ISSN: 0974-5823 Vol. 6 No. 3 December, 2021

International Journal of Mechanical Engineering

HOMOTOPY PERTURBATION METHOD OVER A NON-LINEAR STRETCHING SHEET ON THE SOLUTION OF MHD FLOW PROBLEM

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Abstract

The dilemma of magneto–hydrodynamic (MHD) flow past a non-linear stretching sheet in corporation with transverse magnetic field is analyzed. The governing equations are transformed into non-linear ordinary differential equations and solved by adopting homotopy perturbation method. In the present analysis, homotopy perturbation method shows an excellent agreement with the method implemented by the existing authors.

Keywords: Stretching sheet, MHD, Homotopy perturbation method, Magnetic field.

1. INTRODUCTION

Now-a-days, investigation into magnetohydrodynamic (MHD) flow is of paramount importance in solving problems concerned with its applications in wide ranging domains of applied physical and technological sciences. There arise various problems of MHD induced flows under different situations. Some situations may involve flow over shrinking sheet or stretching sheet. These lead to emerge both shrinking and stretching flow problems which need necessary approaches for solutions. Besides examining, the behaviours of flow over shrinking surface, a good number of researchers have investigated the behaviours of flow caused by stretching of a sheet. Stretching sheet has its considerable influences on boundary layer flow and heat transfer and this has important applications in chemical technology, especially polymer technology. The nature of heat transfer over a stretching surface determines the quality of products of chemical technology.

In recent years, there has been an increasing emphasis on the study of non-linear fluid behaviours. The most common examples of non-linear fluids are various kinds of salt solutions, molten polymers, lubricants, bio fluids etc. Importance of MHD flow study, especially over non-linear stretching sheet are growing due to its relevance and application capabilities in solving the allied problems. Besides other application areas, flows of this kind find proper applications mostly in industry-based manufacturing processes involving production of glass fiber, plastic items, fabrication of metals etc. some notable researchers have examined the various flow models over stretching sheet. Taking MHD flows over a non-linear stretching sheet, some works are pursued by Hayat et al.[4], Eerdunbuhe [3], Cortell[1], Kai et al.[11], Ullah et al.[14] etc.

While solving certain MHD flow problems concerned with non-linear stretching sheet, use of Homotopy Perturbation Method (HPM) adds new dimension to the study of hydrodynamic flow. The HPM helps solve ordinary or partial differential equations of both linear and non-linear nature. Most importantly the HPM provides ways to solve non-linear differential equations without operating linearization process thereby reducing huge computation works. Dr. Ji Huan He was the first to advocate and apply HPM method in 1998. Applications of HPM prominently appear in a good number of scholarly works, of which the worth–mentioning ones include J.H.He [5,6,7], Jhankal[10], Cuce and Cuce[2], Kharrat and Toma[12], Jameel et al.[9], He and El-Dib[8] etc. Present study is carried out in the line of the work done by Motsa and Sibanda [13]. They solved the problem using spectral-homotopy analysis method (SHAM). But the present authors have applied the homotopy perturbation method (HPM) here to acquire solution of MHD flow over a non-linear stretching sheet.

2. MATHEMATICAL FORMULATION

Considered a problem involving two dimensional steady boundary layer flow from an impervious horizontal non-linear stretching sheet passing through incompressible, viscous electrically conducting fluid. The magnetic field generated by motion of electrically conducting fluid is considered nominal as compared with applied magnetic field. Magnetic Reynolds number is taken to be small and no electric field is applied so that Hall effects become negligible. A constant pressure is also assumed and magnetic pressure term is ignored.

Based on above assumptions, the governing continuity and momentum equations (Motsa and Sibanda [13]) are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \rightarrow (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u \qquad \rightarrow (2)$$

Where υ implies kinematic viscosity, ρ and σ imply density and electrical conductivity of fluid respectively. B(x) denotes magnetic field which is set normal to the flow direction.

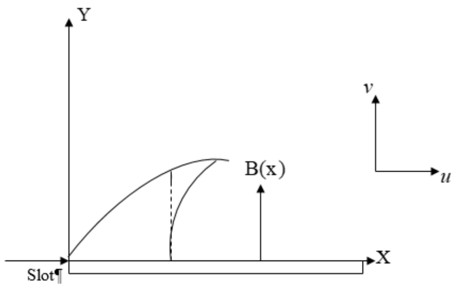

Figure1: Physical Structure of the Problem

As presented in Figure1 the non-linear stretching sheet is parallel with X-axis. Let u and v are velocity components along X-axis and Y- axis respectively.

Let $B(x)=B_o x^{(n-1)/2}$, B_o implies strength of applied magnetic force.

It is assumed that the sheet moves with a power-law velocity for which the boundary conditions become

$$u(x,0)=cx^n$$

$$u(x,y)\rightarrow 0 \;\; as \;\; y\rightarrow\infty$$

Where, n>0 and n<0 are for accelerating and decelerating sheet respectively.

With the help of transformations,

$$u=cx^n f'(\eta) \text{ where } \eta=\sqrt{\frac{(n+1)c}{2v}}\,x^{(n-1)/2}y \qquad \rightarrow (5)$$

$$v=-\sqrt{\frac{cv(n+1)}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right]x^{(n-1)/2} \qquad \rightarrow (6)$$

Here, $f(\eta)$ indicates non-dimensional stream function.

Now the equations (1) and (2) can be converted to following non-linear differential equation:

$$f'''(\eta)+f(\eta)f''(\eta)-\beta(f'(\eta))^2-Mf'(\eta)=0 \qquad \rightarrow (7)$$

Taking boundary conditions as:

$$f(0)=0, \qquad f'(0)=1, \qquad f'(\infty)=0 \qquad \rightarrow (8)$$

Where, β and M indicates stretching parameter and magnetic parameter respectively and their values are:

$$\beta=2n/_{1+n} \quad \text{and} \quad M=2\sigma B_0^2/_{\rho c(1+n)}$$

3. SOLUTION WITH HOMOTOPY PERTURBATION METHOD(HPM)

Applying HPM, the equation (1) takes the following form:

$$(1-p)(f'''-M.f')+p\left(f'''+f.f''-\beta f'^2-M.f'\right)=0 \qquad \rightarrow (9)$$

Where, p is perturbation parameter.

Considering "*f*" as under:

$$f=f_0+pf_1+p^2f_2+\dots\dots\dots\dots \qquad \rightarrow (10)$$

Using (3) in (2) equating the terms free from 'p', the following ordinary differential equation is obtained:

$$f_o'''-M.f_o'=0 \qquad \rightarrow (11)$$

Solving the equation (11) with boundary conditions for zeroth order,

i.e. $f_0(0)=0,\; f_0'(0)=1,\; f_0'(6)=0$, obtained the following solutions:

$$f_o(\eta)=A_1-A_2(e^{\sqrt{M}\eta}+e^{(12-\eta)\sqrt{M}}) \qquad \rightarrow (12)$$

Where,

$$A_1=\frac{e^{12\sqrt{M}}+1}{\sqrt{M}(e^{12\sqrt{M}}-1)} \quad \text{and} \quad A_2=\frac{1}{\sqrt{M}(e^{12\sqrt{M}}-1)}$$

Now,

$$f_0'(\eta)=-A_2\sqrt{M}(e^{\sqrt{M}\eta}-e^{(12-\eta)\sqrt{M}}) \qquad \rightarrow (13)$$

$$f_0''(\eta)=-A_2M(e^{\sqrt{M}\eta}+e^{(12-\eta)\sqrt{M}}) \qquad \rightarrow (14)$$

Again, equating the terms involving coefficients of 'p' from (2), the following ordinary differential equations are obtained:

$$f_1'''-M.f_1'+f_0.f_0''-\beta(f_0')^2=0 \quad \rightarrow (3)$$

$$i.e.\; f_1'''-M.f_1'=-A_3(e^{\sqrt{M}\eta}+e^{\sqrt{M}(12-\eta)})+A_4(e^{2\sqrt{M}\eta}+e^{2(12-\eta)\sqrt{M}})+A_5 \qquad \rightarrow (15)$$

$\rightarrow (4)$

Where,

$$A_3=-A_1A_2M,$$

$$A_4=(1-\beta)A_2^2M,$$

$$A_5=2(1+\beta)A_2^2Me^{12\sqrt{M}}$$

Solving equation (15) with boundary conditions for 1st order

i.e. $f_1(o)=0,\; f_1'(0)=0,\; f_1'(6)=0;$

Solutions are obtained as under:

$$f_1(\eta)=\frac{C_1e^{\sqrt{M}\eta}}{\sqrt{M}}+\frac{C_2e^{-\sqrt{M}\eta}}{\sqrt{M}}+C_3+\frac{A_3\eta}{2M}\left[e^{(12-\eta)\sqrt{M}}-e^{\sqrt{M}\eta}\right]+\frac{A_4}{6M\sqrt{M}}\left[e^{2\sqrt{M}\eta}-e^{2(12-\eta)\sqrt{M}}\right]-\frac{A_5\eta}{M} \qquad \rightarrow (16)$$

$$f_1'(\eta)=C_1e^{\sqrt{M}\eta}-C_2e^{-\sqrt{M}\eta}+\frac{A_3}{2M}\left[e^{(12-\eta)\sqrt{M}}-e^{\sqrt{M}\eta}\right]+\frac{A_3\eta}{2\sqrt{M}}\left[-e^{(12-\eta)\sqrt{M}}-e^{\sqrt{M}\eta}\right]+\frac{A_4}{3M}\left[e^{2\sqrt{M}\eta}+e^{2(12-\eta)\sqrt{M}}\right]-\frac{A_5}{M} \qquad \rightarrow (17)$$

Neglecting higher order perturbed terms, it is finally obtained that

$$f'(\eta) = -A_2\sqrt{M}\,(e^{\sqrt{M}\eta} - e^{(12-\eta)\sqrt{M}}) + p[C_1 e^{\sqrt{M}\eta} - C_2 e^{-\sqrt{M}\eta} + \frac{A_3}{2M}\left(e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right) + \frac{A_3\eta}{2\sqrt{M}}\left(-e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right) + \frac{A_4}{3M}\left(e^{2\sqrt{M}\eta} + e^{2(12-\eta)\sqrt{M}}\right) - \frac{A_5}{M}] \rightarrow (18)$$

$$f(\eta) = f_0 + pf_1$$

$$= A_1 - A_2(e^{\sqrt{M}\eta} + e^{(12-\eta)\sqrt{M}}) + p[\frac{C_1 e^{\sqrt{M}\eta}}{\sqrt{M}} + \frac{C_2 e^{-\sqrt{M}\eta}}{\sqrt{M}} + C_3 + \frac{A_3\eta}{2M}\left(e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right) + \frac{A_4}{6M\sqrt{M}}\left(e^{2\sqrt{M}\eta} - e^{2(12-\eta)\sqrt{M}}\right) - \frac{A_5\eta}{M}] \rightarrow (19)$$

Where,

$$A_6 = \frac{A_4}{6M}(e^{24\sqrt{M}} - 1),$$

$$A_7 = \frac{A_5}{M} - \frac{A_3}{2M}(e^{12\sqrt{M}} - 1) - \frac{A_4}{3M}(e^{24\sqrt{M}} + 1)$$

$$A_8 = \frac{6A_3}{\sqrt{M}} - \frac{2A_4}{3M}e^{6\sqrt{M}} + \frac{A_5}{M}e^{-6\sqrt{M}},\ C_2 = \frac{A_8 - A_7}{1 - e^{-12\sqrt{M}}},\ C_1 = C_2 + A_7$$

$$C_3 = \frac{1}{\sqrt{M}}\left[\frac{A_4}{6M}(e^{24\sqrt{M}} - 1) - C_1 - C_2\right]$$

4. RESULTS AND DISCUSSION

In this study, the problem of boundary layer for MHD flow past a nonlinear stretching sheet in the presence of a transverse magnetic field is considered by Homotopy Perturbation Method. The obtained results are revealed graphically and are compared with the accurate solutions. The numerical results are obtained for different values of parameters M, β and P implanted in the flow system.

Figure 2 and 3 implements the influence of magnetic parameter and stretching parameter on velocity profile. An obstructing performance of the fluid flow velocity is depicted in figure 2 due to the magnetic field applied and its strength that is the fluid motion is obliged on account of magnetic intensity. It usually happens when a magnetic field is present in fluid conducting electrically which generates a *Lorentz force* that acts opposite to the flow when magnetic fields is applied in normal direction. Furthermore, acceleration of velocity profile on account of stretching parameter is observed in figure 3 which indicates the fact that the velocity distribution is directly proportional to the stretching parameter.

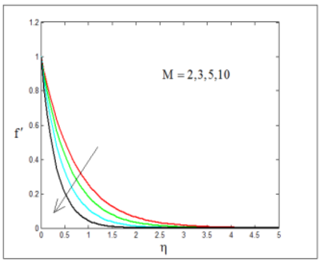

Figure 2: Velocity versus η under β=1.5, P=0.1

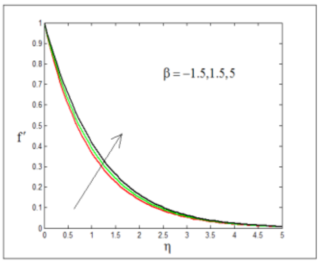

Figure 3: Velocity versus η under M=1, P=0.1

The behaviors of typical velocities against the normal co-ordinate for different values of strength of the applied magnetic field and stretching constraint are depicted in figures 4 and 5. It is empowered from figure 4 that the velocity boundary layer is reduced by virtue of magnetic induction. Moreover, this velocity graph attains its linearity in the infinite direction of the plate. From figure 5, it is revealed that the velocity boundary layer attains its minimum near the plate and greatest value far away from the plate.

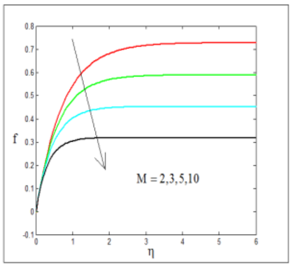

Figure 4: Velocity versus η under β=1.5, P=0.

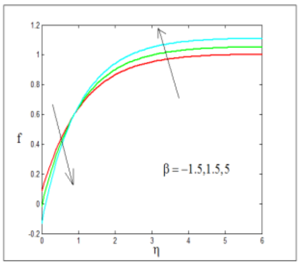

Figure 5: Velocity versus η under M=1, P=0.1

The influence of perturbation parameter on both f' and f are depicted in figure 6 and 7. From these figures, it is observed that the flow pattern of the fluid gets accelerated under the action of perturbation parameter satisfying the boundary restrictions.

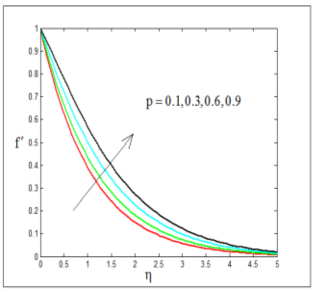

Figure 6: Velocity versus η under β=1.5, M=1

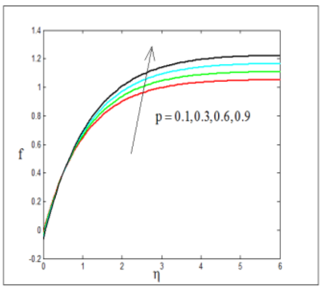

Figure 7: Velocity versus η under β=1.5, M=1

5. COMPARISION OF RESULTS

For comparing outcomes of present research , the results of Motsa S. S. and Sibanda P. [13] are used. Comparing figures 7 with figure 3 (Motsa S. S. and Sibanda P. [13]), it is observed same kind of behaviour due to the implementation of Hartmann Number , i.e. there is a significant effect of magnetic field on velocity profile. With the imposition of Homotopy Perturbation Technique, the velocity profile is almost similar making an admirable fact with the findings investigated by Motsa S. S. and Sibanda P. [13] and the present authors.

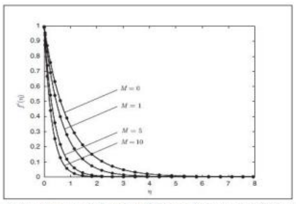

Figure 3. The comparison of the numerical results (solid line) and second-order SHAM solution (filled circles) for $f'(\eta)$ using different values of M when $\beta = 1.5$.

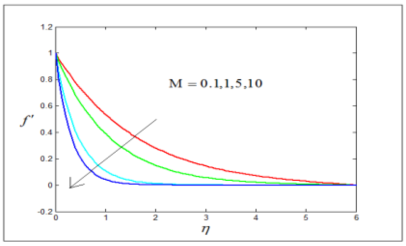

Figure 7: Velocity versus η under β=1.5, P=.1

6. CONCLUSION

Investigation on the current problem leads to the following opinions :

1. An obstructing performance of the fluid flow velocity is observed due to the magnetic field applied and its strength.

2. Acceleration of velocity profile on account of stretching parameter is found which indicates the fact that the velocity distribution is directly proportional to the stretching parameter.

3. The flow pattern of the fluid gets enhanced under the action of perturbation parameter satisfying the boundary restrictions.

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