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Performance Measure of Retrial Queues in Intuitionistic Fuzzy Environment

S. Chandrasekaran[1], Bindu Kumari. V. R.[2]
[1]Assistant Professor, P.G. & Research Department of Mathematics, Pachaiyappa's College,
Affiliated to University of Madras, Chennai.

[2]Assistant Professor, Department of Mathematics, St. Thomas College of Arts and Science,
Affiliated to University of Madras, Chennai.

Abstract

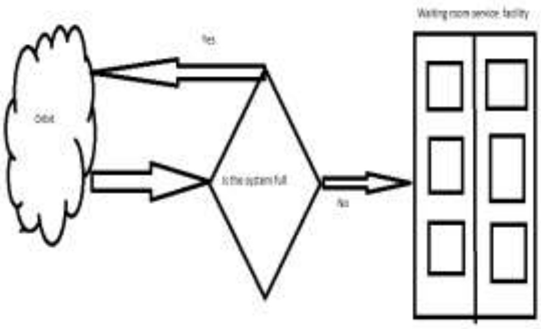
This paper applies intuitionistic fuzzy sets in retrial queues to determine the expected number of customers and the expected waiting time. The arrival rate, service rate and the retrial rate are taken as trapezoidal intuitionistic fuzzy numbers(TrIFN) The concept of cut sets (α and β) and interval arithmetic have been used to find the performance measures of the retrial queue. The performance measures obtained are indeed again TrIFN. The TrIFN are defuzzified into crisp values by defuzzification measure. This approach is illustrated by a numerical example.

Keywords: Retrial queues, Intuitionistic Fuzzy sets, Trapezoidal Fuzzy number, Trapezoidal Intuitionistic Fuzzy number

1. Introduction

In our daily life we observe some blocked customers in the queuing system are not willing to wait and they leave the system temporally. After some random time, they attempt to get the service. A blocked customer is assumed to be waiting in the virtual waiting space known as Orbit before retrying to get service. Such situations for are modelled as retrial queues.

Retrial queues are applied in Call Centres, Communication Networks, Optical networks, Cloud system etc. In general arrival rate λ follows Poisson distribution, the service rate μ follows exponential distribution. The retrial rate r depends on the number of customers in the orbit as the customers in the orbit behave independently of each other. The retrial rate 'r' is independent and identically distributed following an exponential distribution. The following figure shows the general multi server retrial queue.



A number of applications of retrial queues are found in the literature. **Falin** [5] provide an excellent survey on retrial queues and their applications. Artalejo etal [1] discuss a multi-server queuing (M/M/C/C) with constant retrial rate. In [2] they considered Fuzzy retrial queuing system using a computational approach. Phung-Duc and Kawanishi [8] proposed an efficient computational algorithm for multi-server retrial queues.

The uncertainty in real time situation have been tackled in an efficient away by the application of Fuzzy sets introduced by Zadeh [12]. Chen [3,4] analysed fuzzy queues using parametric linear programming. Ke etal [7] discusses retrial queuing model with fuzzy parameters. Viswanathan et.al [9] considered a fuzzy retrial queuing system with coxian -2 vacation. Gang chen et.al [6] analysed strategic customers behaviour in fuzzy queueing system.

This paper is organised as follows: Section 2 contains the basic definitions and preliminaries. Section 3 introduces retrial queues in intuitionistic fuzzy environment. Section 4 presents the performance measures of fuzzy retrial queue. Section 5 illustrates the proposed approach using the numerical example followed by conclusion.

2. Basic Definitions

2.1 Fuzzy Set

Let A be a classical set, $\mu_A(x)$ be a function from A to [0,1]. A Fuzzy set A with the membership function $\mu_A(x)$ is defined as A={(x, $\mu_A(x)$; x ϵ A} and $\mu_A(x)$ ϵ [0.1].

2.2 Intuitionistic Fuzzy Set

Let X be a given set. An Intuitionistic Fuzzy Set B in X is given by, B = {(x, $\theta_B(x)$,$\vartheta_B(x)$) | x ϵ X} where θ_B, ϑ_B : X→[0,1], where $\theta_B(x)$ is the degree of membership of the element x in B and $\vartheta_B(x)$ is the degree of non-membership of x in B and 0≤ $\theta_B(x)$+ $\vartheta_B(x) \leq 1$.
For each x ϵ X, $\pi_B(x)$ = 1 - $\theta_B(x)$ - $\vartheta_B(x)$ is the degree of hesitation.

2.3 Trapezoidal Intuitionistic Fuzzy Number (TrIFN)

A fuzzy number is a TrIFN with parameters $S_1 \le t_1$, $S_2 \le t_2$, $t_3 \le S_3$, $t_4 \le S_4$ denoted by

$\tilde{T}_A = (t_1\ t_2\ t_3\ t_4\ \ S_1\ S_2\ S_3\ S_4)$ whose membership and non-membership is given below

$$\theta_{\tilde{T}}(x) = \begin{cases} 0 & if \quad x \le t_1 \\ \dfrac{x-t_1}{t_2-t_1} & if \quad t_1 < x < t_2 \\ 1 & if \quad t_2 \le x \le t_3 \\ \dfrac{t_4-x}{t_4-t_3} & if \quad t_3 < x < t_4 \\ 0 & if \quad x \ge t_4 \end{cases}$$

$$\vartheta_{\tilde{T}}(x) = \begin{cases} 1 & if \quad x \le s_1 \\ \dfrac{s_2-x}{s_2-s_1} & if \quad s_1 < x < s_2 \\ 1 & if \quad s_2 \le x \le s_3 \\ \dfrac{x-s_3}{s_4-s_3} & if \quad s_3 < x < s_4 \\ 1 & if \quad x \ge s_4 \end{cases}$$

The characteristics of a queuing system are completely described by the membership function and non-membership function of E(N) and E(W) where E(N) and E(W) denote the expected number of customers and the expected waiting time of a customer in the system respectively. The membership function and the non-membership function of E(N) and E(W)are given below:

$$\theta_{E(N)}(x) = \begin{cases} 0 & if \quad x \le n_1 \\ \dfrac{x-n_1}{n_2-n_1} & if \quad n_1 < x < n_2 \\ 1 & if \quad n_2 \le x \le n_3 \\ \dfrac{n_4-x}{n_4-n_3} & if \quad n_3 < x < n_4 \\ 0 & if \quad x \ge n_4 \end{cases}$$

$$\nu_{E(N)}(x) = \begin{cases} 1 & if \quad x \le n_1 \\ \dfrac{n_2-x}{n_2-n_1} & if \quad n_1 < x < n_2 \\ 1 & if \quad n_2 \le x \le n_3 \\ \dfrac{x-n_3}{n_4-n_3} & if \quad n_3 < x < n_4 \\ 1 & if \quad x \ge n_4 \end{cases}$$

$$\theta_{E(w)}(x) = \begin{cases} 0 & if \quad x \le w_1 \\ \dfrac{x-w_1}{w_2-w_1} & if \quad w_1 < x < w_2 \\ 1 & if \quad w_2 \le x \le w_3 \\ \dfrac{w_4-x}{w_4-w_3} & if \quad w_3 < x < w_4 \\ 0 & if \quad x \ge w_4 \end{cases}$$

$$\nu_{E(w)}(x) = \begin{cases} 1 & if \quad x \le w_1 \\ \dfrac{w_2-x}{w_2-w_1} & if \quad w_1 < x < w_2 \\ 1 & if \quad w_2 \le x \le w_3 \\ \dfrac{x-w_3}{w_4-w_3} & if \quad w_3 < x < w_4 \\ 1 & if \quad x \ge w_4 \end{cases}$$

2.4 Cut sets of TrIFN [11]

2.4.1 An (α-β) cut of $\tilde{T}_A = (t_1\ t_2\ t_3\ t_4\ s_1\ s_2\ s_3\ s_4)$ is a crisp set of R denoted as

$\tilde{T}_{\alpha,\beta}$ ={x / $\theta_{\tilde{T}}(x) \ge \alpha, \nu_{\tilde{T}}(x) \le \beta$}--------2.4.1

Where $0 \le \alpha \le 1$ $0 \le \beta \le 1$ and $0 \le \alpha+\beta \le 1$

2.4.2 An α- cut set of $\tilde{T}$ is a crisp subset of R defined by

$\tilde{T}_\alpha$ ={x / $\theta_{\tilde{T}}(x) \ge \alpha$}--------2.4.2

using equation 2.4.1 and 2.4.2, it follows that $\tilde{T}_\alpha$ is s closed interval denoted by $\tilde{T}_\alpha = [L_{\tilde{T}}(\alpha), R_{\tilde{T}}(\alpha)]$

$=[(1-\alpha)t_1+\alpha t_2, (1-\alpha)t_4+ \alpha t_3]$

$=[t_1 +\alpha(t_2 - t_1), t_4 - \alpha(t_4 - t_3)]$-----2.4.3

2.4.3 An β- cut set of $\tilde{T}$ is a crisp subset of R defined by

$\tilde{T}_\alpha$ ={x / $\nu_{\tilde{T}}(x) \le \beta$}--------2.4.4

Using equation 2.4.1 and 2.4.4, it follows that $\tilde{T}_\alpha$ is s closed interval denoted by $\tilde{T}_\beta = [L_{\tilde{T}}(\beta), R_{\tilde{T}}(\beta)]$ $=[(1-\beta)s_2+ \beta s_1, (1- \beta)s_3+ \beta s_4]$

$=[s_2 - \beta(s_2 - s_1), s_3 + \beta(s_4 - s_3)]$ ------2.4.5

3. Retrial Queuing in Intuitionistic Fuzzy Environment

Let $\tilde{\lambda}_I$, $\tilde{\mu}_I$ and $\tilde{r}_I$ be the arrival rate, service rate and retrial rate respectively. These rates can be represented as a convex fuzzy set, as they are not known precisely. Let $\theta(\tilde{\lambda}_I)$, $\theta(\tilde{\mu}_I)$ and $\theta(\tilde{r}_I)$ denote the membership between functions of $\tilde{\lambda}_I$, $\tilde{\mu}_I$ and $\tilde{r}_I$ respectively and let $\nu(\tilde{\lambda}_I)$, $\nu(\tilde{\mu}_I)$ and $\nu(\tilde{r}_I)$ denote the non-membership function of $\tilde{\lambda}_I$, $\tilde{\mu}_I$ and $\tilde{r}_I$ respectively

The I.F. sets are given by

$$\tilde{A}_I = \{ \tilde{\lambda}_I, \theta(\tilde{\lambda}_I), \nu(\tilde{\lambda}_I) \ / \ \tilde{\lambda}_I \in A\}$$

$$\tilde{B}_I = \{ \tilde{\mu}_I, \theta(\tilde{\mu}_I), \nu(\tilde{\mu}_I) \ / \ \tilde{\mu}_I \in B\}$$

$$\tilde{C}_I = \{ \tilde{r}_I, \theta(\tilde{r}_I), \nu(\tilde{r}_I) \ / \ \tilde{r}_I \in C\}$$

where A, B and C are crisp universal sets of arrival rate service rate and retrial rates respectively and

$0 \le \theta(x) + \mu(x) \le 1$ where $x = \tilde{\lambda}_I, \tilde{\mu}_I, \tilde{r}_I$.

For a crisp retrial queuing system if $\frac{\lambda}{\mu} < 1$ the expected number of customers in the orbit and the expected waiting time of a customer are given by

$$E(N) = \frac{\lambda^2}{\mu-\lambda}\left(\frac{1}{\lambda}+\frac{1}{r}\right)$$

$$E(W) = \frac{\lambda}{\mu-\lambda}\left(\frac{1}{\lambda}+\frac{1}{r}\right)$$

4. Performance measures of fuzzy retrial queue

Let arrival rate $\tilde{\lambda}_I$, service rate $\tilde{\mu}_I$ and retrial rate $\tilde{r}_I$ be denoted by TrIFN's

$\tilde{\lambda}_I = (\lambda_1, \lambda_2, \lambda_3, \lambda_4; \lambda'_1, \lambda'_2, \lambda'_3, \lambda'_4)$

$\tilde{\mu}_I = (\mu_1, \mu_2, \mu_3, \mu_4; \mu'_1, \mu'_2, \mu'_3, \mu'_4)$

$\tilde{r}_I = (r_1, r_2, r_3, r_4; r'_1, r'_2, r'_3, r'_4)$

where

$\lambda_1' \le \lambda_1,\ \lambda_2' \le \lambda_2,\ \lambda_3 \le \lambda_3',\ \lambda_4 \le \lambda_4'$

$\mu_1' \le \mu_1,\ \mu_2' \le \mu_2,\ \mu_3 \le \mu_3',\ \mu_4 \le \mu_4'$

$r_1' \le r_1,\ r_2' \le r_2,\ r_3 \le r_3',\ r_4 \le r_4'$

The membership and non-membership function of $\tilde{\lambda}_I$, $\tilde{\mu}_I$ and $\tilde{r}_I$ were respectively given as follows:

$$\theta_{\tilde{\lambda}_I}(x) = \begin{cases} 0 & if\ x \le \lambda_1 \\ \frac{x-\lambda_1}{\lambda_2-\lambda_1} & if\ \lambda_1 < x < \lambda_2 \\ 1 & if\ \lambda_2 \le x \le \lambda_3 \\ \frac{\lambda_4 - x}{\lambda_4-\lambda_3} & if\ \lambda_3 < x < \lambda_4 \\ 0 & if\ x \ge \lambda_4 \end{cases}$$

$$\nu_{\tilde{\lambda}_I}(x) = \begin{cases} 0 & if\ x \le \lambda_1' \\ \frac{\lambda_2'-x}{\lambda_2'-\lambda_1'} & if\ \lambda_1' < x < \lambda_2' \\ 1 & if\ \lambda_2' \le x \le \lambda_3' \\ \frac{x-\lambda_3'}{\lambda_4'-\lambda_3'} & if\ \lambda_3' < x < \lambda_4' \\ 0 & if\ x \ge \lambda_4' \end{cases}$$

$$\theta_{\tilde{\mu}_I}(x) = \begin{cases} 0 & if\ x \le \mu_1 \\ \frac{x-\mu_1}{\mu_2-\mu_1} & if\ \mu_1 < x < \mu_2 \\ 1 & if\ \mu_2 \le x \le \mu_3 \\ \frac{\mu_4-x}{\mu_4-\mu_3} & if\ \mu_3 < x < \mu_4 \\ 0 & if\ x \ge \mu_4 \end{cases}$$

$$\nu_{\tilde{\mu}_I}(x) = \begin{cases} 0 & if\ x \le \mu_1' \\ \frac{\mu_2'-x}{\mu_2'-\mu_1'} & if\ \mu_1' < x < \mu_2' \\ 1 & if\ \mu_2' \le x \le \mu_3' \\ \frac{x-\mu_3'}{\mu_4'-\mu_3'} & if\ \mu_3' < x < \mu_4' \\ 0 & if\ x \ge \mu_4' \end{cases}$$

$$\theta_{\tilde{r}_I}(x) = \begin{cases} 0 & if\ x \le r_1 \\ \frac{x-r_1}{r_2-r_1} & if\ r_1 < x < r_2 \\ 1 & if\ r_2 \le x \le r_3 \\ \frac{r_4-x}{r_4-r_3} & if\ r_3 < x < r_4 \\ 0 & if\ x \ge r_4 \end{cases}$$

$$\nu_{\tilde{r}_I}(x) = \begin{cases} 0 & if\ x \le r_1' \\ \frac{r_2' - x}{r_2' - r_1'} & if\ r_1' < x < r_2' \\ 1 & if\ \mu_2' \le x \le \mu_3' \\ \frac{x - r_3'}{r_4' - r_3'} & if\ r_3' < x < r_4' \\ 0 & if\ x \ge r_4' \end{cases}$$

Using the concept of cut-set

$$\alpha_{\tilde{\lambda}_I} = \left[L_{\tilde{\lambda}_I}(\alpha), R_{\tilde{\lambda}_I}(\alpha)\right]$$
$$= [\ \alpha(\lambda_2 - \lambda_1) + \lambda_1,\ \lambda_4 - \alpha(\lambda_4 - \lambda_3)\ ]$$

$$\beta_{\tilde{\lambda}_I} = \left[L_{\tilde{\lambda}_I}(\beta), R_{\tilde{\lambda}_I}(\beta)\right]$$
$$= [\lambda_2' - \beta\ (\lambda_2' - \lambda_1')\ ,\ \lambda_3' + \beta(\lambda_4' - \lambda_3')\ ]$$

$$\alpha_{\tilde{\mu}_I} = \left[L_{\tilde{\mu}_I}(\alpha), R_{\tilde{\mu}_I}(\alpha)\right]$$
$$= [\ \alpha(\mu_2 - \mu_1) + \mu_1,\ \mu_4 - \alpha\ (\mu_4 - \mu_3)]$$

$$\beta_{\tilde{\mu}_I} = \left[L_{\tilde{\mu}_I}(\beta), R_{\tilde{\mu}_I}(\beta)\right]$$
$$= [\mu_2' - \beta\ (\mu_2' - \mu_1')\ ,\ \mu_3' + \beta(\mu_4' - \mu_3')\ ]$$

$$\alpha_{\tilde{r}_I} = \left[L_{\tilde{r}_I}(\alpha), R_{\tilde{r}_I}(\alpha)\right]$$
$$= [\ \alpha(\ r_2 - r_1) + r_1,\ r_4 - \alpha\ (r_4 - r_3)]$$

$$\beta_{\tilde{r}_I} = \left[L_{\tilde{r}_I}(\beta), R_{\tilde{r}_I}(\beta)\right]$$
$$= [r_2' - \beta(r_2' - r_1')\ ,\ r_3' + \beta(r_4' - r_3')\ ]$$

From interval arithmetic

1. [a, b] + [c, d] = [a + c, b + d]
2. [a, b] – [c, d] = [a – d, b – c]
3. [a, b]. [c, d] = [min (ac, ad, bc, bd), max (ac, ad, bc, bd)] provided $0 \notin$[c, d]
4. [a, b] / [c, d] = [a, b] [1/d, 1/c] = [min (a/c, a/d, b/c, b/d), max(a/c a/d,b/c,b/d)] provided $0 \notin$[c, d]

$$\alpha_{\tilde{\mu}_I - \tilde{\lambda}_I} = [\alpha(\mu_2 - \mu_1) + \mu_1) - (\ \lambda_4 - \alpha(\lambda_4 - \lambda_3)),$$
$$\mu_4 - \alpha(\mu_4 - \mu_3) - (\alpha(\lambda_2 - \lambda_1) + \lambda_1)]$$
$$= [\alpha(\mu_2 - \mu_1 + \lambda_4 - \lambda_3) + \mu_1 - \lambda_4,$$
$$\mu_4 - \lambda_1 - \ \alpha\ (\mu_4 - \mu_3 + \lambda_2 - \lambda_1)]$$
$$= [\mu_1 - \lambda_4 + \alpha(\mu_2 - \mu_1 + \lambda_4 - \lambda_3)\ ,$$
$$\mu_4 - \lambda_1 - \ \alpha\ (\mu_4 - \mu_3 + \lambda_2 - \lambda_1)]$$

$$\beta_{\tilde{\mu}_I - \tilde{\lambda}_I} = [\mu_2' - \beta\ (\mu_2' - \mu_1') - (\lambda_3' + \beta(\lambda_4' - \lambda_3')),$$
$$\mu_3' + \beta(\mu_4' - \mu_3') - (\ \lambda_2' - \beta\ (\lambda_2' - \lambda_1'))]$$
$$= [\mu_2' - \lambda_3' - \beta\ (\mu_2' - \mu_1' + \lambda_4' - \lambda_3')),$$
$$\mu_3' - \lambda_2' + \beta\ (\mu_4' - \mu_3' + \lambda_2' - \lambda_1'))]$$

Hence

$$\alpha_{[\widetilde{\mu_I} - \widetilde{\lambda_I}]} = \begin{bmatrix} (\mu_1 - \lambda_4) + \alpha(\mu_2 - \mu_1 + \lambda_4 - \lambda_3\ ), \\ (\mu_4 - \lambda_1) - \alpha(\lambda_2 - \lambda_1 + \mu_4 - \mu_3\ ) \end{bmatrix}$$

$$\beta_{[\widetilde{\mu_I} - \widetilde{\lambda_I}]} = \begin{bmatrix} (\mu_2' - \lambda_3') - \beta(\mu_2' - \mu_1' + \lambda_4' - \lambda_3'\ ), \\ (\mu_3' - \lambda_2') + \beta(\lambda_2' - \lambda_1' + \mu_4' - \mu_3'\ ) \end{bmatrix}$$

$$\alpha_{\left[\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I} - \widetilde{\lambda_I}}\right]} = \begin{bmatrix} \dfrac{\lambda_1 + \alpha(\lambda_2 - \lambda_1\ )}{(\mu_4 - \lambda_1) - \alpha(\lambda_2 - \lambda_1 + \mu_4 - \mu_3\ )}, \\ \dfrac{\lambda_4 - \alpha(\lambda_4 - \lambda_3\ )}{(\mu_1 - \lambda_4) + \alpha(\mu_2 - \mu_1 + \lambda_4 - \lambda_3\ )} \end{bmatrix}$$

$$\beta_{\left[\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I} - \widetilde{\lambda_I}}\right]} = \begin{bmatrix} \dfrac{\lambda_2' - \beta(\lambda_2' - \lambda_1')}{(\mu_3' - \lambda_2') + \beta(\lambda_2' - \lambda_1' + \mu_4' - \mu_3'\ )}, \\ \dfrac{\lambda_3' + \beta(\lambda_4' - \lambda_3'\ )}{(\mu_2' - \lambda_3') - \beta(\mu_2' - \mu_1' + \lambda_4' - \lambda_3'\ )} \end{bmatrix}$$

$$\alpha_{\left[\frac{1}{\widetilde{\lambda_I}}\right]} = [\lambda_4 - \alpha(\lambda_4 - \lambda_3\ ), \lambda_1 + \alpha(\lambda_2 - \lambda_1\ )]$$

$$\alpha_{\left[\frac{1}{\tilde{r}_I}\right]} = [r_4 - \alpha(r_4 - r_3), r_1 + \alpha(r_2 - r_1)]$$

$$\alpha_{\left[\frac{1}{\tilde{\lambda}_I}+\frac{1}{\tilde{r}_I}\right]} = [\lambda_4 + r_4 - \alpha(\lambda_4 - \lambda_3 + r_4 - r_3), \lambda_1 + r_1 + \alpha(\lambda_2 - \lambda_1 + r_2 - r_1)]$$

$$\beta_{\left[\frac{1}{\tilde{\lambda}_I}\right]} = [\lambda'_3 + \beta(\lambda'_4 - \lambda'_3), \lambda'_2 - \beta(\lambda'_2 - \lambda'_1)]$$

$$\beta_{\left[\frac{1}{\tilde{r}_I}\right]} = [r'_3 + \beta(r'_4 - r'_3), r'_2 - \beta(r'_2 - r'_1)]$$

$$\beta_{\left[\frac{1}{\tilde{\lambda}_I}+\frac{1}{\tilde{r}_I}\right]} = \begin{bmatrix}\lambda'_3 + r'_3 + \beta(\lambda'_4 - \lambda'_3 + r'_4 - r'_3), \\ \lambda'_2 + r'_2 - \beta(\lambda'_2 - \lambda'_1 + r'_2 - r'_1)\end{bmatrix}$$

$$\alpha_{E(W)} = \alpha_{\left[\left(\frac{\tilde{\lambda}_I}{\tilde{\mu}_I - \tilde{\lambda}_I}\right)\left(\frac{1}{\tilde{\lambda}_I}+\frac{1}{\tilde{r}_I}\right)\right]}$$

$$\begin{bmatrix}\dfrac{[\alpha(\lambda_2 - \lambda_1) + \lambda_1][(\lambda_1 + r_1) + \alpha(\lambda_2 - \lambda_1 + r_2 - r_1)]}{(\mu_4 - \lambda_1) - \alpha(\lambda_2 - \lambda_1 + \mu_4 - \mu_3)}, \\ \dfrac{[\lambda_4 - \alpha(\lambda_4 - \lambda_3)][(\lambda_4 + r_4) - \alpha(\lambda_4 - \lambda_3 + r_4 - r_3)]}{(\mu_1 - \lambda_4) + \alpha(\mu_2 - \mu_1 + \lambda_4 - \lambda_3)}\end{bmatrix}$$

$$\alpha_{E(W)} = \left[\frac{[\lambda_1](\lambda_1 + r_1)}{(\mu_4 - \lambda_1)}, \frac{[\lambda_4](\lambda_4 + r_4)]}{(\mu_1 - \lambda_4)}\right]_{\alpha=0}$$

$$\alpha_{E(W)} = \left[\frac{[\lambda_2](\lambda_2 + r_2)}{(\mu_3 - \lambda_2)}, \frac{[\lambda_3](\lambda_3 + r_3)]}{(\mu_2 - \lambda_3)}\right]_{\alpha=1}$$

$$\alpha_{E(N)} = \alpha_{\left[\left(\frac{\tilde{\lambda}_I^2}{\tilde{\mu}_I - \tilde{\lambda}_I}\right)\left(\frac{1}{\tilde{\lambda}_I}+\frac{1}{\tilde{r}_I}\right)\right]}$$

$$= \begin{bmatrix}\dfrac{[\alpha(\lambda_2 - \lambda_1) + \lambda_1]^2[(\lambda_1 + r_1) + \alpha(\lambda_2 - \lambda_1 + r_2 - r_1)]}{(\mu_4 - \lambda_1) - \alpha(\lambda_2 - \lambda_1 + \mu_4 - \mu_3)}, \\ \dfrac{[\lambda_4 - \alpha(\lambda_4 - \lambda_3)]^2[(\lambda_4 + r_4) - \alpha(\lambda_4 - \lambda_3 + r_4 - r_3)]}{(\mu_1 - \lambda_4) + \alpha(\mu_2 - \mu_1 + \lambda_4 - \lambda_3)}\end{bmatrix}$$

$$\alpha_{E(N)} = \left[\frac{[\lambda_1]^2(\lambda_1 + r_1)}{(\mu_4 - \lambda_1)}, \frac{[\lambda_4]^2(\lambda_4 + r_4)}{(\mu_1 - \lambda_4)}\right]_{\alpha=0}$$

$$\alpha_{E(N)} = \left[\frac{[\lambda_2]^2(\lambda_2 + r_2)}{(\mu_3 - \lambda_2)}, \frac{[\lambda_3]^2(\lambda_3 + r_3)}{(\mu_2 - \lambda_3)}\right]_{\alpha=1}$$

$$\beta_{E(W)} = \beta_{\left[\left(\frac{\tilde{\lambda}_I}{\tilde{\mu}_I - \tilde{\lambda}_I}\right)\left(\frac{1}{\tilde{\lambda}_I}+\frac{1}{\tilde{r}_I}\right)\right]}$$

$$= \begin{bmatrix}\dfrac{[\lambda'_2 - \beta(\lambda'_2 - \lambda'_1)][(\lambda'_2 + r'_2) - \beta(\lambda'_2 - \lambda'_1 + r'_2 - r'_1)]}{(\mu'_3 - \lambda'_2) + \beta(\lambda'_2 - \lambda'_1 + \mu'_4 - \mu'_3)}, \\ \dfrac{[\lambda'_3 + \beta(\lambda'_4 - \lambda'_3)][(\lambda'_3 + r'_3) + \beta(\lambda'_4 - \lambda'_3 + r'_4 - r'_3)]}{(\mu'_2 - \lambda'_3) - \beta(\mu'_2 - \mu'_1 + \lambda'_4 - \lambda'_3)}\end{bmatrix}$$

$$\beta_{E(W)} = \left[\frac{[\lambda'_2][\lambda'_2 + r'_2]}{(\mu'_3 - \lambda'_2)}, \frac{[\lambda'_3][\lambda'_3 + r'_3]}{(\mu'_2 - \lambda'_3)}\right]_{\beta=0}$$

$$\beta_{E(W)} = \left[\frac{[\lambda'_1][\lambda'_1 + r'_1]}{(\mu'_4 - \lambda'_1)}, \frac{[\lambda'_4][\lambda'_4 + r'_4]}{(\mu'_1 - \lambda'_4)}\right]_{\beta=1}$$

$$\beta_{E(N)} = \beta_{\left[\left(\frac{\tilde{\lambda}_I^2}{\tilde{\mu}_I - \tilde{\lambda}_I}\right)\left(\left(\frac{1}{\tilde{\lambda}_I}+\frac{1}{\tilde{r}_I}\right)\right)\right]}$$

$$= \begin{bmatrix}\dfrac{[\lambda'_2 - \beta(\lambda'_2 - \lambda'_1)]^2[(\lambda'_2 + r'_2) - \beta(\lambda'_2 - \lambda'_1 + r'_2 - r'_1)]}{(\mu'_3 - \lambda'_2) + \beta(\lambda'_2 - \lambda'_1 + \mu'_4 - \mu'_3)}, \\ \dfrac{[\lambda'_3 + \beta(\lambda'_4 - \lambda'_3)]^2[(\lambda'_3 + r'_3) + \beta(\lambda'_4 - \lambda'_3 + r'_4 - r'_3)]}{(\mu'_2 - \lambda'_3) - \beta(\mu'_2 - \mu'_1 + \lambda'_4 - \lambda'_3)}\end{bmatrix}$$

$$\beta_{E(N)} = \left[\frac{[\lambda'_2]^2[\lambda'_2 + r'_2]}{(\mu'_3 - \lambda'_2)}, \frac{[\lambda'_3]^2[\lambda'_3 + r'_3]}{(\mu'_2 - \lambda'_3)}\right]_{\beta=0}$$

$$\beta_{E(W)} = \left[\frac{[\lambda'_1]^2[\lambda'_1 + r'_1]}{(\mu'_4 - \lambda'_1)}, \frac{[\lambda'_4]^2[\lambda'_4 + r'_4]}{(\mu'_1 - \lambda'_4)}\right]_{\beta=1}$$

Considering all these values of α= 0, α=1, β=0 and β=1 we obtain an approximate TrIFN

E(W)=

$$\begin{bmatrix}\left(\frac{\lambda_1(\lambda_1+r_1)}{\mu_4-\lambda_1}, \frac{\lambda_2(\lambda_2+r_2)}{\mu_3-\lambda_2}, \frac{\lambda_3(\lambda_3+r_3)}{\mu_2-\lambda_3}, \frac{\lambda_4(\lambda_4+r_4)}{\mu_1-\lambda_4}\right) \\ \left(\frac{\lambda_1'(\lambda_1'+r_1')}{\mu_4'-\lambda_1'}, \frac{\lambda_2'(\lambda_2'+r_2')}{\mu_3'-\lambda_2'}, \frac{\lambda_3'(\lambda_3'+r_3')}{\mu_2'-\lambda_3'}, \frac{\lambda_4'(\lambda_4'+r_4')}{\mu_1'-\lambda_4'}\right)\end{bmatrix}$$

$$\approx [(w_1, w_2, w_3, w_4)(w_1', w_2', w_3', w_4')]$$

In a similar manner

E(N)=

$$\begin{bmatrix}\left(\frac{\lambda_1^2(\lambda_1+r_1)}{\mu_4-\lambda_1}, \frac{\lambda_2^2(\lambda_2+r_2)}{\mu_3-\lambda_2}, \frac{\lambda_3^2(\lambda_3+r_3)}{\mu_2-\lambda_3}, \frac{\lambda_4^2(\lambda_4+r_4)}{\mu_1-\lambda_4}\right) \\ \left(\frac{\lambda_1'^2(\lambda_1'+r_1')}{\mu_4'-\lambda_1'}, \frac{\lambda_2'^2(\lambda_2'+r_2')}{\mu_3'-\lambda_2'}, \frac{\lambda_3'^2(\lambda_3'+r_3')}{\mu_2'-\lambda_3'}, \frac{\lambda_4'^2(\lambda_4'+r_4')}{\mu_1'-\lambda_4'}\right)\end{bmatrix}$$

$$\approx [(n_1, n_2, n_3, n_4)(n_1', n_2', n_3', n_4')]$$

5. Numerical example

Arrival at a ticket counter are considered to be Poisson with an average time $\tilde{\lambda}_I$= (2,4,6,8) (1,3,7,9) per hour. The length of the service is distributed exponentially with $\tilde{\mu}_I$= (21, 24, 27, 29) (20, 23,28,30) per hour. If the facility is busy the customers leave the system and after sometime they enter with the following retrial rates $\tilde{r}_I$= (0.25 0.5, 0.625, 0.875) (0.125 0.375 0.75,1). Find the expected waiting time and the number of customers in the orbits.

Solution

Given

$\tilde{\lambda}_I = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)(\lambda_1', \lambda_2', \lambda_3', \lambda_4')$
= (2,4,6,8) (1,3,7,9)

$\tilde{\mu}_I = (\mu_1, \mu_2, \mu_3, \mu_4)(\mu_1', \mu_2', \mu_3', \mu_4')$
= (21, 24, 27, 29) (20, 23,28,30)

$\tilde{r}_I = (r_1, r_2, r_3, r_4)(r_1', r_2', r_3', r_4')$
= (0.25 0.5, 0.625, 0.875)
(0.125 0.375 0.75,1)

Expected waiting time

$$E(\tilde{W}) = \begin{bmatrix}\left(\frac{2(2+0.25)}{29-2}, \frac{4(4+0.5)}{27-4}, \frac{6(6+0.625)}{24-6}, \frac{8(8+0.875)}{21-8}\right) \\ \left(\frac{1(1+0.25)}{30-1}, \frac{3(3+0.375)}{28-3}, \frac{7(7+0.75)}{23-7}, \frac{9(9+1)}{20-9}\right)\end{bmatrix}$$

= [(0.1667, 0.7826, 2.2083, 5.4615),
(0.03879, 0.405, 3.3906, 8.1818)]

$E(\tilde{N})$ = [(0.3334, 3.1304, 13.2498, 43.692),
(0.03879, 1.215, 23.7342, 73.6362)]

Using the defuzzification measures

$$r(\tilde{A}) = \frac{a_1+2a_2+2a_3+a_4+a'_1+2a'_2+2a'_3+a'_4}{12}$$

$E(\tilde{W}) = \frac{27.42179}{12}$= 2.2815 hours

$E(\tilde{N}) = \frac{200.35919}{12}$=16.6965 ≈ 17

6. Conclusion

In this paper, the performance measures of a retrial queuing model are found in an Intuitionistic Fuzzy environment. The arrival, service and retrial rates are considered as TrIFN. The concepts of cut sets and interval arithmetic are applied to find the expected number of customers and their expected waiting time in the system. This approach would be appropriate to deal with retrial queues in an uncertain environment. Future work aims to apply the linguistic variables which would be effective in dealing with qualitative data.

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