HYPERCUBES OF FQ5, FQ6 AND THEIR FAULT TOLERANCE

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ABSTRACT

Let G=FQ5 and FQ6 be a 5-dimensional and6-dimensional fault tolerance of folded hypercubes, Based on Interconnection network. There are several types of connectivity For measuring the fault tolerance of folded hypercubes .But in this paper we use component connectivity. Let G=(V,E) be a connected graph. A r-component is a cut of G then r-component edge connectivity cμr(G) can be defined as r-component edge connectivity is determined by the following condition :

1)cμ2(Q_K)=μ(Q_K)=2k-1 for k ≥2, 4) cμ2(FQ_K)=k+1 for k ≥3,

2) cμ5(Q_K)=4k-2 for k ≥2, 5) cμ5(FQ_K)=4k+1 for k ≥3,

3) cμ6(Q_K)=5k-2 for k ≥2, 6) cμ6(FQ_K)=5k+1 for k ≥3.

Key words: Interconnection networks ,folded hypercube ,r-component edge connectivity

1.Introduction

In this paper we give FQ5 and FQ6 in fault tolerance of folded hypercube .Let G=(V,E) to be connected graph ,with NG(v) representing the neighbours of a vertex v in G (just N(v)) and E(v) representing the edges incident to v.

Furthermore given S$\subset V$,G[s] is the subgraph induced by S, then NG(v)is the neighbours of a vertex v then N_G(S)=$\cup_{v\in s}$N(v)-s ,N_G(s)= N_G(s)$\cup$s and G-S shows a subgraph of G through the vertex set V/S.

If S,T$\in$V ,d(S,T) represents the shortest (S,T) path .Then denote the set of edges of G with one end in A and the other in B by [A B] for A,B $\subset$ V.A set of vertices whose deletion produces a graph with atleast r-component is known as an r-component cut ofG.Similiary ,the r-components edge connectivity ,C$\mu(G)$ can bedefined for each positive integer ,S we can show that CSr+1(G)≥CSr(G). If G-S is not connected and each component of G-S has more than n vertices ,we call S $\subseteq V$.The cardinality of the minimum extra-cuts is the extra connectivity n_n(G) [1].

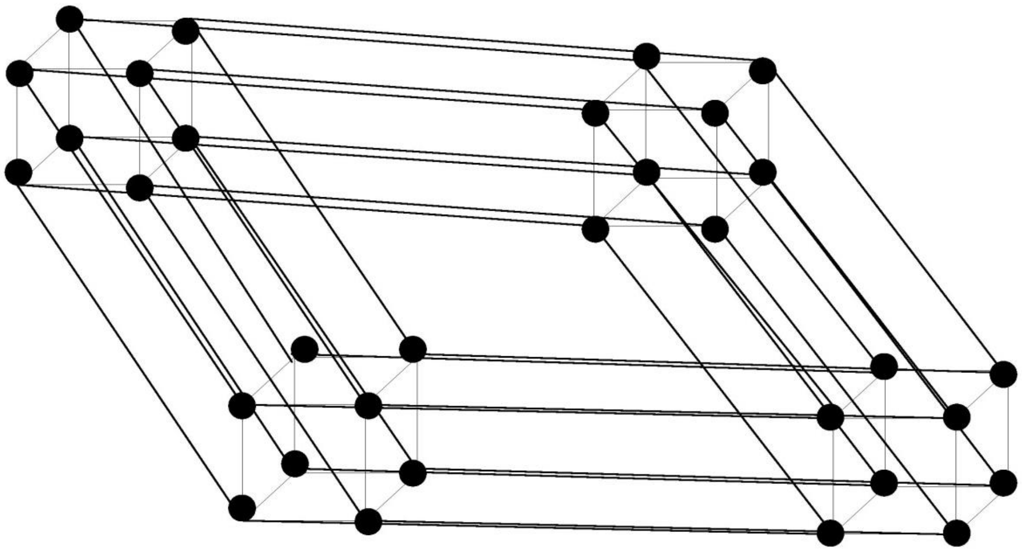

Figure 1:FQ5

2. PRELIMINARIES

Definition 2.1. Graph

The Graph is a pair of(V,E).Where V is a finite set of nodes and E is a finite set of edge.

Definition 2.2. Edge

The line connecting a pair of nodes is called an Edge. It is represents as 'E'.

Definition 2.3. Vertex

Vertices are also called nodes. It is a point or a circle .It is the fundamental unit from which graphs are made .It is represents as 'V'

Definition 2.4. Hypercube

Let k be a positive integer. The k-dimensional balanced hypercube, denoted by BH_k,has 2^{2k} vertices, each labeled by (b_0,b_1,...,b_{i-1},b_i,b_{i+1},...,b_{n-1}),whereb_iϵ{ 0,1,2,3} for all 0≤i≤k-1.An arbitrary vertex (b_0,b_1,...,b_{i-1},b_i,b_{i+1},...,b_{k-1}), is adjacent to the following 2k vertices:

1)((b_0±1)mod4,b_0,b_1,...,b_{i-1},b_i,b_{i+1},...,b_{k-1}) where 0≤i≤k-1

2)((b_0±1)mod4,b_0,b_1,...,b_{i-1} (b_i+$(-1)^{b_0}$mod4,b_{i+1},...,b_{k-1}) where 0≤i≤k-1.[6]

3. SOME PROPERTIES OF FIVE AND SIX DIMENSION HYPERCUBE

The five and six dimensional hypercube networks arerepresented by Q5 and Q6.A k-dimensional hypercube is represented by Q_k.ThenQ_k =(V,E) and |V|= 2^k and |E|=k2^{k-1}.A k-dimensional folded hypercube that represented by FQ_k is proposed by El-Amawy and Latifi [2].FQ_k is obtained from Q_k by adding 2^{k-1} edged called complementary edges. Each edge is between the vertices, y=(y_1,...., y_k)and $\overline{y}$=($\overline{y_1}$,... $\overline{y_k}$),where $\overline{y_i}$=1-y_i then FQ_k is obtained from Q_k by adding a perfect matching M.where M=(y,$\overline{y}$,);y∈ v(Q_k),then Q_k is expressed as $Q^0_{K-1} \odot Q^1_{K-1}$. Where Q^0_{K-1} and $Q^1_{K-1}$are k-1 dimension hypercube with the prefix 0 and 1 respectively.

Furthermore Q_K can be expressed as G($Q_{K-1}^0 : Q_{K-1}^1$,M_0)where M_0={(0s,0s):0s∈v(Q_{K-1}^0),1s∈v(Q_{K-1}^1)}.In this case FQ_k can be expressed as G{($Q_{K-1}^0 : Q_{K-1}^1$,M_0+$\overline{M}$}where{(0s,1s):0s∈v(Q_{K-1}^0),1s∈v(Q_{K-1}^1)}.and $\overline{M_0}$={(0s,1s):0s∈v(Q_{K-1}^0),1$\bar{s}$ ∈v(Q_{K-1}^1)}.[3]

FQ_k is (k+1)-connected and (k+1)-regular.In addition FQ_k is a cayley graph.It has a diameter [k/2], which is nearly half that of Q_k .As a result ,the folded hypercube FQ_k is a better version of the hypercube Q_k.

The analysis of the fault tolerance of folded hypercubes has recently attracted much researches. If r=2,3,.,n+1 determines the r-component connectivity of the folded hypercube Q_k, and r=k+2,k+3,... determines the r-component connectivity of folded hypercubeQ_k.

In this paper ,we obtain that ,

1) cμ2(Q_K)=μ(Q_K)=2k-1 for k ≥2, 4) cμ2(FQ_K)=k+1 for k ≥3,

2) cμ5(Q_K)=4k-2 for k ≥2, 5) cμ5(FQ_K)=4k+1 for k ≥3,

3) cμ6(Q_K)=5k-2 for k ≥2, 6) cμ6(FQ_K)=5k+1 for k ≥3.

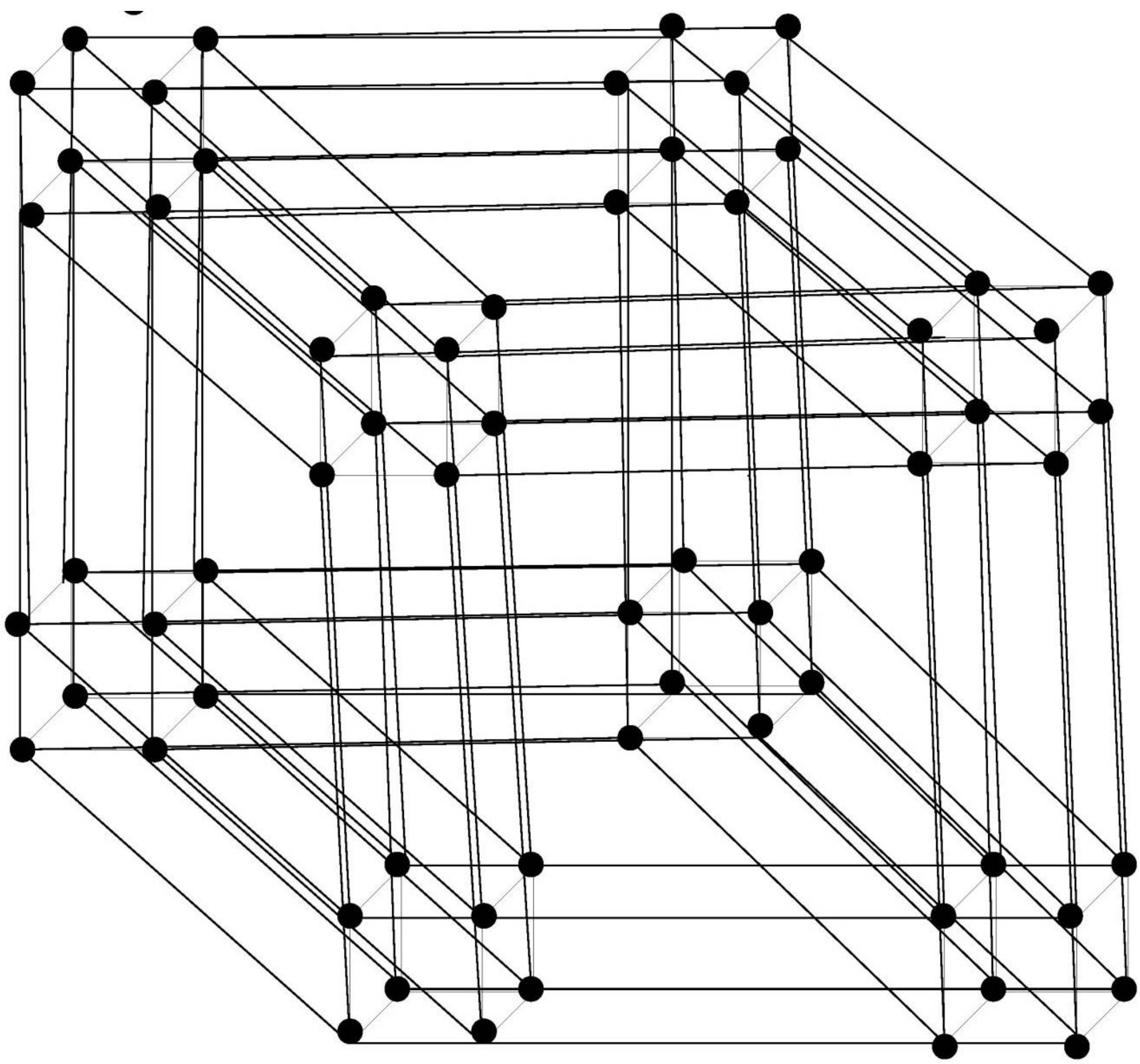

Figure 2:FQ6

4. Connectivity of Hypercube And Folded Hypercube

The vertices that pass through the k^{th} coordinate of a binary string represented differently v's through Uk. similarly vjk is the vertex where the k bit binary string is clearly v_{kk}=u.

Lemma 4.1.

Any two vertices of FQ_k have exactly two common neighbours for k≥4 if they have any.[4]

Proof.

Prove any two vertices,FQ_k =G{Q^0_{K-1}:Q^1_{K-1},Mo+$\overline{M}$} has two adjacencies incommon with k ≥3.It is known that any two nodes in a Q_k have two commonneighbours only if they exist.

Case 1:

Both vertices at V(Q^0_{K-1}) or V(Q^1_{K-1}) suppose the two nodes are 0s and 0t.Suppose 0s and 0t have two neighbors in common with Q^0_{K-1}.The definition of Q^0_{K-1} means that 0s and 0t are exactly different at the two bit positions.Then {1s,1$\bar{s}$}⋂{1t,1$\bar{t}$}=ϕ for k≥3.Therefore 0s and 0t have exactly two common adjacencies to FQ_k because 0s and 0t do not have a common adjacency to Q^1_{K-1}.

Suppose that Q^0_{K-1} has no neighbors in common 0s and 0t.For s=$\bar{t}$ then{1s,1$\bar{s}$}={1t,1$\bar{t}$}.Therefore 0s and 0t have exactly two common neighbours in FQ_k.For s≠ $\bar{t}$,Then {1s,1$\bar{s}$}⋂{1t,1$\bar{t}$}=ϕ Therefore 0s and 0t have no neighbours in common with FQ_k.

Case 2:

One of the two nodes is at V(Q^0_{K-1})and the other is at V (Q^1_{K-1}).Suppsoe 0s∈V(Q^0_{K-1})and 1t∈V(Q^1_{K-1})without loss of generality. If there exist an j such that V∈{sj,$\bar{s}$j} then |NFQ_k (0s)⋂NFQ_k (1t)|-|{ 0s_1,...,0s_n,1s,1$\bar{s}$}⋂{ 1t_1,...,1t_n,0t,0$\bar{s}$}|=2

Therefore 0s and 1t have two neighbours in common with FQ_k If v not belongs to{sj,$\bar{s}$j},for (j=1,2,..,n) then|NFQ_k (0s)⋂NFQ_k(1t)|=|{ 0s_1,...,0s_n,1s,1$\bar{s}$}⋂{ 1t_1,...,1t_n,0t,0$\bar{s}$}|=0.That is 0s and1t have a common neighbor for FQ_k.

THEOREM4.1

cμ5(Q_k)=4k-2 for k ≥2

Proof:

By using theorem 2.7[1], Take an edge P_3=stu then |E(s)⋃E(t)⋃E(u)|=4k-2.and Q_k-E(s)-E(t)-E(u) has at least 4 connected components.

i.e)cμ5(Q_k)≤4k-2.

Next show that cμ5(Q_k)≥4k-2 by Mathematical induction .it is true for n=2,3,4,5,6.suppose n≥7 assume that true for all k<n.prove that k=n.Let F ⊆ E(Q_k)with |F|≤4k-3,and Q_k-F has atleast 4 components .since Q_k= $Q^0_{K-1} \odot Q^1_{K-1}$.

Case 1:

$Q^0_{K-1} - F$ is connected. If Q^1_{K-1} –F has atleast 4 components ,thencμ5(Q_k-1)≥4k-7 by the hypothesis ,atmost two edges since the vertex Q^1_{K-1}has a neighbor in $Q^0_{K-1}$and Q_k-F has atmost 3 components

Case 2:

$Q^0_{K-1} - F$ has only two connected component,Then |E(Q^0_{K-1})$\cap F$|≥μ(Q_k-1)=k-1and |E(Q^1_{K-1})$\cap F$|≤2k-2 that cμ3(Qk-1)=2k-3.

If Q^1_{K-1} –F has atleast 3 components,then |E(Q^0_{K-1})$\cap F$|≥2k-3and |E(Q^0_{K-1})$\cap F$|≤k.then Q_k-F has atmost two components Hence Q^1_{K-1} –F has atmost two components ,we have |[Q^0_{K-1}, Q^1_{K-1}]|>3k-3(n≥7),and Q_k-F has atmost 3 components.

THEOREM 4.2:

cμ6(Q_k)=5k-2 for k ≥2

Proof:

Take an edge P_4=stuv then|E(s)∪E(t)∪E(u)∪E(v)|=5k-2.and Q_k-E(s)-E(t)-E(u)-E(v) hasatleast 5 connected components.

ie)cμ6(Q_k)≤5k-2.

Next show that cμ5(Q_k)≥5k-2 by Mathematical induction .it is true for n=2,3,4,5,6.suppose n≥7 assume that true for all k<n.prove that k=n.

Case 1:

$Q^0_{K-1} - F$ is connected, If Q^1_{K-1} –F has atleast 5 components, then cμ6(Q_k-1)≥5k-7 by the hypothesis ,atmost two edges since the vertex Q^1_{K-1}has a neighbor in Q^0_{K-1}andQ_k-F has atmost 4 components

Case 2:

$Q^0_{K-1} - F$ has only two connectedcomponentsThen |E(Q^0_{K-1})$\cap F$|≥μ(Q_k-1)=k-1and |E(Q^1_{K-1})$\cap F$|≤5k-2 that cμ3(Q_k-1)=5k-3. If Q^1_{K-1} –F has atleast 4 components,then |E(Q^0_{K-1})$\cap F$|≥5k-3and |E(Q^0_{K-1})$\cap F$|<k.then Q_k-F has atmost two components

Hence Q^1_{K-1} –F has atmost two components ,we have |[Q^0_{K-1}, Q^1_{K-1}]|>5k-3(n≥7),and Q_k-F has atmost 3 components.

And because the hypercube Q_k is the subgraph of folded hypercube FQ_k .we can apply the similar method to FQ_k

5. CONCLUSION

Hypercube network Q_k has become one of the most famous interconnect networks.The folded hypercube FQ_k could be a variant of Q_k.If r=2,3,4,5,6 determines the r-components connectivity of folded hypercube Q_k and FQ_k. Future research on this topic the folded hypercube of FQ7 and FQ8.

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