 \times 30) + (7 \times 40) + (15 \times 50) = 3460 \text{ units}$$

Example 2 [1]: Depending to the data of the following table:

Table 5. Data of TP

Source	Destination I	II	III	Supply
S_1	24	28	21	65
S_2	27	25	26	110
S_3	23	22	29	75
Demand	100	70	80	250

The given transportation table is balanced, so we get:

Table 6. Reduced column matrix

Source	Destination I	II	III
S_1	1	6	0
S_2	4	3	5
S_3	0	0	8

Table 7. Represents the solution by ZT

Source	Destination						Supply
	I		II		III		
S_1	✖	24	✖	28	65	21	65
S_2	25	27	70	25	15	26	110
S_3	75	23		22	✖	29	75
Demand	100		70		80		250

The total cost of the (IBFS) of (TP) is:

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}\, x_{ij}$$

$$\therefore\ Z = (21 \times 65) + (27 \times 25) + (25 \times 70) + (26 \times 15) + (23 \times 75) = 5905 \text{ units}$$

5. Result Analysis

We placed the total cost of the (TP) indicated above in the following table to compare the performance of the new suggested approach (ZT) with the three classical methods (NWCM), (MCM), and (VAM), the comparison is based on the lowest cost. The optimal algorithm is one that provides the lowest possible cost answer.

Also, mention the results of (OS) to the above se mentioned examples.

Table 8. A comparison between the new approach and the classical methods

	NWCM	MCM	VAM	ZT	OS
Ex ❶	3680	3790	3520	3460	3510
Ex ❷	6560	5975	5905	5905	5905

Table 8 shows that the new approach (ZT) provides the best (IBFS) compared to the three classical methods because it has the lowest cost and closed to (OS).

Our proposed method has been applied to many examples. The results of the solutions have always been better than the three classic methods for solving transportation problems. A few cases have even equaled to the results of Vogel's approximation method, but it has never given the worst results compared to any of the three methods. We have mentioned two examples in this work and a table comparing the outcomes.

6. Conclusion

For the balanced table of (TP), having an initial basic feasible solution (IBFS) is critical. (VAM) is widely acknowledged as the best. For solving (TP), we modified an effective heuristic approach over (VAM). Compared to the solutions provided by standard algorithms (NWCM), (MCM), and (VAM). The (IBFS) that we obtained using the new method (ZT) is the best or almost the best solution, and in certain situations, it is even so closed to the (OS). The proposed approach has a high chance of resolving the transportation problem, it works well in both large and small sizes. This strategy is quite beneficial for decision-makers who deal with supply chain and logistics issues. The proposed approach is simple to comprehend, requires a few calculations, and saves time. As a result, this it may be recommended over existing methods.

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