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THREE DIMENIONAL FIBONACCI SEQUENCE AND DIFFERENCE EQUATION

[1]Brightlin. D, [2] Dominic Babu. G

[1]Ph.D Scholar (Reg.No:19213012092007) *and* [2] Associate Proffessor

[1,2]P.G and Research Department of Mathematics,

Annai Vellankanni College,

Tholayavattam - 629187, Kanyakumari District,Tamil Nadu, S.India.

Affiliated to Manonmaniam Sundaranar University,

Abishekapatti,Tirunelveli-627012,Tamilnadu,India;

Abstract: In this paper, we introduce three dimensional difference operator and its inverse of the three dimensional Fibonacci sequence. Also the summation solution and the closed form solution of polynomial using three dimensional difference equation have been obtained this work also include generalized product formula for polynomial, reciprocal of polynomial using inverse of three dimensional difference operator.

Key words: Fibonacci number, three dimensional difference operator, three dimensional Fibonacci sequence, Higher order difference operator and summation solution.

1 INTRODUCTION

In 1984, Jerzy popenda introduced a particular type of difference operator Δ_α defined on $u(k)$ as $\Delta_\alpha u(k) = u(k+1) - \alpha u(k)$. Recently, G.Britto Anthony Xavier et al have got the solution of the generalized q-difference equation $\Delta_q^t v(k) = u(k), \in (-\alpha, \alpha)$ and $q \neq 1$. In the authours introduced q-alpha difference operator defined as

$$\Delta_{(q)\alpha} u(k) = u(qk) - \alpha u(k) \quad (1)$$

and then extended to generalized higher order q-alpha difference equation

$$\Delta_{(q_1)\alpha_1}(\Delta_{(q_t)\alpha_2}(\cdots \Delta_{(q_t)\alpha_t}(u(k))\cdots)) = u(k), k \in (-\alpha, \alpha) \quad (2)$$

and obtained finite q-alpha multi-series formula and finite higher order q-alpha series formula.

2 THREE DIMENSIONAL q-DIFFERENCE OPERATOR

Before stating and proving our results, we present basic definitions and preliminary results which will be used for the subsequent discussion.

Definition 2.1 *let β_1,and β_2 be fixed reals and $(e^{k1}, e^{k2}, e^{k3}) \in R^3$. Then the three dimensional q difference operator $\underset{(\beta_1,\beta_2)}{\Delta}q$ is defined as*

$$\underset{(\beta_1,\beta_2,\beta_3)}{\Delta_q} v(e^{k1}, e^{k2}, e^{k3}) = v(q^2 e^{k1}, q^2 e^{k2}, q^2 e^{k3}) - \beta_1 v(q e^{k1}, q e^{k2}, q e^{k3}) - \beta_2 v(e^{k1}, e^{k2}, e^{k3})$$

and its inverse denoted by $\underset{(\beta_1,\beta_2)}{\Delta_q} \Delta_q{}^{-1}$ is defined as below.

If, $\underset{(\beta_1,\beta_2)}{\Delta_q} \Delta_q v(e^{k1}, e^{k2}, e^{k3}) = u(e^{k1}, e^{k2}, e^{k3})$ Then,

$$v(e^{k1}, e^{k2}, e^{k3}) = \underset{(\beta_1,\beta_2)}{\Delta_q} \Delta_q{}^{-1} u(e^{k1}, e^{k2}, e^{k3}) \quad (4)$$

Lemma 2.2 *If $q^{2n} - \beta_1 q^n - \beta_2 \neq 0$ for $n = 0,1,2,\ldots$.then*

$$\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} (e^{k1}, e^{k2}, e^{k3}) = \frac{(e^{k1}, e^{k2}, e^{k3})^n}{q^{2n} - \beta_1 q^n - \beta_2} \text{ and}$$

$$\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} (1) = \frac{1}{1 - \beta_1 - \beta_2} \quad (5)$$

Proof. Replacing $v(e^{k1}, e^{k2}, e^{k3})$ by $(e^{k1}, e^{k2}, e^{k3})^n$ in ⇒ 2

$$\underset{(\beta_1,\beta_2)}{\Delta_q} (e^{k1}, e^{k2}, e^{k3})^n = (q^2(e^{k1}, e^{k2}, e^{k3}))^n - \beta_1(q^n(e^{k1}, e^{k2}, e^{k3})^n) - \beta_2(e^{k1}, e^{k2}, e^{k3})^n$$

$$(e^{k1}, e^{k2}, e^{k3})^n = (q^{2n} - \beta_1 q^n - \beta_2) \underset{(\beta_1,\beta_2)}{\Delta^{-1}{}_q} (e^{k1}, e^{k2}, e^{k3})^n \quad (6)$$

Again replacing $u(e^{k1}, e^{k2}, e^{k3})$ by $(e^{k1}, e^{k2}, e^{k3})^0$ in ⇒ 2

$$\underset{(\beta_1,\beta_2)}{\Delta_q} (e^{k1}, e^{k2}, e^{k3})^0 = (q^2(e^{k1}, e^{k2}, e^{k3}))^0 - \beta_1(q^0(e^{k1}, e^{k2}, e^{k3})^0) - \beta_2(e^{k1}, e^{k2}, e^{k3})^0$$

$$\underset{(\beta_1,\beta_2)}{\Delta_q}(1) = (1-\beta_1-\beta_2)(1)$$

$$\underset{(\beta_1,\beta_2)}{\Delta^{-1}{}_q}(1) = \frac{1}{1-\beta_1-\beta_2} \quad (7)$$

Lemma 2.3 *Let* $(e^{k1}, e^{k2}, e^{k3}) \in R^3$ *and* $q \neq 0$, *Then we have*

$$\Delta^2_{q(\beta)} v(e^{k1}, e^{k2}, e^{k3}) = \underset{(2\beta-\beta^2)}{\Delta_q} v(e^{k1}, e^{k2}, e^{k3})$$

Proof. then we using previous Definition 3

$$\Rightarrow \underset{(\beta_1,\beta_2)}{\Delta_q} v(e^{k1}e^{k2}e^{k3}) = v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) - \beta_1 v(qe^{k1}, qe^{k2}, qe^{k3}) - \beta_2 v(e^{k1}, e^{k2}, e^{k3})$$

putting $\beta_1 = 2\beta$ and $\beta_2 = -\beta^2$

$$\underset{(\beta_1,\beta_2)}{\Delta_q} v(e^{k1}e^{k2}e^{k3}) = v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) - 2\beta_1 v(qe^{k1}, qe^{k2}, qe^{k3}) + \beta^2 v(e^{k1}, e^{k2}, e^{k3})$$

$$\Delta^2_{(q)\beta} v(e^{k1}e^{k2}e^{k3}) = v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) - 2\beta v(qe^{k1}, qe^{k2}, qe^{k3}) + \beta^2 v(e^{k1}, e^{k2}, e^{k3})$$

$$\Delta^2_{(q)\beta} v(e^{k1}e^{k2}e^{k3}) = \underset{(2\beta,-\beta^2)}{\Delta_q} v(e^{k1}e^{k2}e^{k3})$$

3 Fibonacci sequence Using three Dimensional q-difference Equation

In this section, we introduce three dimensional Fibonacci sequence and its sum.

Definition 3.1 *For each pair* $(\beta_1, \beta_2 \in R^2)$, *the three dimensional Fibonacci sequence is defined as,*

$$F_{(\beta_1,\beta_2)} = \{F_n\}_{n=0}^{\infty} \quad (8)$$

where $F_0 = 1, F_1 = \beta_1$ and $F_n = \beta_1 F_{n-1} + \beta_2 F_{n-2}$ for $n \geq 2$ when $\beta_1 = \beta_2 = 18$ becomes the usual Fibonacci sequence

Theorem 3.2 *(Three dimensional Finite q series), Let* $F_n \in F_{(\beta_1,\beta_2)}$ *and* $e^{k1}, e^{k2}, e^{k3} \in R^3$ *then we have.*

$$\sum_{d=0}^{m} F_d u\left(\frac{e^{k1}}{q^{d+2}}, \frac{e^{k2}}{q^{d+2}}, \frac{e^{k3}}{q^{d+2}}\right) = \underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} u(e^{k1}, e^{k2}, e^{k3}) - F_{m+1} \underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} u\left(\frac{e^{k1}}{q^{m+1}}, \frac{e^{k2}}{q^{m+1}}, \frac{e^{k3}}{q^{m+1}}\right)$$
$$-\beta_2 F_m \Delta_q^{-1}(\beta_1, \beta_2) u\left(\frac{e^{k1}}{q^{m+2}}, \frac{e^{k2}}{q^{m+2}}, \frac{e^{k3}}{q^{m+2}}\right) \quad (9)$$

Proof. Taking $\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} u(e^{k1}, e^{k2}, e^{k3}) = v(e^{k1}, e^{k2}, e^{k3})$

$$\underset{(\beta_1,\beta_2)}{\Delta_q} v(e^{k1}, e^{k2}, e^{k3}) = u(e^{k1}, e^{k2}, e^{k3})$$

By 3 we write

$$v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) = u(e^{k1}, e^{k2}, e^{k3}) + \beta_1 v(e^{k1}, e^{k2}, e^{k3}) + \beta_2 v(e^{k1}, e^{k2}, e^{k3}) \quad (10)$$

Replacing $v(e^{k1}, e^{k2}, e^{k3})$ by $\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right)$ in eqn 10, we obtain.

$$v(qe^{k1}, qe^{k2}, e^{k3}) = u\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right) + \beta_1 v(e^{k1}, e^{k2}, e^{k3}) + \beta_2 v\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k1}}{q}\right) \quad (11)$$

Substituting the value of $v(qe^{k1}, qe^{k2}, qe^{k3})$ in eqn 10 we get

$$v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) = u(e^{k1}, e^{k2}, e^{k3}) + \beta_1\left[v\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right)\right] + \beta_1 v(e^{k1}, e^{k2}, e^{k3}) + \beta_1\left[v\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right)\right] + \beta_2 v(e^{k1}, e^{k2}, e^{k3})$$

$$v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) = u(e^{k1}, e^{k2}, e^{k3}) + \beta_1 u\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right)$$
$$+(\beta_1^2 + \beta_2) v(e^{k1}, e^{k2}, e^{k3}) + \beta_1 + \beta_2) v\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right) \quad (12)$$

Replace (e^{k1}, e^{k2}, e^{k3}) by $\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right)$ in 11 and putting the value of $v(e^{k1}, e^{k2}, e^{k3})$ in eqn 12 we obtain.

$$v(q^2e^{k1}, q^2e^{k2}, q^2e^{k3}) = u(e^{k1}, e^{k2}, e^{k3}) + \beta_1 u\left(\frac{e^{k1}}{q}, \frac{e^{k2}}{q}, \frac{e^{k3}}{q}\right)$$

$+(\beta_1^2+\beta_2)[u\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right)+\beta_1 v\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$

$+\beta_2 v\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right)]+\beta_1\beta_2 v\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$

$+\beta_1(\beta_1^2\beta_2)v\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right)+\beta_2(\beta_1^2\beta_2)v\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$

$+\beta_1\beta_2 v\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$

$$v(q^2e^{k1},q^2e^{k2},q^2e^{k3}) = u(e^{k1},e^{k2},e^{k3}) + \beta_1 u\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)(\beta_1^2 + \beta_2)u\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right)$$

$+(\beta_1(\beta_1^2+\beta_2)+\beta_1\beta_2)v\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$

$$+\beta_2(\beta_1^2+\beta_2)v\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right) \quad (13)$$

Since $F_n \in F_{(\beta_1,\beta_2}$ we get,

$$v(q^2e^{k1},q^2e^{k2},q^2e^{k3}) = F_0u(e^{k1},e^{k2},e^{k3})+F_1u\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$$

$F_3u\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$

$+\beta_2F_2v\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right)$

Proceeding like this we arrive,

$$v(q^2e^{k1},q^2e^{k2},q^2e^{k3}) = F_0u(e^{k1},e^{k2},e^{k3}) + F_1u\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)+\cdots + F_{m+1}v\left(\frac{e^{k1}}{q^{m-1}},\frac{e^{k2}}{q^{m-1}},\frac{e^{k3}}{q^{m-1}}\right)+\beta_2F_mv\left(\frac{e^{k1}}{q^m},\frac{e^{k2}}{q^m},\frac{e^{k3}}{q^m}\right) \quad (14)$$

Now replace (e^{k1},e^{k2},e^{k3}) by $\left(\frac{e^{k1}}{q},\frac{e^{k2}}{q},\frac{e^{k3}}{q}\right)$ in eqn 14 we get.

$$v(e^{k1},e^{k2},e^{k3}) = F_0u\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right) + F_1u\left(\frac{e^{k1}}{q^3},\frac{e^{k2}}{q^3},\frac{e^{k3}}{q^3}\right)+\cdots$$

$+F_mu\left(\frac{e^{k1}}{q^{m+2}},\frac{e^{k2}}{q^{m+2}},\cdots\right)$

$F_{m+1}v\left(\frac{e^{k1}}{q^{m+1}},\frac{e^{k2}}{q^{m+1}},\frac{e^{k3}}{q^{m+1}}\right)$

$+\beta_2F_mv\left(\frac{e^{k1}}{q^{m+2}},\cdots\right)$

$$\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}(e^{k1},e^{k2},e^{k3}) - F_{m+1}\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}u\left(\frac{e^{k1}}{q^{m+1}},\frac{e^{k2}}{q^{m+1}},\frac{e^{k3}}{q^{m+1}}\right) -\beta_2F_m\underset{(\beta_1,\beta_2)}{\Delta}{}_q^{-1}u\left(\frac{e^{k1}}{q^{m+2}},\frac{e^{k2}}{q^{m+2}},\frac{e^{k3}}{q^{m+2}}\right) = \sum_{d=0}^{m}F_du\left(\frac{e^{k1}}{q^{d+2}},\frac{e^{k2}}{q^{d+2}},\frac{e^{k3}}{q^{d+2}}\right) \quad (15)$$

Hence the proof theorem.

Corollary 3.3 *Assume that* $P_1+P_2\neq 1$ *and* $F_n\in F_{(p_1,P_2)}$ *then we have.*

$$\sum_{d=0}^{m}F_d = \frac{1-F_{m+1}-\beta_2F_m}{1-\beta_1-\beta_2} \quad (16)$$

Proof. From eqn 9 replacing $u(e^k)$ by e^{k^0} we find.

$$\sum_{d=0}^{m}F_d\left(\frac{e^{k1}}{q^{d+2}},\frac{e^{k2}}{q^{m+2}},\frac{e^{k3}}{q^{m+2}}\right)^0 = \underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}(e^{k1},e^{k2},e^{k3})^0 - F_{m+1}\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}\left(\frac{e^{k1}}{q^{m+1}},\frac{e^{k2}}{q^{m+1}},\frac{e^{k3}}{q^{m+1}}\right)^0 -\beta_2F_m\underset{(\beta_1,\beta_2)}{\Delta}{}_q^{-1}\left(\frac{e^{k1}}{q^{m+2}},\frac{e^{k2}}{q^{m+2}},\frac{e^{k3}}{q^{m+2}}\right)^0$$

$$\sum_{d=0}^{m}F_d = \left(\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} - F_{m+1}\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} - \beta_2F_m\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}\right)$$

Hence the Proof.

$+F_2u\left(\frac{e^{k1}}{q^2},\frac{e^{k2}}{q^2},\frac{e^{k3}}{q^2}\right)+$

4 PRODUCT FORMULA OF THREE DIMENSIONAL q-DIFFERENCE EQUATION

We introduce the product formula of three dimensional difference equation

Theorem 4.1 *For the real valued functions* $u(e^{k1},e^{k2},e^{k3})$ *and* $v(e^{k1},e^{k2},e^{k3})$ *we have,*

$$\underset{\beta_1,\beta_2}{\Delta_q^{-1}}(u(e^{k1},e^{k2},e^{k3})v(e^{k1},e^{k2},e^{k3})) = \frac{1}{\beta_2}[u(e^{k1},e^{k2},e^{k3})\underset{(0,1)}{\Delta_q^{-1}}v(e^{k1},e^{k2},e^{k3})$$

$$-\underset{\beta_1,\beta_2}{\Delta_q^{-1}}\left(\underset{\beta_1,\beta_2}{\Delta_q} u(e^{k1},e^{k2},e^{k3})\underset{(0,1)}{\Delta_q^{-1}} v(e^{k1},e^{k2},e^{k3})\right)$$

$$-\beta_1 \underset{\beta_1,\beta_2}{\Delta_q^{-1}}\left(u(q^{k1},q^{k2},q^{k3})\underset{(0,1)}{\Delta_q}\left(\underset{(0,1)}{\Delta_q^{-1}} v(e^{k1},e^{k2},e^{k3})\right)\right)] \quad (17)$$

Proof. we find that.

$$\underset{(\beta_1,\beta_2)}{\Delta_q} u(e^{k1},e^{k2},e^{k3}) = u(q^2e^{k1},q^2e^{k2},q^2e^{k3})$$

$$-\beta_1 u(qe^{k1},qe^{k2},qe^{k3}) -$$

$$\beta_2 u(e^{k1},e^{k2},e^{k3})$$

$$\underset{(\beta_1,\beta_2)}{\Delta_q} u(e^{k1},e^{k2},e^{k3})w(e^{k1},e^{k2},e^{k3}) = u(q^2e^{k1},q^2e^{k2},q^2e^{k3})w(q^2e^{k1},q^2e^{k2},q^2e^{k3})$$

$$-\beta_1 u(qe^{k1},qe^{k2},qe^{k3})w(qe^{k1},qe^{k2},qe^{k3})$$

$$-\beta_2 u(e^{k1},e^{k2},e^{k3})w(e^{k1},e^{k2},e^{k3})$$

$$\Rightarrow u(e^{k1},e^{k2},e^{k3})w(e^{k1},e^{k2},e^{k3}) = \Delta_q^{-1}[w(q^2e^{k1},q^2e^{k2},q^2e^{k3})(\Delta_q u(e^{k1},e^{k2},e^{k3})]$$

$$+\beta_1 u(qe^{k1},qe^{k2},qe^{k3})\Delta_q(\Delta_q^{-1} w(e^{k1},e^{k2},e^{k3}))$$

$$+\beta_2[u(e^{k1},e^{k2},e^{k3})v(e^{k1},e^{k2},e^{k3})]$$

$$\Rightarrow u(e^{k1},e^{k2},e^{k3})\Delta_q^{-1} v(e^{k1},e^{k2},e^{k3}) = \Delta_q^{-1}[\Delta_q^{-1} v(q^2e^{k1},q^2e^{k2},q^2e^{k3})][\Delta_q u(e^{k1},e^{k2},e^{k3})]$$

$$+\beta_1 u(qe^{k1},qe^{k2},qe^{k3})\Delta_q[\Delta_q^{-1} v(e^{k1},e^{k2},e^{k3})$$

$$+\beta_2 u(e^{k1},e^{k2},e^{k3}), v(e^{k1},e^{k2},e^{k3})]$$

$$\Rightarrow u(e^{k1},e^{k2},e^{k3})\Delta_q^{-1} v(e^{k1},e^{k2},e^{k3}) = \Delta_q^{-1}[\Delta_q u(e^{k1},e^{k2},e^{k3})\Delta_q^{-1} v(q^2e^{k1},q^2e^{k2},q^2e^{k3})$$

$$+\beta_1 \Delta_q^{-1}(u(qe^{k1},qe^{k2},qe^{k3})\Delta_q(\Delta_q^{-1}(v(e^{k1},e^{k2},e^{k3})))$$

$$\beta_2 \Delta_q^{-1} u(^{k1},e^{k2},e^{k3})v(e^{k1},e^{k2},e^{k3})$$

$$-\beta_2 \Delta_q^{-1}(u(e^{k1},e^{k2},e^{k3})v(e^{k1},e^{k2},e^{k3}))$$

$$\Rightarrow u(e^{k1},e^{k2},e^{k3})\Delta_q^{-1} v(e^{k1},e^{k2},e^{k3}) = -u(e^{k1},e^{k2},e^{k3})\Delta_q^{-1} v(e^{k1},e^{k2},e^{k3})$$

$$+\Delta_q^{-1}(\Delta_q u(e^{k1},e^{k2},e^{k3})\Delta_q^{-1} v(e^{k1},e^{k2},e^{k3}))$$

Hence the Proof.

Example 4.2

To find the $\beta_1 = 1, \beta_2 = 2, m = 1, n = 2, q = 5, k_1 = 1, k_2 = 3, k_3 = 2$. Then we have able to find $F_0 = 1, F_1 = \beta_1 = 2, F_2 = 3, F_3 = 4, F_4 = 5,$

$$u(e^{k1},e^{k2},e^{k3}) = (e^{k1},e^{k2},e^{k3})^n$$

Solution:

$$\sum_{d=0}^{m} F_d\left(\frac{e^{k1}}{q^{d+2}},\frac{e^{k2}}{q^{m+2}},\frac{e^{k3}}{q^{m+2}}\right)^n = \underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}(e^{k1},e^{k2},e^{k3})^n$$

$$-F_{m+1} \Delta$$

$$-\beta_2 F_m$$

L.H.S $=\sum_{d=0}^{m} F_d[\frac{e^{k1}}{q^{d+2}},\frac{e^{k2}}{q^{m+2}},\frac{e^{k3}}{q^{m+2}}]^n$

L.H.S = 231.2010633

$$\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} u(e^{k1},e^{k2},e^{k3})^n = \underset{(\beta_1,\beta_2)}{\Delta_q^{-1}}(u(e^{k1},e^{k2},e^{k3})^n)^n$$

$$= \frac{(e^{k1},e^{k2},e^{k3})^n}{q^n-\beta_1 q^n-\beta_2}$$

$$= \frac{(2.718*20.08*7.30)^2}{598}$$

$$= 232.5727081$$

$$\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} u\left(\frac{e^{k1}}{q^{m+1}},\frac{e^{k2}}{q^{m+1}},\frac{e^{k3}}{q^{m+1}}\right)^n = 0.434064825$$

$$\underset{(\beta_1,\beta_2)}{\Delta_q^{-1}} u\left(\frac{e^{k1}}{q^{m+2}},\frac{e^{k2}}{q^{m+2}},\frac{e^{k3}}{q^{m+2}}\right)^n = 0.017362593$$

$R.H.S = 232.5727081 - F_2$ x $0.434064825 - 2$ x $F_1(0.17362593)$

$= 232.5727081 - 3$ x $0.434064825 - 2$ x 2 x0.017362593

$= 231.2010633$

Example 4.3

To find the $m = 5, \beta_1 = 15, \beta_2 = 20, F_0 = 1, F_1 = \beta_1 = 20,$

$$F_2 = 200, F_3 = 250, F_4 = 14000, F_5 = 156750, F_6 = 1788375$$

Solution :

$$\sum_{d=0}^{m} F_d = \frac{1 - F_{m+1} - \beta_2 F_m}{1 - \beta_1 - \beta_2}$$

$$L.H.S = 144805$$

Now$R.H.S = \frac{1 - F_6 - \beta_2 F_5}{1 - \beta_1 - \beta_2}$

$$= \frac{1 - 1788375 - 20 * 156750}{1 - 15 - 20}$$

$$= 144805$$

Conclusion:

In this paper the summation solution and the closed form solutions of polynomials using three dimensional difference equation have been obtained. Also several results using three dimensional q-difference operator are derived. Moreover generalized product formula for polynomial reciprocal of polynomial using inverse of three dimensional q difference operator is obtained.

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