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To Study $^{122-130}Te$ (Tellurium) nuclei with the help of Casimir Operators and IBM-2

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Abstract- The Interacting Boson Model-2 is used to find out whether Hamiltonian is best for studying Tellurium nuclei. The IBM-2 has been extensively used to describe the medium heavy nuclei quadrupole collective states. When this version of the model is used, the proton and neutron variables are distinguished. Because it is essential to explicitly describe the proton and neutron variables. Using the best-fitted values of parameters in the Hamiltonian of the IBM-2, we have calculated energy levels and electric transition probabilities for $^{122-130}Te$ isotopes. The theoretical results are compared to experimental data, they are found to be in good agreement. The $^{122-130}Te$ isotopes are of the U (5) symmetry. Also, there is no sign of triaxiality in $^{122-130}Te$ nuclei.

Key words: Interacting Boson Model, Even-Even Tellurium, Casimir operators, Energy levels, Electric transition probabilities.

INTRODUCTION

The vibrational model, the quasiparticle model, and the interacting boson approximation model [1-2] have all been used to calculate theoretical calculations on the even Te (Tellurium) nuclei [3], which show general collective nuclei conclusion. Beta and gamma spectroscopic measurements, as well as neutron capture and inelastic scattering experiments, have all been used to study the spectra of these nuclei [4].

In recent years, the even-mass tellurium isotopes have been extensively studied both theoretically and experimentally, with a particular focus on the interpretation of experimental data using collective models [5-14]. The semi-microscopic model, the two-proton core coupling model [15-16], and the dynamic deformation model [17] and the interacting boson model-2 [18] have all been used to investigate energy levels, electric quadrupole moments, and B(E2) values.

The IBM-2 confirms the geometric distinctions while also broadening them to include all nuclei, not only spherical and deformed ones, but also γ- unstable and transitional ones. This model was introduced in which seems to be relevant for the explanation of deformed nuclei showing triaxial features.

The aim of this work is to calculate the energy levels and reduced electric transition probabilities B(E2) in $^{122-130}Te$ isotopes, using the IBM-2 with the help of Casimir operators, and to compare the results with the experimental data.

IBM-2 HAMILTONIAN

The IBM-2 formalism is a general model for describing the Hamiltonian parameters, with the neutrons and protons degrees of freedom explicitly taken into consideration. It has the advantage of being closer to a microscopic theory, but it requires significantly larger matrices to be diagonalized.

The pairs are treated as bosons in which proton boson with J = 0 are denoted by $s_\pi(s_v)$ and J = 2 are denoted by $d_\pi(d_v)$. The number of protons is designed by N_π, and the neutron N_v, in the closed shells of the particle space in which more than half of the shell is filled.

The creation operators for neutron and proton bosons

$b^\dagger_{\pi, jm} = s^\dagger_\pi, d^\dagger_{\pi, m}$ (m = -2, -1, …,2) and $b^\dagger_{v, jm} = s^\dagger_v, d^\dagger_{v, m}$ (m = -2, -1, …,2)

The operators for bosons annihilation like this

$b^\sim_{\pi, jm} = s_\pi, d^\sim_{\pi, m}$ (m = -2, -1, …,2) and $b^\sim_{v, jm} = s^\sim_v, d^\sim_{v, m}$ (m = -2, -1, …,2)

$$N_\pi = \sqrt{5}[d^\dagger_\pi \times d^\sim_\pi]^{(0)} + s^\dagger_\pi s_\pi$$

The Hamiltonian for Interacting boson model-2 is written as [19]

$$H = H_\pi + H_v + V_{\pi v} \quad (1)$$

Where H_π, H_v are the proton and neutron boson Hamiltonian, the last term $V_{\pi v}$ is the neutron-proton interaction.

A basic Hamiltonian [20] which can be written as

$$H = \varepsilon_\pi d^\dagger_\pi d^\sim_\pi + \varepsilon_v d^\dagger_v d^\sim_v + V_{\pi\pi} + V_{vv} + k\, Q_\pi . Q_v + M_{\pi v} \quad (2)$$

Here ε is the d-boson energy and ε_π, ε_v are proton and neutron energy respectively, and are thought to be equal to $\varepsilon_v = \varepsilon_\pi = \varepsilon$, k is the strength of the quadrupole interaction between neutron and proton bosons.

In the IBM-2, the quadrupole moment operator is given by [21]:

$$Q_{\rho\rho} = (s^\dagger d^\sim + d^\dagger s)^{(2)}_\rho + \chi_\rho (d^\dagger d^\sim)^{(2)}_\rho \quad (3)$$

Where ρ = π or v, $Q_{\rho\rho}$ is the quadrupole deformation parameter for protons (ρ = π) and neutrons (ρ = v). V_{vv} is the neutron-neutron d-boson interactions and $V_{\pi\pi}$ is the proton-proton d-boson interactions.

This term is given by:

$$V_{\rho\rho} = \sum_{J = 0,2,4} (1/2)\, C_{L\rho}\, (2J + 1)^{1/2} \{[d^\dagger \times d^\dagger]^{(2)}_\rho \times [d^\sim \times d^\sim]^{(2)}_\rho\}^{(0)} \quad (4)$$

The last term $M_{\pi v}$ is the Majorana interaction, which accounts for symmetry energy which has the form

$M_{\pi\nu} = -\sum_{k = 1,3} 2\xi_k \{[d^\dagger_\pi \times d^\dagger_\pi]^{(k)} \times [\tilde{d}_\pi \times \tilde{d}_\pi]^{(k)}\} + \xi_2 (s^\dagger_\nu d^\dagger_\pi - d^\dagger_\nu s^\dagger_\pi)^{(2)} (\tilde{s}_\nu \tilde{d}_\pi - \tilde{d}_\nu \tilde{s}_\pi)^{(2)}$ (5)

Now, the IBM-2 Hamiltonian in terms of Casimir operators

$$H = \varepsilon [C_{1, U(5)}] + [C_{2, U(5)}]_{\nu\nu} + [C_{2, U(5)}]_{\pi\pi} + k [C_{2, SU(3)}]_{\rho\rho} + [C_{2, SO(6)}] \quad (6)$$

The Casimir invariant operators of U (6) and its subgroups in the pattern are given below:

$C_{1, U(5)} = n_d$, $C_{2, U(5)} = n_d (n_d + 4)$,

$C_{2, SO(6)} = N (N + 4) - \{\sqrt{5} [d^\dagger \times d^\dagger]^{(0)} - s^\dagger s^\dagger\} \{\sqrt{5} [\tilde{d} \times \tilde{d}]^{(0)} - ss\}$

$C_{2, SU(3)} = \sum_\mu (-1)^\mu Q_\mu Q_{-\mu}$, Where $Q_\mu = \{d_\mu^\dagger \tilde{s} + s^\dagger \tilde{d}_\mu - \sqrt{7}/2 [d^\dagger \times \tilde{d}]_\mu^{(2)}\}$

The E2 transitions that are one of the important factors within the collective nuclear structure. So, the reduced electric transitions can also be analysed in the framework of the IBM-2 and the most general E2 transition operator can be written as [22]

$$T(E2) = e_\rho [s^\dagger_\rho \times \tilde{d}_\rho + d^\dagger_\rho \times \tilde{s}_\rho]^{(2)} + \chi_\rho (d^\dagger_\rho \times \tilde{d}_\rho)^{(2)}$$

$$= e_\nu Q_\nu + e_\pi Q_\pi \quad (7)$$

Where χ_ρ is a dimensional coefficient and e_ρ is the effective quadrupole charges depending on the boson number N and they can any value to fit the experimental result.

The B(E2) strength for the E2 transitions is given by:

$$B (E2; L_i \rightarrow L_f) = 1/ (2L_i + 1)^{1/2} |< L_f \| T_m(E2) \| L_i>|^2 \quad (8)$$

RESULTS AND DISCUSSION

In theoretically, all factors in fitting the energy spectrum of a single nucleus can be adjusted independently. However, in order to limit the number of free parameters and in accordance with Subber et al [23] microscopic estimates, only ε and k are varying as a function to both of N_π and N_ν i.e., $\varepsilon = \varepsilon (N_\pi, N_\nu)$ and $k = k (N_\pi, N_\nu)$ is allowed. The other parameters depend only on N_π or N_ν i.e.,

$\chi_\pi = \chi_\pi (N_\pi)$, $\chi_\nu = \chi_\nu (N_\nu)$, $C_{L\pi} = C_{L\pi} (N_\pi)$ and $C_{L\nu} = C_{L\nu} (N_\nu)$

Thus, in isotopes chain, χ_π is kept constant, whereas for two isotonic Te isotopes, χ_ν, $C_{L\pi}$ and $C_{L\nu}$ are kept constant in Table-1. The isotopes $^{122-130}Te$ have $N_\pi = 1$, and N_ν varies from 6 to 1, while the parameters k, χ_π, χ_ν and ε treated as free parameters and their values are estimated by fitting to the measured level energies. The best fit values for the Hamiltonian parameters are given in Table-1.

Table-1: IBM-2 Hamiltonian parameters, all parameters in MeV units.

Nuclei	N_π	N_ν	N	ε	k	χ_ν	χ_π	$C_{0\nu}$	$C_{2\nu}$	$C_{4\nu}$	$C_{0\pi}$	$C_{2\pi}$	$C_{4\pi}$
$_{52}Te^{122}$	1	6	7	0.503	0.008	0.02	0.01	0.002	0.003	0.002	0.002	0.003	0.002
$_{52}Te^{124}$	1	5	6	0.508	0.010	0.02	0.01	0.002	0.003	0.002	0.002	0.003	0.002
$_{52}Te^{126}$	1	4	5	0.602	0.020	0.03	0.02	0.002	0.003	0.002	0.002	0.003	0.002
$_{52}Te^{128}$	1	3	4	0.621	0.022	0.03	0.02	0.002	0.003	0.002	0.002	0.003	0.002
$_{52}Te^{130}$	1	2	3	0.820	0.00	0.9	1.2	0.002	0.003	0.002	0.002	0.003	0.002

Using the parameters in Table-1, the estimated energy levels are shown in Table-2, along with experimental energy levels. As can be observed, there is a wide range of views between experiment and theory, and the general features are well reproduced. For high spin states, we observe a contradiction between theory and experiment.

The symmetry of the nucleus is determined by the energy ratio $R_{4/2} = E (4^+)/E (2^+)$ of the energies of the first 4^+ and 2^+ states are good criteria for the shape transition [24]. The value of the energy ratio has the limiting value 2 for a quadrupole vibrator, 2.5 for a non-axial γ-soft rotor and 3.33 for an ideally symmetric rotor. The estimated values change from about 2.18 to about 2.26, meaning that their structure seems to be varying from axial γ-soft to quadrupole vibrator O (6) → SU (5).

Because the Te nucleus has a vibrational-like character, we used the multiple expansion form of the Hamiltonian for our approximation, taking into account the dynamic symmetry location of the even-even Te nuclei at the IBM phase casten triangle, where their parameter seta is at the O (6) → SU (5) transition region and closer to SU (5) character.

Table-2: Comparison of theoretical data with experimental results of IBM-2 energies for $^{122-130}Te$ isotopes.

J^π_i	$_{52}Te^{122}$		$_{52}Te^{124}$		$_{52}Te^{126}$		$_{52}Te^{128}$		$_{52}Te^{130}$	
	E_{exp}	E_{cal}	E_{exp}	E_{cal}	E_{exp}	E_{cal}	E_{exp}	E_{cal}	E_{exp}	E_{cal}
0^+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2^+	0.5641	0.503	0.602	0.508	0.666	0.602	0.743	0.621	0.840	0.820
4^+	1.1813	1.006	1.248	1.016	1.316	1.204	1.498	1.242	1.633	1.640
6^+	1.7514	1.509	1.747	1.524	1.776	1.806	1.811	1.863	1.815	2.460

Now, the following table-3 is related to B(E2) values of some transitions for $^{122-130}Te$ isotopes. In order to find the value of effective charge we have fitted the calculated absolute strength $B (E2; 2_1^+ \rightarrow 0_1^+)$ the transitions ground state band to the experimental ones. The values of the boson effective charges for all isotopes are determined by the experimental $B (E2; 2_1^+ \rightarrow 0_1^+)$. The $B (E2; 2_1^+ \rightarrow 0_1^+)$ and $B (E2; 4_1^+ \rightarrow 2_1^+)$ values decreases as neutron number increases toward the middle of the shell. The value of $B (E2; 2_1^+ \rightarrow 0_1^+)$ is small because this transition is from quasi beta to ground state band. The energy levels and B(E2) values in $^{122-130}Te$ isotopes, theoretical and experimental results [25-27] are good in agreement.

Table-3: Electric transition probabilities for $^{122-130}$Te in $e^2\ b^2$ units.

Spin Parity $J_i^+ \rightarrow J_f^+$	$_{52}Te^{122}$		$_{52}Te^{124}$		$_{52}Te^{126}$	
	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)
$2_1^+ \rightarrow 0_1^+$	0.132	0.133	0.1138	0.12	0.094	0.092
$4_1^+ \rightarrow 2_1^+$	0.19	0.22	0.14	0.18	0.159	0.150
$2_2^+ \rightarrow 2_1^+$	0.0350	0.0340	0.0340	0.0328	0.0200	0.028
$2_2^+ \rightarrow 0_1^+$	0.0390	0.0380	0.0033	0.0028	0.0013	0.0018

Spin Parity $J_i^+ \rightarrow J_f^+$	$_{52}Te^{128}$		$_{52}Te^{130}$	
	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)
$2_1^+ \rightarrow 0_1^+$	0.076	0.078	0.290	0.290
$4_1^+ \rightarrow 2_1^+$	-	0.114	-	0.392
$2_2^+ \rightarrow 2_1^+$	-	0.028	-	0.392
$2_2^+ \rightarrow 0_1^+$	-	0.0019	-	0.0

CONCLUSION

We have described the results of calculations for $^{122-130}$Te isotopes in terms of the neutron-proton interacting boson model-2 with the help of Casimir operators. There is good agreement between the theoretical and experimental results. Using the best fitted values of parameters in the Hamiltonian of the IBM-2, we have calculated energy levels and B(E2) values for $^{122-130}$Te isotopes. In IBM-2, electric transition probability B (E2; $J_i^+ \rightarrow J_f^+$) calculations for even-even Te isotopes better in agreement with the experimental data. In IBM-2, all $^{122-130}$Te isotopes are of the U (5) symmetry in both cases of energy levels and B(E2) values. Also, in IBM-2, there is no signature of triaxiality in the $^{122-130}$Te isotopes.

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