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Near Plithogenic Neutrosophic Hypersoft Heronian Mean Aggregation Operators and their Application in Multiple Attribute Decision Making

Dhivya. E [1,a*], A. Arokia Lancy[2,b]

[1]Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India

[2]Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India

[a]*dhivyaeugene.14@gmail.com*, [b]*aarokia.lancy@gmail.com*

Abstract

In this paper, the concept of Near Plithogenic Neutrosophic Hypersoft number, its operational laws, score and accuracy functions are defined. Also, the Heronian Mean aggregation operator under Near Plithogenic Neutrosophic Hypersoft environment is studied. First the Near Plithogenic Neutrosophic Hypersoft Geometric Heronian mean operator (NPNHsGHM) is proposed and its desirable properties and special cases are investigated. Further, the Near Plithogenic Neutrosophic Hypersoft Geometric Weighted Heronian Mean operator (NPNHsGWHM) is defined and its properties are also studied. Then, the effectiveness of the developed approaches is verified with a numerical illustration.

Keywords Near Plithogenic Neutrosophic Hypersoft number; Near Plithogenic Neutrosophic Hypersoft score and accuracy function; Near Plithogenic Neutrosophic Hypersoft Heronian Mean; Near Plithogenic Neutrosophic Hypersoft Geometric Heronian Mean; Near Plithogenic Neutrosophic Hypersoft Geometric Weighted Heronian Mean

1. Introduction

James F. Peters [16] introduced the near sets as a generalization of rough sets introduced by Zdzislaw Pawlak [40] in 1982. The notion of soft sets was first commenced by Molodtsov [23] to deal with uncertainty. Chiranjibe Jana and Madhumangal Pal [17] gave some soft aggregation operators for single valued neutrosophic sets. Abhishek Guleria and Rakesh Kumar [4] used the aggregation operators for T-spherical fuzzy soft sets in decision making problems. Many authors [6,19,20,32] have contributed their work on soft aggregation operators

Neutrosophic sets are powerful logics designed to understand the inconsistent and indeterminate information. Wan et al. [38] introduced Frank Choquet Bonferroni mean operators and utilized this operator to develop MCDM problems in single-valued bipolar neutrosophic environment. Shi and Ye [35] introduced Dombi aggregation operator to originate neutrosophic cubic Dombi (NCD) aggregation functions. Wei and Zhang [39] utilized combination of power averaging and Bonferroni mean operator to develop SVN Bonferroni power aggregation operators. Ulucay et al. [37] developed a decision-making problem using similarity measure method under bipolar neutrosophic environment. Abdel-Basset et al. [2] studied MCGDM based on neutrosophic hierarchy method. Abdel-Basset et al. [1] proposed strategic planning and decision-making based on neutrosophic AHP-SWOT analysis. Dalapati et al. [10] proposed cross entropy based MAGDM based on interval neutrosophic information. In [7], Bausys and Zavadskas provided VIKOR method based MCDM problems using interval neutrosophic numbers. Biswas et al. [8] utilized TOPSIS method for MCDM problems under single valued neutrosophic environment.

The concept of hypersoft set was initiated by Florentin Smarandache [12]. He defines hypersoft set as a multi-augmented function, where one can have multiple parameters and so it can be used in several applications. Muhammad Saeed et al [27] contributed a development on complex multi-fuzzy hypersoft set based on entropy and similarity measure. Saqlain et al [28] proposed tangent similarity measure for single valued neutrosophic sets. Muhammad Naveed Jafar et al [25] gave the trigonometric similarity measures for neutrosophic hypersoft sets. Abdul Samad, et al [3] extended the TOPSIS technique based on correlation coefficient under neutrosophic hypersoft environment. Few authors [26,30,31] have contributed their work on hypersoft sets.

Florentin Smarandache [12,13,14] introduces the plithogenic set as a generalization of crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets. A plithogenic set is characterized by one or more parameters and each parameter may have several values. Shazia et al [34] constructed operators for plithogenic fuzzy whole hypersoft set and used it in multi attribute decision making technique. Abdel et al [22] proposed plithogenic TOPSIS-CRIRIC model for sustainable supply chain risk management. Abdel et al [21] proposed a hybrid plithogenic decision making approach. Nivetha et al[29] used the extended plithogenic hypersoft sets with dual dominant attributes in Covid-19 decision making model.

Aggregation operators are mathematical functions that combines 'n' numerical values to a single value. The Heronian mean is a mean type aggregation technique, which is developed to deal with the exact numerical values. Dejian Yu [11] gave the geometric heronian mean and geometric weighted heronian mean operators for intuitionistic fuzzy numbers. Z.Li et al[41] studied generalized heronian mean operators under Pythagorean fuzzy environment.

The paper is organized as follows. In Section 2, the basic definitions of the set and the heronian mean operator is given. In Section 3, Near Plithogenic Neutrosophic Hypersoft Number is defined and its operations laws are studied. In Section 4, the geometric Heronian mean operator for near plithogenic neutrosophic hypersoft numbers is proposed and its properties are discussed. In Section 5, the geometric weighted heronian mean operator for near plithogenic neutrosophic hypersoft numbers is introduced and its properties are studied. In Section 6, the developed approaches are verified with a numerical illustration.

2. Preliminaries

Definition 2.1[16] Let U be the Global (Universal) set of objects, A, B⊆U and Þ be the set of all functions representing object features (probe functions), D⊆ Þ. Sets A and B are said to be near if a ϵA, b ϵB and α_i ϵ D, $1 \leq \alpha \leq n$ and $a \sim_{\{\alpha i\}} b$.

Definition 2.2 [16] A nearness approximation space is a collection NAS= (U, Þ, $\sim_{Dq}$, Γ_q, $\zeta_{\Gamma q}$) where U represents the global set of objects, Þ denotes the probe functions, $\sim_{Dq}$ is the similarity relation on $D_q \subseteq D \subseteq$ Þ, Γ_q denotes the pile of partitions (collection of neighborhoods) and $\zeta_{\Gamma q}$ denotes the neighborhood overlap function.

The lower and upper near approximations of A with respect to NAS is given by,

$\underline{\Gamma q(D)}(A) = \cup_{a:[a]Dq \subseteq A} [a]Dq$ and

$\overline{\Gamma q(D)}(A) = \cup_{a:[a]Dq \cap A \neq \emptyset} [a]Dq$ respectively

The boundary of A with respect to NAS is given by, $B_{\Gamma q(D)}(A) = \overline{\Gamma q(D)}(A) - \underline{\Gamma q(D)}(A)$

If $B_{\Gamma q(D)}(A) \geq 0$, then A is a near set.[By Neighbourhoods Approximation Boundary Theorem].

Definition 2.3 [12] Let U be the global set of objects, P(U) the power set of U. Let $n_1, n_2, \ldots n_m$, $m \geq 1$ be the parameters whose values belong to the sets $N_1, N_2, \ldots N_m$ respectively and $N_i \cap N_j = \emptyset$, $i \neq j$, i, j ϵ{1,2,3,…n}. Then the set $(F, N_1 x N_2 x \ldots N_m)$ where $F: N_1 x N_2 x \ldots N_m \longrightarrow P(U)$ is the hypersoft set over U.

Definition 2.4 [13,14] Let U be the universal set of objects, A⊆U and let $n_1, n_2, \ldots n_m$, $m \geq 1$ be the parameters, R be the range of values of the parameter and among the range of parameter values, there is a dominant attribute value d which is the most essential value that one is interested in. Also, let d_a be the degree of appurtenance of each parameter value to the set A and d_c is the degree of contradiction between values of the parameter.

Then the tuple (A, n_m, R, d_a, d_c) is the plithogenic set.

Definition 2.5 Let U be the universal set of objects, A⊆U and P(U) the power set of U. Let $n_1,n_2,\ldots n_m$, m≥1 be the parameters whose values belong to the sets $N_1,N_2,\ldots N_m$ respectively and $N_i \cap N_j = \emptyset$, i ≠ j, i, j ϵ{1,2,3,…n}, R be the range of values of the parameter, d_a be the degree of appurtenance of each parameter value to the set A and d_c be the degree of contradiction between values of the parameter.. Then the set (F_p, $N_1 x N_2 x \ldots N_m$) where F_p: $N_1 x N_2 x \ldots N_m \longrightarrow P(U)$ is the plithogenic hypersoft set (PH_s) over U.

Definition 2.6 Let U be the universal set of objects, A⊆U, **Ω** be a plithogenic hypersoft set whose degree of appurtenance of each parameter is a neutrosophic set over U and NAS= (U, Þ, $\sim_{Dq}$, Γ_q, $\zeta_{\Gamma q}$) be the nearness approximation space. The lower and upper near approximations of **Ω** with respect to NAS is given by,

$\underline{\Gamma q(D)}(\Omega) = \bigcup_{a:[a]Dq \subseteq \Omega} [a]Dq$ and

$\overline{\Gamma q(D)}(\Omega) = \bigcup_{a:[a]Dq \cap \Omega \neq \emptyset} [a]Dq$

respectively. The boundary of Ω with respect to NAS is given by, $B_{\Gamma q(D)}(\Omega) = \overline{\Gamma q(D)}(\Omega) - \underline{\Gamma q(D)}(\Omega)$. If $\underline{\Gamma q(D)}(\Omega) \neq \emptyset$ and $B_{\Gamma q(D)}(\Omega) \geq 0$, then Ω is a near plithogenic neutrosophic hypersoft set.

Definition 2.7 [42] Let a_i (i=1,2,…n) be a collection of non-negative numbers. If

$HM(a_1,a_2,\ldots a_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \sqrt{a_i a_j}$

then HM is called the heronian mean.

Definition 2.8 [11] Let p,q ≥ 0 and p, q do not take the value 0 simultaneously and a_i (i=1,2,…n) be a collection of non-negative numbers. If

$GHM^{p,q}(a_1,a_2,\ldots a_n) = \frac{1}{p+q} \left(\prod_{i=1,j=i}^{n} (pa_i + qa_j)^{\frac{2}{n(n+1)}}\right)$

then GHM is called the geometric heronian mean.

3. Near Plithogenic Neutrosophic Hypersoft Numbers

Definition 3.1

For two Near Plithogenic Neutrosophic Hypersoft Numbers (NPNHsN) F_{K_1} and F_{K_2} some operational laws are given as follows,

1. $F_{K_1} \oplus F_{K_2} = <(1-c_i)[T_{F_{K_1}} + T_{F_{K_2}} - T_{F_{K_1}} T_{F_{K_2}}, I_{F_{K_1}} I_{F_{K_2}}, F_{F_{K_1}} F_{F_{K_2}}] + c_i[T_{F_{K_1}} T_{F_{K_2}}, I_{F_{K_1}} + I_{F_{K_2}} - I_{F_{K_1}} I_{F_{K_2}}, F_{F_{K_1}} + F_{F_{K_2}} - F_{F_{K_1}} F_{F_{K_2}}]>$
2. $F_{K_1} \otimes F_{K_2} = <(1-c_i)[T_{F_{K_1}} T_{F_{K_2}}, I_{F_{K_1}} + I_{F_{K_2}} - I_{F_{K_1}} I_{F_{K_2}}, F_{F_{K_1}} + F_{F_{K_2}} - F_{F_{K_1}} F_{F_{K_2}}] + c_i[T_{F_{K_1}} + T_{F_{K_2}} - T_{F_{K_1}} T_{F_{K_2}}, I_{F_{K_1}} I_{F_{K_2}}, F_{F_{K_1}} F_{F_{K_2}}]>$
3. $\lambda F_{K_1} = <(1-c_i)[1-(1-T_{F_{K_1}})^{\lambda}, I_{F_{K_1}}^{\lambda}, F_{F_{K_1}}^{\lambda}] + c_i [T_{F_{K_1}}^{\lambda}, 1-(1-I_{F_{K_1}})^{\lambda}, 1-(1-F_{F_{K_1}})^{\lambda}]>; \lambda>0$
4. $F_{K_1}^{\lambda} = <(1-c_i)[T_{F_{K_1}}^{\lambda}, 1-(1-I_{F_{K_1}})^{\lambda}, 1-(1-F_{F_{K_1}})^{\lambda}] + c_i [1-\left(1-T_{F_{K_1}}\right)^{\lambda}, I_{F_{K_1}}^{\lambda}, F_{F_{K_1}}^{\lambda}]>; \lambda>0$

Definition 3.2

Let $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ be a NPNHsN, then the score function is defined as

$S(F_{K_i}) = \frac{1}{3}\left(2 + \frac{\sum_{i=1}^{n} T_{F_{K_i}}}{n} - \frac{\sum_{i=1}^{n} I_{F_{K_i}}}{n} - \frac{\sum_{i=1}^{n} F_{F_{K_i}}}{n}\right)$

Definition 3.3

The accuracy function of a NPNHsN, $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ is defined as,

$H(F_k) = \frac{\sum_{i=1}^{n} T_{F_{K_i}}}{n} - \frac{\sum_{i=1}^{n} F_{F_{K_i}}}{n}$

Definition 3.4

Based on the score function and accuracy function, the order relation on two NPNHsNs F_{K_1} and F_{K_2} are defined as,

I. If $S(F_{K_1}) < S(F_{K_2})$, then $F_{K_1} < F_{K_2}$
II. If $S(F_{K_1}) > S(F_{K_2})$, then $F_{K_1} > F_{K_2}$
III. If $S(F_{K_1}) = S(F_{K_2})$, then

i. If $H(F_{K_1}) < H(F_{K_2})$, then $F_{K_1} < F_{K_2}$
ii. If $H(F_{K_1}) > H(F_{K_2})$, then $F_{K_1} > F_{K_2}$
iii. If $H(F_{K_1}) = H(F_{K_2})$, then $F_{K_1} \sim F_{K_2}$

4. Near Plithogenic Neutrosophic Hypersoft Geometric Heronian Mean

Definition 4.1

Let F_{K_i} be a collection of NPNHsNs, then

$NPNHsGHM^{u,v}(F_{K_1}, F_{K_2}, \ldots F_{K_n}) = \frac{1}{u+v}\left(\otimes_{i=1,j=i}^{n} (uF_{K_i} + vF_{K_j})^{2/n(n+1)}\right)$

Theorem 4.2

The aggregated value by using NPNHsGHM is also a NPNHsN, where

$(1 - c_i)\,[1-(1-\prod_{i=1,j=i}^{n}(1-(1-T_{F_{K_i}})^{u}\,(1-T_{F_{K_j}})^{v})^{2/n(n+1)})^{1/u+v}, (1-\prod_{i=1,j=i}^{n}(1-I_{F_{K_i}}{}^{u}\, I_{F_{K_j}}{}^{v})^{2/n(n+1)})^{1/u+v}, (1-\prod_{i=1,j=i}^{n}(1-F_{F_{K_i}}{}^{u}\, F_{F_{K_j}}{}^{v})^{2/n(n+1)})^{1/u+v}] +$

$c_i[(1-\prod_{i=1,j=i}^{n}(1-T_{F_{K_i}}{}^{u}\, T_{F_{K_j}}{}^{v})^{2/n(n+1)})^{1/u+v}, 1-(1-\prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}})^{u}\,(1-I_{F_{K_j}})^{v})^{2/n(n+1)})^{1/u+v}, 1-(1-\prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}})^{u}\,(1-F_{F_{K_j}})^{v})^{2/n(n+1)})^{1/u+v}]$ …..(I)

$NPNHsGHM^{u,v}(F_{K_1}, F_{K_2}, \ldots F_{K_n}) =$

Proof

From the definition (3.1) of the operational laws,

$uF_{K_i} = (1 - c_i)[1-(1-T_{F_{K_i}})^{u}, I_{F_{K_i}}{}^{u}, F_{F_{K_i}}{}^{u}] + c_i[T_{F_{K_i}}{}^{u}, 1-\left(1-I_{F_{K_i}}\right)^{u}, 1-(1-F_{F_{K_i}})^{u}]$ …...(1)

$vF_{K_j} = (1 - c_i)(1-(1-T_{F_{K_j}})^{v}, I_{F_{K_j}}{}^{v}, F_{F_{K_j}}{}^{v}) + c_i[T_{F_{K_j}}{}^{v}, 1-\left(1-I_{F_{K_j}}\right)^{v}, 1-(1-F_{F_{K_j}})^{v}]$……(2)

$uF_{K_i} \oplus vF_{K_j} = (1 - c_i)\,[1-(1-T_{F_{K_i}})^{u}(1-T_{F_{K_j}})^{v}, I_{F_{K_i}}{}^{u} I_{F_{K_j}}{}^{v}, F_{F_{K_i}}{}^{u} F_{F_{K_j}}{}^{v}] + c_i[T_{F_{K_i}}{}^{u} T_{F_{K_j}}{}^{v}, 1-\left(1-I_{F_{K_i}}\right)^{u}\left(1-I_{F_{K_j}}\right)^{v}, 1-(1-F_{F_{K_i}})^{u}(1-F_{F_{K_j}})^{v}]$ …….(3)

Then, $(uF_{K_i} \oplus vF_{K_j})^{2/(n(n+1)} = (1 - c_i)\,[\,(1-(1-T_{F_{K_i}})^u(1-T_{F_{K_j}})^v)^{2/(n(n+1)}, 1-(1-I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{2/(n(n+1)}, 1-(1-F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{2/(n(n+1)}] + c_i[1-\left(1-T_{F_{K_i}}{}^u T_{F_{K_j}}{}^v\right)^{2/(n(n+1)}, (1-(1-I_{F_{K_i}})^u\left(1-I_{F_{K_j}}\right)^v)^{2/(n(n+1)}, (1-(1-F_{F_{K_i}})^u(1-F_{F_{K_j}})^v)^{2/(n(n+1)}]$ …….(4)

$\otimes_{i=1,j=i}^{n}(uF_{K_i} \oplus vF_{K_j})^{2/(n(n+1)} = (1 - c_i)(\prod_{i=1,j=i}^{n}(1-(1-T_{F_{K_i}})^u(1-T_{F_{K_j}})^v)^{2/n(n+1)}, 1-\prod_{i=1,j=i}^{n}(1-I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{2/n(n+1)}, 1-\prod_{i=1,j=i}^{n}(1-F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{2/n(n+1)}) + c_i[1-\prod_{i=1,j=i}^{n}(1-T_{F_{K_i}}{}^u T_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}}, \prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}})^u\left(1-I_{F_{K_j}}\right)^v)^{\frac{2}{n(n+1)}}, \prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}})^u(1-F_{F_{K_j}})^v)^{2/n(n+1)}]$ ……..(5)

Then NPNHsGHM$^{u,v}(F_{K_1}, F_{K_2}, \dots F_{K_n}) = \frac{1}{u+v}(\otimes_{i=1,j=i}^{n}(uF_{K_i} + vF_{K_j})^{2/n(n+1)})$

$(1 - c_i)\,[1-(1-\prod_{i=1,j=i}^{n}(1-(1-T_{F_{K_i}})^u(1-T_{F_{K_j}})^v)^{2/n(n+1)})^{1/u+v}, (1-\prod_{i=1,j=i}^{n}(1-I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{2/n(n+1)})^{1/u+v},$

$(1-\prod_{i=1,j=i}^{n}(1-F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{2/n(n+1)})^{1/u+v}] +$

$c_i[(1-\prod_{i=1,j=i}^{n}(1-T_{F_{K_i}}{}^u T_{F_{K_j}}{}^v)^{2/n(n+1)})^{1/u+v}, 1-(1-\prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}})^u(1-I_{F_{K_j}})^v)^{2/n(n+1)})^{1/u+v}, 1-(1-\prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}})^u(1-F_{F_{K_j}})^v)^{2/n(n+1)})^{1/u+v}]$

=

which completes the proof of the theorem.

Theorem 4.3 (Idempotency)

If all F_{K_i}, $i = 1,2,\dots,n$ are equal, i.e., $F_{K_i} = F_k = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \dots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ for all i, l=1,2,…,n, u,v ≥ 0 and u, v do not take the value 0 simultaneously. Then,

NPNHsGHM$^{u,v}(F_{K_1}, F_{K_2}, \dots F_{K_n})$ = NPNHsGHM$^{u,v}(F_K, F_K, \dots F_K) = F_K$.

Proof

Since all F_{K_i} are equal,

$F_{K_i} = F_k = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \dots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$, ∀ i, then

NPNHsGHM$^{u,v}(F_{K_1}, F_{K_2}, \dots F_{K_n})$ = NPNHsGHM$^{u,v}(F_K, F_K, \dots F_K)$

$= \frac{1}{u+v}(\otimes_{i=1,j=i}^{n}(uF_K + vF_K)^{2/n(n+1)})$

$= \frac{1}{u+v}(\otimes_{i=1,j=i}^{n}((u+v)F_K)^{2/n(n+1)}$

$= \frac{1}{u+v}((u+v)F_K)^{2/n(n+1)/(2/n(n+1))}$

$= F_K$

Theorem 4.4 (Monotonicity)

Let u,v ≥ 0 and u, v do not take the value 0 simultaneously, $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}),\ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ and $G_{K_i} = b_l[(T_{\beta_1}, I_{\beta_1}, F_{\beta_1}), (T_{\beta_2}, I_{\beta_2}, F_{\beta_2}),\ldots (T_{\beta_n}, I_{\beta_n}, F_{\beta_n})]$ be two collections of NPNHsNs . If $T_{F_{K_i}} \le T_{G_{K_i}}$, $I_{F_{K_i}} \ge I_{G_{K_i}}$, $F_{F_{K_i}} \ge F_{G_{K_i}}$, then

$NPNHsGHM^{u,v}(F_{K_1}, F_{K_2}, \ldots F_{K_n}) = NPNHsGHM^{u,v}(G_{K_1}, G_{K_2}, \ldots G_{K_n})$

Proof

Since $T_{F_{K_i}} \le T_{G_{K_i}}$, $I_{F_{K_i}} \ge I_{G_{K_i}}$, $F_{F_{K_i}} \ge F_{G_{K_i}}$, ∀ i, then

$(1 - T_{F_{K_i}})^u(1 - T_{F_{K_j}})^v \ge (1 - T_{G_{K_i}})^u(1 - T_{G_{K_j}})^v$, $I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v \ge I_{G_{K_i}}{}^u I_{G_{K_j}}{}^v$, and $F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v \ge F_{G_{K_i}}{}^u F_{G_{K_j}}{}^v$ …….(6)

From (6)

$1-(1 - T_{F_{K_i}})^u(1 - T_{F_{K_j}})^v \le (1 - T_{G_{K_i}})^u(1 - T_{G_{K_j}})^v$, $1 - I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v \le 1 - I_{G_{K_i}}{}^u I_{G_{K_j}}{}^v$, and $1 - F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v \le 1 - F_{G_{K_i}}{}^u F_{G_{K_j}}{}^v$ …….(7)

$\prod_{i=1,j=i}^{n}(1 - (1 - T_{F_{K_i}})^u \left(1 - T_{F_{K_j}}\right)^v)^{\frac{2}{n(n+1)}} \le \prod_{i=1,j=i}^{n}(1 - (1 - T_{G_{K_i}})^u \left(1 - T_{G_{K_j}}\right)^v)^{\frac{2}{n(n+1)}}$,

$\prod_{i=1,j=i}^{n}(1 - I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}} \le \prod_{i=1,j=i}^{n}(1 - I_{G_{K_i}}{}^u I_{G_{K_j}}{}^v)^{\frac{2}{n(n+1)}}$ and

$\prod_{i=1,j=i}^{n}(1 - F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}} \le \prod_{i=1,j=i}^{n}(1 - F_{G_{K_i}}{}^u F_{G_{K_j}}{}^v)^{\frac{2}{n(n+1)}}$ ……(8)

$(1 - \prod_{i=1,j=i}^{n}(1 - (1 - T_{F_{K_i}})^u \left(1 - T_{F_{K_j}}\right)^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}} \ge (1 - \prod_{i=1,j=i}^{n}(1 - (1 - T_{G_{K_i}})^u \left(1 - T_{G_{K_j}}\right)^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$,

$(1 - \prod_{i=1,j=i}^{n}(1 - I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}} \le (1 - \prod_{i=1,j=i}^{n}(1 - I_{G_{K_i}}{}^u I_{G_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$ and

$(1 - \prod_{i=1,j=i}^{n}(1 - F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}} \le (1 - \prod_{i=1,j=i}^{n}(1 - F_{G_{K_i}}{}^u F_{G_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$ ……(9)

Also,

$1 - (1 - \prod_{i=1,j=i}^{n}(1 - (1 - T_{F_{K_i}})^u \left(1 - T_{F_{K_j}}\right)^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}} \ge 1 - (1 - \prod_{i=1,j=i}^{n}(1 - (1 - T_{G_{K_i}})^u \left(1 - T_{G_{K_j}}\right)^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$,

$(1 - \prod_{i=1,j=i}^{n}(1 - I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}} \le (1 - \prod_{i=1,j=i}^{n}(1 - I_{G_{K_i}}{}^u I_{G_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$ and

$(1 - \prod_{i=1,j=i}^{n}(1 - F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}} \le (1 - \prod_{i=1,j=i}^{n}(1 - F_{G_{K_i}}{}^u F_{G_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$ ……(10)

$(1 - c_i)[1-(1 - \prod_{i=1,j=i}^{n}(1 - (1 - T_{F_{K_i}})^u \left(1 - T_{F_{K_j}}\right)^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}, (1 - \prod_{i=1,j=i}^{n}(1 - I_{F_{K_i}}{}^u I_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}, (1 - \prod_{i=1,j=i}^{n}(1 - F_{F_{K_i}}{}^u F_{F_{K_j}}{}^v)^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}] \le (1 - c_i)[1- (1 - \prod_{i=1,j=i}^{n}(1 - (1 - T_{G_{K_i}})^u \Big(1 -$

$T_{G_{K_j}} \Big)^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}, (1-\prod_{i=1,j=i}^{n}(1-I_{G_{K_i}}{}^{u}I_{G_{K_j}}{}^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}, (1-\prod_{i=1,j=i}^{n}(1-F_{G_{K_i}}{}^{u}F_{G_{K_j}}{}^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$] ……(11)

Similarly, we can prove,

$c_i\,[(1-\prod_{i=1,j=i}^{n}(1-T_{F_{K_i}}{}^{u}T_{F_{K_j}}{}^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$, 1-$(1-\prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}})^{u}\Big(1-I_{F_{K_j}}\Big)^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$, , $1-(1-\prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}})^{u}\Big(1-F_{F_{K_j}}\Big)^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$] $\leq$ $c_i\,[(1-\prod_{i=1,j=i}^{n}(1-T_{G_{K_i}}{}^{u}T_{G_{K_j}}{}^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$, 1-$(1-\prod_{i=1,j=i}^{n}(1-(1-I_{G_{K_i}})^{u}\Big(1-I_{G_{K_j}}\Big)^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$, 1- $(1-\prod_{i=1,j=i}^{n}(1-(1-F_{G_{K_i}})^{u}\Big(1-F_{G_{K_j}}\Big)^{v})^{\frac{2}{n(n+1)}})^{\frac{1}{u+v}}$] ……(12)

Theorem 4.5 (Permutation)

Let $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}),\dots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ be a collection of NPNHsNs, then, NPNHsGHM^{u,v} $(F_{K_1}, F_{K_2}, \dots F_{K_n})$ = NPNHsGHM^{u,v} $(\dot{F}_{K_1}, \dot{F}_{K_2}, \dots \dot{F}_{K_n})$ where $(\dot{F}_{K_1}, \dot{F}_{K_2}, \dots \dot{F}_{K_n})$ is any permutation of $(F_{K_1}, F_{K_2}, \dots F_{K_n})$.

Proof

Since $(\dot{F}_{K_1}, \dot{F}_{K_2}, \dots \dot{F}_{K_n})$ is any permutation of $(F_{K_1}, F_{K_2}, \dots F_{K_n})$,

$$\text{NPNHsGHM}^{u,v}(F_{K_1}, F_{K_2}, \dots F_{K_n}) = \frac{1}{u+v}\left(\otimes_{i=1,j=i}^{n}(uF_{K_i}+vF_{K_j})^{2/n(n+1)}\right)$$

$$= \frac{1}{u+v}\left(\otimes_{i=1,j=i}^{n}(u\dot{F}_{K_i}+v\dot{F}_{K_j})^{2/n(n+1)}\right)$$

$$= \text{NPNHsGHM}^{u,v}(\dot{F}_{K_1}, \dot{F}_{K_2}, \dots \dot{F}_{K_n})$$

Theorem 4.6 (Boundary)

Let $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}),\dots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ be a collection of NPNHsNs and

$$F_K^- = (\min_i\{T_{F_{K_i}}\}, \max_i\{I_{F_{K_i}}\}, \max_i\{F_{F_{K_i}}\})$$

$$F_K^+ = (\max_i\{T_{F_{K_i}}\}, \min_i\{I_{F_{K_i}}\}, \min_i\{F_{F_{K_i}}\})$$

Then $F_K^- \leq \text{NPNHsGHM}^{u,v}(F_{K_1}, F_{K_2}, \dots F_{K_n}) \leq F_K^+$ which can be obtained by monotonicity.

By imposing different values to u and v, some special cases of NPNHsGHM is obtained.

Case 1: If u=v=$\frac{1}{2}$, then NPNHsGHM reduces to

$$\text{NPNHsGHM}^{\frac{1}{2},\frac{1}{2}}(F_{K_1}, F_{K_2}, \dots F_{K_n}) = (1-c_i)\Big[\prod_{i=1,j=i}^{n}\Big(1-\sqrt{\Big(1-T_{F_{K_i}}\Big)(1-T_{F_{K_j}})}\Big)^{\frac{2}{n(n+1)}}\Big),\ 1-\prod_{i=1,j=i}^{n}\Big(1-\sqrt{I_{F_{K_i}}I_{F_{K_j}}}\Big)^{\frac{2}{n(n+1)}}, 1-\prod_{i=1,j=i}^{n}\Big(1-\sqrt{F_{F_{K_i}}F_{F_{K_j}}}\Big)^{\frac{2}{n(n+1)}}\Big] + c_i\Big[1-\prod_{i=1,j=i}^{n}\Big(1-\sqrt{T_{F_{K_i}}T_{F_{K_j}}}\Big)^{\frac{2}{n(n+1)}}, \prod_{i=1,j=i}^{n}\Big(1-\sqrt{\Big(1-I_{F_{K_i}}\Big)(1-I_{F_{K_j}})}\Big)^{\frac{2}{n(n+1)}}\Big), \prod_{i=1,j=i}^{n}\Big(1-\sqrt{\Big(1-F_{F_{K_i}}\Big)(1-F_{F_{K_j}})}\Big)^{\frac{2}{n(n+1)}}\Big)\Big]\ \Big]$$

Case 2: If v→0, then

$\lim_{v\to 0} \text{NPNHsGHM}^{u,v} (F_{K_1}, F_{K_2}, \dots F_{K_n}) = \frac{1}{u} (\sum_{i=1}^{n} (uF_{K_i})^{\frac{1}{n}})$

$= (1\text{-}c_i)[(1\text{-}(1-\prod_{i=1}^{n}(1-(1-T_{F_{K_i}})^u)^{\frac{1}{n}})^{\frac{1}{u}}, (1-\prod_{i=1}^{n}(1-I_{F_{K_i}}{}^u)^{\frac{1}{n}})^{\frac{1}{u}}, (1-\prod_{i=1}^{n}(1-F_{F_{K_i}}{}^u)^{\frac{1}{n}})^{\frac{1}{u}}) + c_i[(1-\prod_{i=1}^{n}(1-T_{F_{K_i}}{}^u)^{\frac{1}{n}})^{\frac{1}{u}}), (1\text{-}(1-\prod_{i=1}^{n}(1-(1-I_{F_{K_i}})^u)^{\frac{1}{n}})^{\frac{1}{u}}, (1-(1-\prod_{i=1}^{n}(1-(1-F_{F_{K_i}})^u)^{\frac{1}{n}})^{\frac{1}{u}}]$

Case 3: If u=1 and v→0, then

$$\text{NPNHsGHM}^{1,0}(F_{K_1}, F_{K_2}, \dots F_{K_n}) = \sum_{i=1}^{n} (F_{K_i})^{\frac{1}{n}}$$

$$= (1\text{-}c_i)[\prod_{i=1}^{n} T_{F_{K_i}}{}^{\frac{1}{n}}, 1-\prod_{i=1}^{n} I_{F_{K_i}}{}^{\frac{1}{n}}, 1-\prod_{i=1}^{n} F_{F_{K_i}}{}^{\frac{1}{n}})] + c_i[1-\prod_{i=1}^{n} T_{F_{K_i}}{}^{\frac{1}{n}}, \prod_{i=1}^{n} I_{F_{K_i}}{}^{\frac{1}{n}}, \prod_{i=1}^{n} F_{F_{K_i}}{}^{\frac{1}{n}}]$$

5. Near Plithogenic Neutrosophic Hypersoft Geometric Weighted Heronian Mean

Definition 5.1

Let F_{K_i} be a collection of NPNHsNs, w = $(w_1, w_2, \dots, w_n)^T$ is the weight vector of F_{K_i} where w_i indicates the importance degree of F_{K_i} satisfying $w_i \geq 0$, i=1,2,…n and $\sum_{i=1}^{n} w_i = 1$. Then,

$\text{NPNHsGWHM}_w^{u,v}(F_{K_1}, F_{K_2}, \dots F_{K_n}) = \frac{1}{u+v} (\otimes_{i=1,j=i}^{n} ((uF_{K_i})^{w_i} \oplus (vF_{K_j})^{w_i})^{2/n(n+1)}$

Theorem 5.2

The aggregated value by using NPNHsGWHM is also a NPNHsN, where

$\text{NPNHsGWHM}_w^{u,v} (F_{K_1}, F_{K_2}, \dots F_{K_n}) = (1\text{-}c_i) [(1\text{-}(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-T_{F_{K_i}}{}^{w_i})^u ((1-T_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}\prod_{i=1,j=i}^{n}(1-(1-\left(1-I_{F_{K_i}}\right)^{w_i})^u (1-(1-I_{F_{K_j}})^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}\prod_{i=1,j=i}^{n}(1-(1-\left(1-F_{F_{K_i}}\right)^{w_i})^u (1-(1-F_{F_{K_j}})^{w_j})^v)^{2/n(n+1)})^{1/u+v}] + c_i [(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-\left(1-T_{F_{K_i}}\right)^{w_i})^u (1-(1-T_{F_{K_j}})^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}}{}^{w_i})^u ((1-I_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}}{}^{w_i})^u ((1-F_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)})^{1/u+v}]$ ……(II)

Proof

From the definition (3.1) of the operational laws,

$uF_{K_i} = (1-c_i)[(1-(1-T_{F_{K_i}})^u, I_{F_{K_i}}{}^u, F_{F_{K_i}}{}^u)] + c_i[T_{F_{K_i}}{}^u, (1-\left(1-I_{F_{K_i}}\right)^u, (1-(1-F_{F_{K_i}})^u]$ ……(13)

$vF_{K_j} = (1-c_i)[(1-(1-T_{F_{K_j}})^v, I_{F_{K_j}}{}^v, F_{F_{K_j}}{}^v)] + c_i[T_{F_{K_j}}{}^v, (1-\left(1-I_{F_{K_j}}\right)^v, (1-(1-F_{F_{K_j}})^v]$…….(14)

$(uF_{K_i})^{w_i} = (1-c_i)[(1-(1-T_{F_{K_i}}{}^{w_i})^u, (1\text{-}(1\text{-}I_{F_{K_i}})^{w_i})^u, (1\text{-}(1\text{-}F_{F_{K_i}})^{w_i})^u)] + c_i[(1-\left(1-T_{F_{K_i}})^{w_i})^u\right), (1-\left(1-I_{F_{K_i}}{}^{w_i}\right)^u, (1-\left(1-F_{F_{K_i}}{}^{w_i}\right)^u]$……(15)

$(vF_{K_j})^{w_j} = (1-c_i)[(1-(1-T_{F_{K_j}}{}^{w_j})^v, (1\text{-}(1\text{-}I_{F_{K_j}})^{w_j})^v, (1\text{-}(1\text{-}F_{F_{K_j}})^{w_j})^v)] + c_i[(1\text{-}(1\text{-}T_{F_{K_j}})^{w_j})^v), (1-(1-I_{F_{K_j}}{}^{w_j})^v, (1-(1-F_{F_{K_j}}{}^{w_j})^v]$ ……(16)

$(uF_{K_i})^{w_i} \oplus (vF_{K_j})^{w_j} = (1-c_i)[(1-(1-T_{F_{K_i}}{}^{w_i})^u(1-T_{F_{K_j}}{}^{w_j})^v, (1\text{-}(1\text{-}I_{F_{K_i}})^{w_i})^u(1-(1-I_{F_{K_j}})^{w_j})^v, (1-(1-F_{F_{K_i}})^{w_i})^u)(1-(1-F_{F_{K_j}})^{w_j})^v))] + c_i[(1-(1-T_{F_{K_i}})^{w_i})^u)(1-(1-T_{F_{K_j}})^{w_j})^v)), (1-(1-I_{F_{K_i}}{}^{w_i})^u(1-I_{F_{K_j}}{}^{w_j})^v, (1-(1-F_{F_{K_i}}{}^{w_i})^u(1-F_{F_{K_j}}{}^{w_j})^v]$ …. (17)

Then, $((uF_{K_i})^{w_i} \oplus (vF_{K_j})^{w_j})^{2/(n(n+1))} = (1-c_i)[(1-(1-T_{F_{K_i}}{}^{w_i})^u(1-T_{F_{K_j}}{}^{w_j})^v)^{2/(n(n+1))}, 1-(1-(1-(1-I_{F_{K_i}})^{w_i})^u(1-(1-I_{F_{K_j}})^{w_j})^v)^{2/(n(n+1))}, 1-(1-(1-(1-F_{F_{K_i}})^{w_i})^u(1-(1-F_{F_{K_j}})^{w_j})^v)^{2/(n(n+1))}] + c_i[1-(1-(1-(1-T_{F_{K_i}})^{w_i})^u(1-(1-T_{F_{K_j}})^{w_j})^v)^{2/(n(n+1))}, (1-(1-I_{F_{K_i}}{}^{w_i})^u(1-I_{F_{K_j}}{}^{w_j})^v)^{2/(n(n+1))}, (1-(1-F_{F_{K_i}}{}^{w_i})^u(1-F_{F_{K_j}}{}^{w_j})^v)^{2/(n(n+1))}]$……..(18)

$\otimes_{i=1,j=i}^{n}((uF_{K_i})^{w_i} \oplus (vF_{K_j})^{w_j})^{2/(n(n+1))} = (1-c_i)[\prod_{i=1,j=i}^{n}(1-(1-T_{F_{K_i}}{}^{w_i})^u(1-T_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)}, 1\text{-}\prod_{i=1,j=i}^{n}(1-(1-(1-I_{F_{K_i}})^{w_i})^u(1-(1-I_{F_{K_j}})^{w_j})^v)^{2/n(n+1)}, 1\text{-}\prod_{i=1,j=i}^{n}(1-(1-(1-F_{F_{K_i}})^{w_i})^u(1-(1-F_{F_{K_j}})^{w_j})^v)^{2/n(n+1)}] + c_i[1\text{-}\prod_{i=1,j=i}^{n}(1-(1-(1-T_{F_{K_i}})^{w_i})^u(1-(1-T_{F_{K_j}})^{w_j})^v)^{2/n(n+1)}, \prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}}{}^{w_i})^u(1-I_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)}, \prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}}{}^{w_i})^u(1-F_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)}]$……..(19)

Then $NPNHsGWHM_w^{u,v}(F_{K_1}, F_{K_2}, \ldots F_{K_n}) =$ $(1\text{-}c_i)[(1\text{-}(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-T_{F_{K_i}}{}^{w_i})^u((1-T_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}\prod_{i=1,j=i}^{n}(1-(1-\left(1-I_{F_{K_i}}\right)^{w_i})^u(1-(1-I_{F_{K_j}})^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}\prod_{i=1,j=i}^{n}(1-(1-\left(1-F_{F_{K_i}}\right)^{w_i})^u(1-(1-F_{F_{K_j}})^{w_j})^v)^{2/n(n+1)})^{1/u+v}] + c_i[(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-\left(1-T_{F_{K_i}}\right)^{w_i})^u(1-(1-T_{F_{K_j}})^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-I_{F_{K_i}}{}^{w_i})^u((1-I_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)})^{1/u+v}, (1\text{-}(1\text{-}\prod_{i=1,j=i}^{n}(1-(1-F_{F_{K_i}}{}^{w_i})^u((1-F_{F_{K_j}}{}^{w_j})^v)^{2/n(n+1)})^{1/u+v}]$

Theorem 5.3 (Idempotency)

If all F_{K_i} , $i = 1,2, \ldots, n$ are equal, i.e., $F_{K_i} = F_k = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ for all i, l=1,2,..n, u,v ≥ 0 and u, v do not take the value 0 simultaneously and $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of F_{K_i} where w_i indicates the importance degree of F_{K_i} satisfying $w_i \geq 0$, i=1,2,…n and $\sum_{i=1}^{n} w_i = 1$. Then,

$\text{NPNHsGWHM}_w^{u,v}$ $(F_{K_1}, F_{K_2}, \ldots F_{K_n})$ = $\text{NPNHsGWHM}_w^{u,v}$ $(F_K, F_K, \ldots F_K)$ = F_K.

Theorem 5.4 (Monotonicity)

Let u,v ≥ 0 and u, v do not take the value 0 simultaneously, $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ and $G_{K_i} = b_l[(T_{\beta_1}, I_{\beta_1}, F_{\beta_1}), (T_{\beta_2}, I_{\beta_2}, F_{\beta_2}), \ldots (T_{\beta_n}, I_{\beta_n}, F_{\beta_n})]$ be the collection of NPNHsNs . If $T_{F_{K_i}} \leq T_{G_{K_i}}$, $I_{F_{K_i}} \geq I_{G_{K_i}}$, $F_{F_{K_i}} \geq F_{G_{K_i}}$, then

$\text{NPNHsGWHM}_w^{u,v}$ $(F_{K_1}, F_{K_2}, \ldots F_{K_n}) \leq$ $\text{NPNHsGWHM}_w^{u,v}$ $(G_{K_1}, G_{K_2}, \ldots G_{K_n})$

Theorem 5.5 (Permutation)

Let $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ be a collection of NPNHsNs, then,

$\text{NPNHsGWHM}_w^{u,v}$ $(F_{K_1}, F_{K_2}, \ldots F_{K_n})$ = $\text{NPNHsGWHM}_w^{u,v}$ $(\dot{F}_{K_1}, \dot{F}_{K_2}, \ldots \dot{F}_{K_n})$ where $(\dot{F}_{K_1}, \dot{F}_{K_2}, \ldots \dot{F}_{K_n})$ is any permutation of $(F_{K_1}, F_{K_2}, \ldots F_{K_n})$.

Theorem 5.6 (Boundary)

Let $F_{K_i} = a_l[(T_{\alpha_1}, I_{\alpha_1}, F_{\alpha_1}), (T_{\alpha_2}, I_{\alpha_2}, F_{\alpha_2}), \ldots (T_{\alpha_n}, I_{\alpha_n}, F_{\alpha_n})]$ be a collection of NPNHsNs and

$$F_K^- = (\min_i \{T_{F_{K_i}}\}, \max_i \{I_{F_{K_i}}\}, \max_i \{F_{F_{K_i}}\})$$
$$F_K^+ = (\max_i \{T_{F_{K_i}}\}, \min_i \{I_{F_{K_i}}\}, \min_i \{F_{F_{K_i}}\})$$

Then $F_K^- \leq$ $\text{NPNHsGWHM}_w^{u,v}$ $(F_{K_1}, F_{K_2}, \ldots F_{K_n}) \leq F_K^+$ which can be obtained by monotonicity.

6. Models for MADM with Near Plithogenic Neutrosophic Hypersoft numbers

In this section we propose the model for MADM with NPNHsNs based on NPNHsGHM (NPNHsGWHM) operator. Let $A=\{A_1,A_2,\ldots,A_m\}$ be the set of alternatives and $E=E_1,E_2,\ldots,E_n$ be the set of attributes and each attributes have their attribute values $(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n)$, $w_i=(w_1,w_2,\ldots,w_n)$ be the weight vector of the $(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n)$ where $w_i \geq 0$, i=1,2,…n and $\sum_{i=1}^{n} w_i = 1$. The decision matrix is obtained as D= $\left(f_{k_{ij}}\right)_{mxn}$. The steps of the decision making based on NPNHsNs are given as follows:

6.1 Algorithm

Step 1 The Decision Makers take their analysis of each alternative based on assumed criteria. The values are taken in the form of matrix.

Step 2 Calculate the NPNHsGHM NPNHsGWHM) of alternatives using (I) and (II)

Step 3 Calculate the scores to rank the alternatives. If there is no difference between two scores, then the accuracy function must be calculated and then the alternatives are ranked accordingly.

Step 4 Rank all the alternatives and select the best alternative.

Step 5 End

6.2 Numerical Illustration

Using the developed approaches, we illustrate an example for supplier selection.

A company wants to select an appropriate supplier according to their requirements. A group of three decision makers D_1, D_2 and D_3 will judge the suppliers A_1, A_2 and A_3 on the basis of certain parameters E={Production cost, Production quality, Service} and their corresponding attribute values {High, Reasonable, Low}, {1st class, 2nd class, 3rd class} and {Very good, Good, Medium, Poor}. Here the attributes (High,1st class, Very good), (Reasonable, 2nd class, Good) (Low, 2nd class, Good) are considered and the values are given in terms of near plithogenic neutrosophic numbers.

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	(0.5,0.6,0.3) (0.7,0.3,0.7) (0.9,0.4,0.7)	(0.4,0.5,0.7) (0.5,0.6,0.2) (0.4,0.3,0.1)	(0.5,0.6,0.9) (0.5,0.6,0.9) (0.4,0.3,0.1)
A_2	(0.3,0.2,0.7) (0.2,0.3,0.8) (0.5,0.6,0.9)	(0.1,0.2,0.3) (0.7,0.5,0.4) (0.8,0.2,0.1)	(0.3,0.5,0.7) (0.7,0.5,0.4) (0.8,0.2,0.1)
A_3	(0.7,0.3,0.6) (0.7,0.8,0.7) (0.7,0.3,0.2)	(0.9,0.8,0.6) (0.3,0.2,0.5) (0.8,0.6,0.3)	(0.7,0.5,0.4) (0.3,0.2,0.5) (0.8,0.6,0.3)

Table 1: Near Plithogenic neutrosophic values by Decision maker D_1

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	(0.7,0.2,0.1) (0.8,0.3,0.5) (0.8,0.6,0.2)	(0.9,0.6,0.3) (0.7,0.5,0.1) (0.6,0.5,0.3)	(0.7,0.5,0.2) (0.7,0.5,0.1) (0.6,0.5,0.3)
A_2	(0.8,0.6,0.9) (0.9,0.8,0.7) (0.7,0.6,0.5)	(0.8,0.2,0) (0,1,1) (1,0,1)	(1,1,0) (0,1,1) (1,0,1)
A_3	(0.2,0,0) (0.7,0.8,0.3) (0.4,0.3,0.2)	(0.8,0,0.6) (0.8,0.2,0.1) (0.9,0.6,0.3)	(0.1,0.2,0.1) (0.8,0.2,0.1) (0.9,0.6,0.3)

Table 2: Near Plithogenic neutrosophic values by Decision maker D_2

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	(0.7,0.2,0.1) (0.8,0.3,0.5) (0.8,0.6,0.2)	(0.1,0.2,0.7) (0.5,0.6,0.9) (0.4,0.3,0.1)	(0.5,0.6,0.9) (0.5,0.6,0.9) (0.4,0.3,0.1)
A_2	(0.3,0.2,0.7) (0.2,0.3,0.6) (0.5,0.6,0.9)	(0.8,0.2,0) (0,1,1) (1,0,1)	(0.9,0.6,0.2) (0,1,1) (1,0,1)
A_3	(0.7,0.3,0.6) (0.7,0.8,0.7) (0.7,0.3,0.2)	(0.7,0.5,0.3) (0.8,0.2,0.1) (0.9,0.6,0.3)	(0.1,0.2,0.1) (0.8,0.2,0.1) (0.9,0.6,0.3)

Table 3: Near Plithogenic neutrosophic values by Decision maker D_3

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	(0.7236,0.2905,0.1376) (0.8299,0.2146,0.4339) (0.8766,0.4039,0.1234)	(0.4548,0.3483,0.4625) (0.6748,0.4229,0.4986) (0.5961,0.2764,0.1376)	(0.6748,0.4229,0.4973) (0.6748,0.4229,0.6532) (0.5961,0.2764,0.1376)

A_2	(0.5451,0.2905,0.4504) (0.452,0.4278,0.5484) (0.6748,0.4476,0.6728)	(0.4975,0.1423,0.1249) (0.8751,0.2166,0.1694) (0.9175,0.0825,0.0412)	(0.6715,0.3396,0.3354) (0.8751,0.2166,0.1694) (0.9175,0.0825,0.0412)
A_3	(0.5722,0.1759,0.3721) (0.7854,0.633,0.4625) (0.6829,0.2146,0.1423)	(0.8452,0.4432,0.3897) (0.6557,0.1423,0.2239) (0.8991,0.4476,0.2147)	(0.3407,0.2446,0.1789) (0.6557,0.1423,0.2239) (0.8991,0.4476,0.2147)

Table 4: Aggregated value of the suppliers by NPNHsGHM operators

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	(0.7536,0.3012,0.1570) (0.8320,0.2017,0.4379) (0.8965,0.4506,0.1107)	(0.4735,0.3824,0.2456) (0.7245,0.4362,0.3956) (0.5793,0.2468,0.1401)	(0.6748,0.4229,0.4973) (0.6748,0.4229,0.6532) (0.6200,0.1905,0.1420)
A_2	(0.5456,0.2935,0.4602) (0.4525,0.5201,0.5735) (0.7829,0.4103,0.6792)	(0.5375,0.1379,0.1239) (0.8556,0.2563,0.1664) (0.9279,0.133,0.0679)	(0.6735,0.3379,0.3279) (0.8889,0.2264,0.1735) (0.9245,0.0756,0.0352)
A_3	(0.5829,0.1689,0.2987) (0.7758,0.6533,0.4878) (0.6689,0.2285,0.1327)	(0.8500,0.4567,0.7689) (0.6865,0.1346,0.2663) (0.8897,0.4457,0.2237)	(0.3569,0.2457,0.1246) (0.6557,0.1423,0.2748) (0.8941,0.4476,0.2147)

Table 5: Aggregated value of the suppliers by NPNHsGWHM operators

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	0.7585	0.6199	0.6150
A_2	0.5372	0.8348	0.8088
A_3	0.6801	0.7265	0.7159
Ranking	$A_1>A_3>A_2$	$A_2>A_3>A_2$	$A_2>A_3>A_1$

Table 6: Score function and ranking of the suppliers by NPNHsGHM operators

	(e_1,e_4,e_7)	(e_2,e_5,e_8)	(e_3,e_5,e_8)
A_1	0.7385	0.6453	0.6578
A_2	0.5678	0.8895	0.8179
A_3	0.6965	0.7645	0.7432
Ranking	$A_1>A_3>A_2$	$A_2>A_3>A_2$	$A_2>A_3>A_1$

Table 7: Score function and ranking of the suppliers by NPNHsGWHM operators

From table 6 and 7 it is seen that the ranking is same by NPNHsGHM and NPNHsGWHM operators. Considering the parameter (e_1,e_4,e_7) the best supplier would be A_1 and for the parameter (e_2,e_5,e_8) it would be best for the company to choose the supplier A_2 and similarly for the parameter (e_3,e_5,e_8) the best choice would be A_2.

Conclusion

Thus, in this paper we have studied the near plithogenic neutrosophic hypersoft Heronian mean aggregation operators. Also, the NPNHsGHM and NPNHsGWHM operators for near plithogenic neutrosophic hypersoft numbers are proposed and its properties are studied. Then we have used the two operators in multi criteria decision making problem. A practical example of supplier selection is given to show its effectiveness. Since NPNHsNs are better tool to define uncertain information, it can be used in many practical problems. Also, the Heronian mean operators focuses on the aggregated arguments which can be used to get accurate results.

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