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$\mathrm{Ng}^{\#}-$ Homeomorphism in Neutrosophic Topological Spaces

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Abstract

The aim of this paper is to introduce the concepts of $\mathcal{N}\mathrm{g}^{\#}-$ homeomorphism and strongly $\mathcal{N}\mathrm{g}^{\#}-$ homeomorphism in Neutrosophic Topological Space. Further, the work establishes some of their related attributes.

Keywords: $\mathcal{N}\mathrm{g}^{\#}-$ closed set, $\mathcal{N}\mathrm{g}^{\#}-$ continuous function, $\mathcal{N}\mathrm{g}^{\#}-$ irresolute function, $\mathcal{N}\mathrm{g}^{\#}-$ homeomorphism, strongly $\mathcal{N}\mathrm{g}^{\#}-$ homeomorphism.

AMS Mathematics Subject Classification – 06D72; 03E72

1 Introduction

Smarandache [9] introduced the idea of Neutrosophic set, and in 2014 Salama et.al. [20] initiated further studies into Neutrosophic closed sets and Neutrosophic continuous functions. Recently Pious Missier et.al.[15],[16],[13] introduced the concept of $\mathcal{N}\mathrm{g}^{\#}-$ closed sets, continuous and irresolute mappings, closed and open mappings in Neutrosophic Topological Spaces. In this paper, we introduce $\mathcal{N}\mathrm{g}^{\#}-$ homeomorphism and strongly $\mathcal{N}\mathrm{g}^{\#}-$ homeomorphism in Neutrosophic Topological Spaces and investigate their properties.

2 Preliminaries

Definition 2.1 [9]

A Neutrosophic set ($\mathcal{NS}$) $\mathcal{A}_{\mathcal{N}}$ is an object having the form $\mathcal{A}_{\mathcal{N}}=\{\langle \mathrm{x}, \mu_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \sigma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \gamma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x})\rangle : \mathrm{x} \in \mathcal{X}\}$ where $\mu_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \sigma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x})$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x})$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $\mathrm{x} \in \mathcal{X}$ to the set $\mathcal{A}_{\mathcal{N}}$. A Neutrosophic set $\mathcal{A}_{\mathcal{N}} = \{\langle \mathrm{x}, \mu_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \sigma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \gamma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x})\rangle : \mathrm{x} \in \mathcal{X}\}$ can be identified as an ordered triple $\langle \mu_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \sigma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x}), \gamma_{\mathcal{A}_{\mathcal{N}}}(\mathrm{x})\rangle$ in $]-0,1+[$ on $\mathcal{X}$.

Definition 2.2 [20] A Neutrosophic topology ($\mathcal{NT}$) on a non-empty set $\mathcal{X}$ is a family τ of Neutrosophic subsets in $\mathcal{X}$ satisfies the following axioms:

1. $\mathbf{0}_{\mathcal{N}}, \mathbf{1}_{\mathcal{N}} \in \tau$
2. $\mathrm{R}_{\mathrm{N}_1} \cap \mathrm{R}_{\mathrm{N}_2} \in \tau$ for any $\mathrm{R}_{\mathrm{N}_1}, \mathrm{R}_{\mathrm{N}_2} \in \tau$
3. $\cup\, \mathrm{R}_{\mathrm{N}_i} \in \tau \quad \forall \quad \mathrm{R}_{\mathrm{N}_i} : i \in I \subseteq \tau$

Definition 2.3 [20] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathrm{X}_{\mathcal{N}}$. Then

1.$\mathcal{N}\mathrm{int}(\mathcal{A}_{\mathcal{N}}) = \cup\{\mathrm{G} : \mathrm{G}$ is a $\mathcal{NOS}$ in $\mathrm{X}_{\mathcal{N}}$ and $\mathrm{G} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic interior of $\mathcal{A}_{\mathcal{N}}$.

2. $\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}) = \cap\{\mathrm{K} : \mathrm{K}$ is a $\mathcal{NCS}$ in $\mathrm{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathrm{K}\}$ is called Neutrosophic closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.4 [11] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}, \tau)$ is called a neutrosophic $\mathcal{N}\alpha\mathrm{gCS}$ if $\mathcal{N}\alpha\mathrm{cl}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{U}_{\mathcal{N}}$, whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{U}_{\mathcal{N}}$ and $\mathcal{U}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}$. The complement of $\mathcal{N}\alpha\mathrm{gCS}$ is $\mathcal{N}\alpha\mathrm{gOS}$.

Definition 2.5 [15] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}, \tau)$ is called a Neutrosophic $\mathrm{g}^{\#}-$closed ($\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$) if $\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{Q}_{\mathcal{N}}$ whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{Q}_{\mathcal{N}}$ and $\mathcal{Q}_{\mathcal{N}}$ is $\mathcal{N}\alpha\mathrm{gOS}$ in $\mathcal{X}$. The complement of $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ is $\mathcal{N}\mathrm{g}^{\#}\mathrm{OS}$.

Definition 2.6 [17] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}$. Then

1. $\mathcal{N}\mathrm{g}^{\#}\mathrm{int}(\mathcal{A}_{\mathcal{N}}) = \cup\{\mathrm{G} : \mathrm{G}$ is a $\mathcal{N}\mathrm{g}^{\#}\mathrm{OS}$ in $\mathcal{X}$ and $\mathrm{G} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic $\mathrm{g}^{\#}-$ interior of $\mathcal{A}_{\mathcal{N}}$.

2. $\mathcal{N}\mathrm{g}^{\#}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}) = \cap\{\mathrm{K} : \mathrm{K}$ is a $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ in $\mathcal{X}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathrm{K}\}$ is called Neutrosophic $\mathrm{g}^{\#}-$ closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.7 [16] A function $\mathrm{f}_{\mathcal{N}} : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be $\mathcal{N}g^{\#}-$ continuous function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X},\tau)$ for every neutrosophic closed set $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y},\zeta)$.

Definition 2.8 [16] A function $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is said to be Neutrosophic $g^{\#}-$ irresolute function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ of $(\mathcal{X},\tau)$ for every $\mathcal{N}g^{\#}CS$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y},\zeta)$.

Definition 2.9 [17] A Neutrosophic Topological space $(\mathcal{X},\tau)$ is called a $T_{\mathcal{N}}g^{\#}-$ space if every $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$ is $\mathcal{NCS}$ in $(\mathcal{X},\tau)$.

Definition 2.10 [18] Let $(\mathcal{X},\tau)$ and $(\mathcal{Y},\zeta)$ be two Neutrosophic topological spaces. A mapping $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is called $\mathcal{N}g^{\#}-$ closed mapping ($\mathcal{N}g^{\#}CM$ for short) if $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$ for every $\mathcal{NCS}$ $\mathcal{A}_{\mathcal{N}}$ of $(\mathcal{X},\tau)$.

Definition 2.11 [18] Let $(\mathcal{X},\tau)$ and $(\mathcal{Y},\zeta)$ be two Neutrosophic topological spaces. A mapping $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is called $\mathcal{N}g^{\#}-$ open mapping ($\mathcal{N}g^{\#}OM$ for short) if $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y},\zeta)$ for every $\mathcal{NOS}$ $\mathcal{A}_{\mathcal{N}}$ of $(\mathcal{X},\tau)$.

Definition 2.12 [13] Let $(\mathcal{X},\tau)$ and $(\mathcal{Y},\zeta)$ be two Neutrosophic Topological Spaces. A bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is called Neutrosophic homeomorphism ($\mathcal{N}-\mathcal{HOM}$ for short) if $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}-$ continuous.

3 Neutrosophic $g^{\#}$- Homeomorphism

Definition 3.1 Let $(\mathcal{X},\tau)$ and $(\mathcal{Y},\zeta)$ be two Neutrosophic Topological Spaces. A bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is called Neutrosophic $g^{\#}-$ homeomorphism ($\mathcal{N}g^{\#}-\mathcal{HOM}$ for short) if $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous.

Theorem 3.2 Every $\mathcal{N}-\mathcal{HOM}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{N}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}-$ continuous. Since every $\mathcal{N}-$ continuous function is $\mathcal{N}g^{\#}-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Example 3.3 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.3,0.4,0.6)\rangle,\langle m,(0.4,0.3,0.6)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.2,0.3,0.7)\rangle,\langle q,(0.3,0.2,0.7)\rangle\}$.

Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\}$ and $\zeta=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here $\mathcal{NCS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$, $\mathcal{NCS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X})=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})$. Here $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$. Now $\mathcal{M}_{\mathcal{N}_2}^{c}$ is a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$ but $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}_2}^{c})$ is not a $\mathcal{NCS}$ in $(\mathcal{X},\tau)$. Therefore $f_{\mathcal{N}}$ is not $\mathcal{N}-$ continuous and hence $f_{\mathcal{N}}$ is not $\mathcal{N}-\mathcal{HOM}$.

Theorem 3.4 Every $\mathcal{NR}-\mathcal{HOM}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{NR}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}-$ continuous. Since every $\mathcal{NR}-$ continuous function is $\mathcal{N}g^{\#}-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 3.5 Every $\mathcal{N}g^{\#}-\mathcal{HOM}$ is $\mathcal{NG}-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{N}g^{\#}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Since every $\mathcal{N}g^{\#}-$ continuous function is $\mathcal{NG}-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{NG}-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{NG}-\mathcal{HOM}$.

Example 3.6 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.1,0.2,0.8)\rangle,\langle m,(0.2,0.3,0.8)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.2,0.3,0.7)\rangle,\langle q,(0.3,0.3,0.7)\rangle\}$.

Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\}$ and $\zeta=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here, $\mathcal{NCS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X})$, $\mathcal{NCS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{NGCS}(\mathcal{Y})=\mathcal{NGCS}(\mathcal{X})$. Here $f_{\mathcal{N}}$ is $\mathcal{NG}-\mathcal{HOM}$. Now $\mathcal{M}_{\mathcal{N}_2}^{c}$ is a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$ but $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}_2}^{c})$ is not a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$. Therefore $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-$ continuous and hence $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 3.7 Every $\mathcal{N}g^{\#}-\mathcal{HOM}$ is $\mathcal{N}\alpha g-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{N}g^{\#}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Since every $\mathcal{N}g^{\#}-$ continuous function is $\mathcal{N}\alpha g-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}\alpha g-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{N}\alpha g-\mathcal{HOM}$.

Example 3.8 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.2,0.3,0.7)\rangle,\langle m,(0.3,0.4,0.7)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.3,0.4,0.6)\rangle,\langle q,(0.4,0.5,0.6)\rangle\}$.

Now $(\mathcal{X},\tau)=\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}$ and $(\mathcal{Y},\zeta)=\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}$ are Neutrosophic topological spaces. Then $\tau=\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}$ and $\zeta=\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection

$f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here, $\mathcal{NCS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X}),\mathcal{NCS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\},\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}\alpha g\mathcal{CS}(\mathcal{Y})$, $\mathcal{N}\alpha g\mathcal{CS}(\mathcal{X})$. Here $f_{\mathcal{N}}$ is $\mathcal{N}\alpha g-\mathcal{HOM}$. Now $\mathcal{M}_{\mathcal{N}_2}^{c}$ is a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$ but $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}_2}^{c})$ is not a $\mathcal{N}g^{\#}$CS in $(\mathcal{X},\tau)$. Therefore $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-$ continuous and hence $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 3.9 Every $\mathcal{N}g^{\#}-\mathcal{HOM}$ is $\mathcal{NGP}-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{N}g^{\#}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Since every $\mathcal{N}g^{\#}-$ continuous function is $\mathcal{NGP}-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{NGP}-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{NGP}-\mathcal{HOM}$.

Example 3.10 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.2,0.2,0.8)\rangle,\langle m,(0.2,0.3,0.8)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.3,0.3,0.7)\rangle,\langle q,(0.3,0.3,0.7)\rangle\}$.

Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\}$ and $\zeta=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here $\mathcal{NCS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X})$, $\mathcal{NCS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{NGPCS}(\mathcal{Y})=\mathcal{NGPCS}(\mathcal{X})$. Here $f_{\mathcal{N}}$ is $\mathcal{NGP}-\mathcal{HOM}$. Now $\mathcal{M}_{\mathcal{N}_2}^{c}$ is a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$ but $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}_2}^{c})$ is not a $\mathcal{N}g^{\#}$CS in $(\mathcal{X},\tau)$. Therefore $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-$ continuous and hence $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 3.11 Every $\mathcal{N}g^{\#}-\mathcal{HOM}$ is $\mathcal{NGS}-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{N}g^{\#}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Since every $\mathcal{N}g^{\#}-$ continuous function is $\mathcal{NGS}-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{NGS}-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{NGS}-\mathcal{HOM}$.

Example 3.12 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.1,0.2,0.9)\rangle,\langle m,(0.1,0.3,0.8)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.3,0.3,0.7)\rangle,\langle q,(0.4,0.3,0.7)\rangle\}$.

Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\}$ and $\zeta=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$ Here, $\mathcal{NCS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X})$, $\mathcal{NCS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\},\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{NGSCS}(\mathcal{Y})=\mathcal{NGSCS}(\mathcal{X})$. Here $f_{\mathcal{N}}$ is $\mathcal{NGS}-\mathcal{HOM}$. Now $\mathcal{M}_{\mathcal{N}_2}^{c}$ is a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$ but $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}_2}^{c})$ is not a $\mathcal{N}g^{\#}-$CS in $(\mathcal{X},\tau)$. Therefore $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-$ continuous and hence $f_{\mathcal{N}}$ is not $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Remark 3.13 The following diagram shows the relationships of $\mathcal{N}g^{\#}-\mathcal{HOM}$ with some other Neutrosophic homeomorphisms discussed in this section.

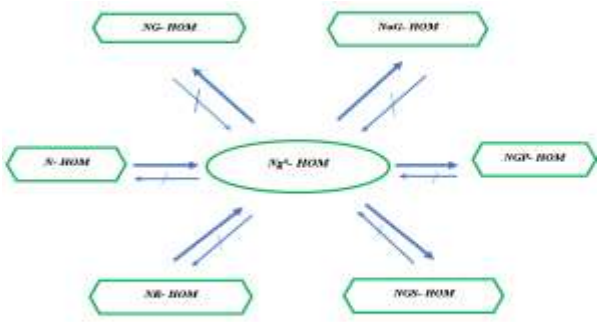

Figure 1

Here A → B means A implies B and Here A → B means A not implies B

Remark 3.14 Composition of two $\mathcal{N}g^{\#}-$ homeomorphism mappings need not be a $\mathcal{N}g^{\#}-$ homeomorphism.

Example 3.15 Let $\mathcal{X}=\{l,m\},\mathcal{Y}=\{u,v\}$ and $\mathcal{Z}=\{p,q\}$.

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.2,0.2,0.8)\rangle,\langle m,(0.3,0.3,0.7)\rangle$

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.9,0.8,0.1)\rangle,\langle q,(0.8,0.9,0.2)\rangle\}$.

Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\},(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Z},\eta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\},\zeta=\{\mathbf{0}_{\mathcal{N}},\mathbf{1}_{\mathcal{N}}\}$ and $\eta=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X},\mathcal{Y}$ and $\mathcal{Z}$ respectively. Define a function $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=u$ and $f_{\mathcal{N}}(m)=v$ and define a function $g_{\mathcal{N}}:(\mathcal{Y},\zeta)\longrightarrow(\mathcal{Z},\eta)$ by $g_{\mathcal{N}}(u)=p$ and $g_{\mathcal{N}}(v)=q$. Then $f_{\mathcal{N}}$ and $g_{\mathcal{N}}$ are $\mathcal{N}g^{\#}-\mathcal{HOM}$s. Now define a function $(g_{\mathcal{N}}\circ f_{\mathcal{N}}):(\mathcal{X},\tau)\longrightarrow(\mathcal{Z},\eta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here $\mathcal{M}_{\mathcal{N}_1}^{c}=\{\langle l,(0.8,0.8,0.4)\rangle,\langle m,(0.7,0.8,0.2)\rangle\}$ is a $\mathcal{NCS}$ in $(\mathcal{X},\tau)$. But $(g_{\mathcal{N}}\circ f_{\mathcal{N}})(\mathcal{M}_{\mathcal{N}_1}^{c})=\{\langle p,(0.8,0.8,0.4)\rangle,\langle q,(0.7,0.8,0.2)\rangle\}$ is not a $\mathcal{N}g^{\#}$CS in $(\mathcal{Z},\eta)$. Hence $(g_{\mathcal{N}}\circ f_{\mathcal{N}})^{-1}$ is not a $\mathcal{N}g^{\#}-$ continuous map. Therefore $(g_{\mathcal{N}}\circ f_{\mathcal{N}})$ is not $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 3.16 Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a bijective mapping. If $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ continuous then the following statements are equivalent:

1. $f_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ closed mapping.
2. $f_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ open mapping.
3. $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ homeomorphism.

Proof. (1) $\Longrightarrow$ (2) Let us assume that $f_{\mathcal{N}}$ be a bijective mapping and $\mathcal{N}g^{\#}CM$. Hence $f_{\mathcal{N}}^{-1}$ is $\mathcal{N}g^{\#}-$ continuous. Clearly every $\mathcal{NOS}$ in $(\mathcal{X},\tau)$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y},\zeta)$. Hence $f_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ open mapping.

(2) $\Longrightarrow$ (3) Let $f_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OM$. Then $f_{\mathcal{N}}^{-1}$ is $\mathcal{N}g^{\#}-$ continuous. Hence $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous mappings. Therefore $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ homeomorphism.

(3) $\Longrightarrow$ (1) Let $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ homeomorphism. Then $f_{\mathcal{N}}$ and $(f_{\mathcal{N}}^{-1})$ both are $\mathcal{N}g^{\#}-$ continuous mappings. Since every $\mathcal{NCS}$ in $(\mathcal{X},\tau)$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$, $f_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ closed mapping. Hence proved.

Theorem 3.17 Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a $\mathcal{N}g^{\#}-\mathcal{HOM}$. Then $f_{\mathcal{N}}$ is a $\mathcal{N}-\mathcal{HOM}$ if $(\mathcal{X},\tau)$ and $(\mathcal{Y},\zeta)$ are $T_{\mathcal{N}}g^{\#}-$ spaces.

Proof. Let $\mathcal{M}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$. By hypothesis, $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$. Since $(\mathcal{X},\tau)$ is $T_{\mathcal{N}}g^{\#}-$ space, $f_{\mathcal{N}}^{-1}(\mathcal{M}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{X},\tau)$. Which implies $f_{\mathcal{N}}$ is $\mathcal{N}-$ continuuous. Since $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$, $f_{\mathcal{N}}^{-1}$ is $\mathcal{N}g^{\#}-$ continuous. Let $\mathcal{P}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{X},\tau)$. By hypothesis, $(f_{\mathcal{N}}^{-1})^{-1}(\mathcal{P}_{\mathcal{N}})=f_{\mathcal{N}}(\mathcal{P}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$. Since $\mathcal{Y}$ is $T_{\mathcal{N}}g^{\#}-$ space, $f_{\mathcal{N}}(\mathcal{P}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$. Which implies $f_{\mathcal{N}}^{-1}$ is $\mathcal{N}-$ continuuous. Hence $f_{\mathcal{N}}$ is $\mathcal{N}-\mathcal{HOM}$.

Neutrosophic Strongly $g^{\#}$- Homeomorphism

Definition 4.1 Let $(\mathcal{X},\tau)$ and $(\mathcal{Y},\zeta)$ be two Neutrosophic topological spaces. A bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is called strongly Neutrosophic $g^{\#}-$ homeomorphism (strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ for short) if $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ irresolute functions.

Example 4.2 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.2,0.3,0.8)\rangle,\langle m,(0.3,0.3,0.7)\rangle\}$, Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\}$ and $\zeta=\{\mathbf{0}_{\mathcal{N}},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here $\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X})$. Here $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ irresolute. Hence $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 4.3 Every strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$ but not conversely.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$, then $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ irresolute. Since every $\mathcal{N}g^{\#}-$ irresolute function is $\mathcal{N}g^{\#}-$ continuous, $f_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}$ both are $\mathcal{N}g^{\#}-$ continuous. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Example 4.4 Let $\mathcal{X}=\{l,m\}$ and $\mathcal{Y}=\{p,q\}$. Consider the Neutrosophic sets

$\mathcal{M}_{\mathcal{N}_1}=\{\langle l,(0.2,0.2,0.8)\rangle,\langle m,(0.2,0.3,0.8)\rangle\}$,

$\mathcal{M}_{\mathcal{N}_2}=\{\langle p,(0.9,0.8,0.1)\rangle,\langle q,(0.8,0.8,0.1)\rangle\}$.

Now $(\mathcal{X},\tau)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y},\zeta)=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1},\mathbf{1}_{\mathcal{N}}\}$ and $\zeta=\{\mathbf{0}_{\mathcal{N}},\mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}$s on $\mathcal{X}$ and $\mathcal{Y}$ respectively. Define a bijection $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ by $f_{\mathcal{N}}(l)=p$ and $f_{\mathcal{N}}(m)=q$. Here, $\mathcal{NCS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathbf{1}_{\mathcal{N}}\}=\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y})$, $\mathcal{NCS}(\mathcal{Y})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_2}^{c},\mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X})=\{\mathbf{0}_{\mathcal{N}},\mathcal{M}_{\mathcal{N}_1}^{c},\mathcal{M}_{\mathcal{N}_2},\mathbf{1}_{\mathcal{N}}\}$. Here $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-\mathcal{HOM}$. Now $\mathcal{M}_{\mathcal{N}_2}$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$ but $({f_{\mathcal{N}}}^{-1})^{-1}(\mathcal{M}_{\mathcal{N}_2})$ is not a $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$. Therefore $f_{\mathcal{N}}^{-1}$ is not $\mathcal{N}g^{\#}-$ irresolute and hence $f_{\mathcal{N}}$ is not strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$.

Theorem 4.5 Composition of two strongly $\mathcal{N}g^{\#}-$ homeomorphism mappings is again a strongly $\mathcal{N}g^{\#}-$ homeomorphism.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ and $g_{\mathcal{N}}:(\mathcal{Y},\zeta)\longrightarrow(\mathcal{Z},\eta)$ are strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$s. Let $\mathcal{W}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{Z},\eta)$. Since $g_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}, g_{\mathcal{N}}^{-1}(\mathcal{W}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$. Since $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}, (g_{\mathcal{N}}\circ f_{\mathcal{N}})(\mathcal{W}_{\mathcal{N}})=f_{\mathcal{N}}^{-1}(g_{\mathcal{N}}^{-1}(\mathcal{W}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$. Therefore, $(g_{\mathcal{N}}\circ f_{\mathcal{N}})$ is $\mathcal{N}g^{\#}-$ irresolute. Now, Let $\mathcal{W}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$. Since $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$, $f_{\mathcal{N}}(\mathcal{W}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$. Since $g_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ $(g_{\mathcal{N}}\circ f_{\mathcal{N}})(\mathcal{W}_{\mathcal{N}})=g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{W}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Z},\eta)$. Hence, $(g_{\mathcal{N}}\circ f_{\mathcal{N}})^{-1}$ is $\mathcal{N}g^{\#}-$ irresolute. Therefore, $(g_{\mathcal{N}}\circ f_{\mathcal{N}})$ is strongly $\mathcal{N}g^{\#}-$ homeomorphism.

Theorem 4.6 A mapping $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ then $\mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))\subseteq f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}))$ for each $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{Y},\zeta)$.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ and $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $(\mathcal{Y},\zeta)$. Then $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{NCS}$ in $(\mathcal{X},\tau)$. Since every $\mathcal{NCS}$ is $\mathcal{N}g^{\#}CS, \mathcal{N}cl(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y},\zeta)$. Now by hypothesis, $f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X},\tau)$. Which implies that, $\mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})))=f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}))$. Here, $\mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))\subseteq\mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})))=f_{\mathcal{N}}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}))$. Hence, $\mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))\subseteq f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}))$ for each $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{Y},\zeta)$.

Theorem 4.7 A mapping $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ is strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ then $\mathcal{N}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))=f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}))$ for each $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{Y},\zeta)$.

Proof. Let $f_{\mathcal{N}}:(\mathcal{X},\tau)\longrightarrow(\mathcal{Y},\zeta)$ be a strongly $\mathcal{N}g^{\#}-\mathcal{HOM}$ then $f_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ irresolute mapping. And let $\mathcal{A}_{\mathcal{N}}$ be a

$\mathcal{NS}$ in $(\mathcal{Y},\zeta)$. Then $\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{NCS}$ in $(\mathcal{Y},\zeta)$. Since every $\mathcal{NCS}$ is $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$, $\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ in $(\mathcal{Y},\zeta)$. Now by hypothesis, $\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ in $(\mathcal{X},\tau)$. Since $\mathrm{f}_{\mathcal{N}}^{-1}((\mathcal{A}_{\mathcal{N}})) \subseteq \mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}))$, $\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}((\mathcal{A}_{\mathcal{N}}))) \subseteq \mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}))) = \mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}))$. Therefore, $\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) \subseteq \mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}))$. Let $\mathrm{f}_{\mathcal{N}}$ be a strongly $\mathcal{N}\mathrm{g}^{\#}-\mathcal{HOM}$ then $\mathrm{f}_{\mathcal{N}}^{-1}$ is a $\mathcal{N}\mathrm{g}^{\#}-$ irresolute mapping. Let us assume that $\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$ be a $\mathcal{NS}$ in $(\mathcal{X},\tau)$, Which implies that, $\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ in $(\mathcal{X},\tau)$. Hence $\mathcal{N}\mathrm{g}^{\#}\mathrm{cl}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))$ is a $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ in $(\mathcal{X},\tau)$. This implies that, $(\mathrm{f}_{\mathcal{N}}^{-1})^{-1}(\mathcal{N}\mathrm{g}^{\#}\mathrm{cl}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))) = \mathrm{f}_{\mathcal{N}}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))$ is a $\mathcal{N}\mathrm{g}^{\#}\mathrm{CS}$ in $(\mathcal{Y},\zeta)$. This proves, $\mathcal{A}_{\mathcal{N}} = (\mathrm{f}_{\mathcal{N}}^{-1})^{-1}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) \subseteq (\mathrm{f}_{\mathcal{N}}^{-1})^{-1}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))) = \mathrm{f}_{\mathcal{N}}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))$. Therefore, $\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))) = \mathrm{f}_{\mathcal{N}}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))$, since $\mathrm{f}_{\mathcal{N}}^{-1}$ is $\mathcal{N}\mathrm{g}^{\#}-$ irresolute. Hence, $\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}})) \subseteq \mathrm{f}_{\mathcal{N}}^{-1}(\mathrm{f}_{\mathcal{N}}(\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})))) = \mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$. That is, $\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$. Hence $\mathcal{N}\mathrm{cl}(\mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) = \mathrm{f}_{\mathcal{N}}^{-1}(\mathcal{N}\mathrm{cl}(\mathcal{A}_{\mathcal{N}}))$ for each $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{Y},\zeta)$.

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