

ADAMS-BASHFORTH CORRECTOR PREDICTOR METHOD USING MATLAB

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ABSTRACT

In this paper, we present Adams-Bashforth method of ordinary differential equation. Adams-Bashforth method is quite efficient and practically well suited for solving differential equations with initial condition. All mathematical calculations performed by MATLAB software for better accuracy and result. The result obtained from numerical examples, shows that this method is more efficient and accurate. These methods are preferable to some existing methods because of their simplicity and less computational cost involved.

KEYWORDS: Adams-Bashforth method, MATLAB, Initial value problem, ordinary differential equation.

INTRODUCTION

Many scientific and engineering problems are described in the form of ordinary differential equations. Its derivative of only one independent variable are called Ordinary Differential Equations and may be classified as either Initial Value Problem (IVP) or Boundary Value Problem (BVP). If such equations cannot be solved analytically, we use computers and numerical methods to approximate them, usually performing all calculations in floating point arithmetic.

In this paper, we propose to join explicit interval methods of Adams-Bashforth type with implicit. In this case, we obtain interval of predictor-corrector methods when we use the solution obtained for the previous integration step as the first approximation in an implicit method, taking the solution from an explicit method multistep method (a predictor) as such an approximation, we can significantly reduce the number of iterations needed in the implicit method (a corrector).

PRELIMINARIES

Initial value problem

Numerical methods for ordinary differential equations approximate solutions to Initial value problems of the form,
 $y' = f(t, y), y(t_0) = y_0$

Adams Bashforth Method

The Adams Bashforth methods are explicit methods. The coefficients $a_{s-1} = -1$ and $a_{s-2} = \dots = a_0 = 0$, while the b_j are chosen such that the methods have order s . (this determines the methods uniquely).

The Adams Bashforth methods with $s=1,2,3,4,5$ are,

$$y_{n+1} = y_n + hf(t_n, y_n) \quad (\text{This is the Euler method})$$

$$y_{n+2} = y_{n+1} + h\left(\frac{3}{2}f(t_{n+1}, y_{n+1}) - \frac{1}{2}f(t_n, y_n)\right),$$

$$y_{n+3} = y_{n+2} + h\left(\frac{23}{12}f(t_{n+2}, y_{n+2}) - \frac{16}{12}f(t_{n+1}, y_{n+1}) + \frac{5}{12}f(t_n, y_n)\right),$$

$$y_{n+4} = y_{n+3} + h\left(\frac{55}{24}f(t_{n+3}, y_{n+3}) - \frac{59}{24}f(t_{n+2}, y_{n+2}) + \frac{37}{24}f(t_{n+1}, y_{n+1}) - \frac{9}{24}f(t_n, y_n)\right),$$

$$y_{n+5} = y_{n+4} + h \left(\frac{1901}{720} f(t_{n+4}, y_{n+4}) - \frac{2774}{720} f(t_{n+3}, y_{n+3}) + \frac{2616}{720} f(t_{n+2}, y_{n+2}) - \frac{1274}{720} f(t_{n+1}, y_{n+1}) + \frac{251}{720} f(t_n, y_n) \right)$$

The coefficients b_j can be determined as follows, use polynomial interpolation to find the polynomial p of degree $s-1$ such that $p(t_{n+i}) = f(t_{n+i}, y_{n+i})$, for $i = 0, 1, \dots, s-1$

Numerical illustration

Consider the ordinary differential equation $\frac{dy}{dx} = f(x,y)$ with initial condition $y(x_0) = y_0$

To find: $y(x_n)$

Note: To apply both Milne and Adams-Bashforth methods we need four starting value of y .

Find $y(x_1), y(x_2), y(x_3)$ using any one of the methods like Picard's method, Taylor series, Euler method, Modified Euler method or Runge-kutta method.

Then calculate, $f_0 = f(x_0, y_0)$

$$f_1 = f(x_1, y_1)$$

$$f_2 = f(x_2, y_2)$$

$$f_3 = f(x_3, y_3)$$

By Adams-Bashforth predictor formula, $y_4 = y_3 + \frac{h}{24} (55f_3 - 59f_2 + 37f_1 - 9f_0)$

Then find $f_4 = f(x_4, y_4)$ since, $x_4 = (x_3 + h)$

By Adams-Bashforth corrector formula, $y_4 = y_3 + \frac{h}{24} (9f_4 + 19f_3 - 5f_2 + f_1)$

An improved value of f_4 is again computed and again Adams-Bashforth corrector formula is applied to find a still better value of y_4 . We repeat this step until y_4 remains unchanged.

Using Adams-Bashforth methods, obtain the solution of $\frac{dy}{dx} = x - y^2$ at $x=0.8$ given that,

x	0	0.2	0.4	0.6
y	0	0.02	0.795	0.1762

Here, $f(x,y) = x - y^2$, $h=0.2$

$$x_0=0, y_0=0, f_0 = f(x_0, y_0)=0$$

$$x_1=0.2, y_1=0.02, f_1 = f(x_1, y_1)=0.1996$$

$$x_2 = 0.4, y_2=0.0795, f_2 = f(x_2, y_2)=0.3937$$

$$x_3=0.6, y_3=0.1762, f_3 = f(x_3, y_3)=0.5690$$

Using Adams-Bashforth predictor formula, $y_4 = y_3 + \frac{h}{24} (55f_3 - 59f_2 + 37f_1 - 9f_0)$

$$y_4 = 0.3050, x_4 = 0.8$$

$$f_4 = f(x_4, y_4) = 0.7072$$

Again using Adams-Bashforth corrector formula, $y_4 = y_3 + \frac{h}{24} (9f_4 + 19f_3 - 5f_2 + f_1)$

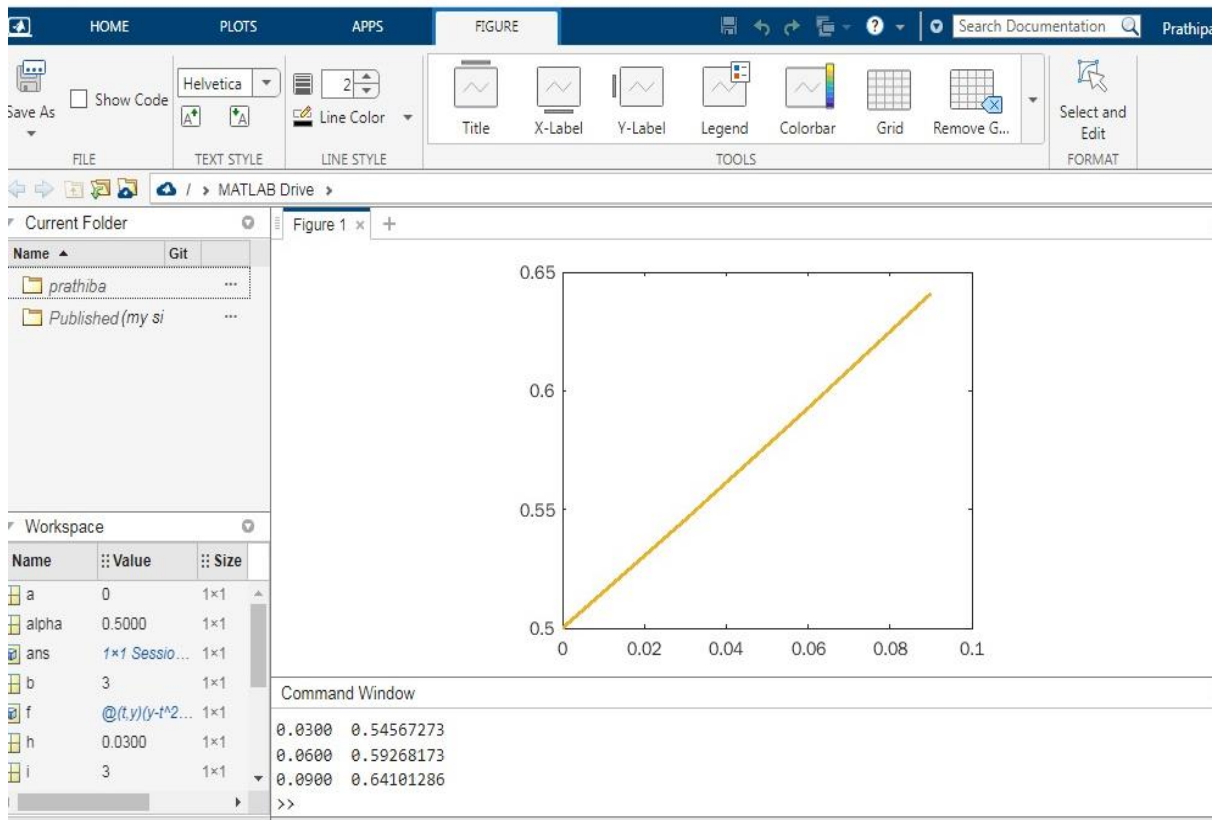
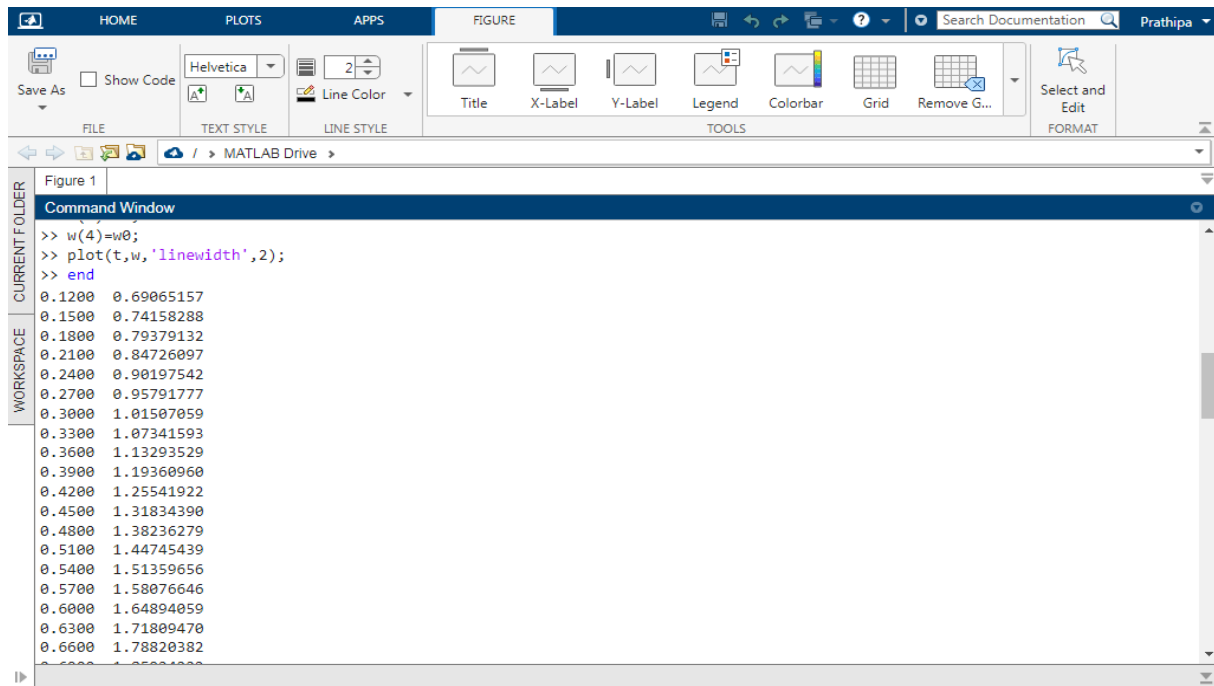
$$y_4 = 0.3046$$

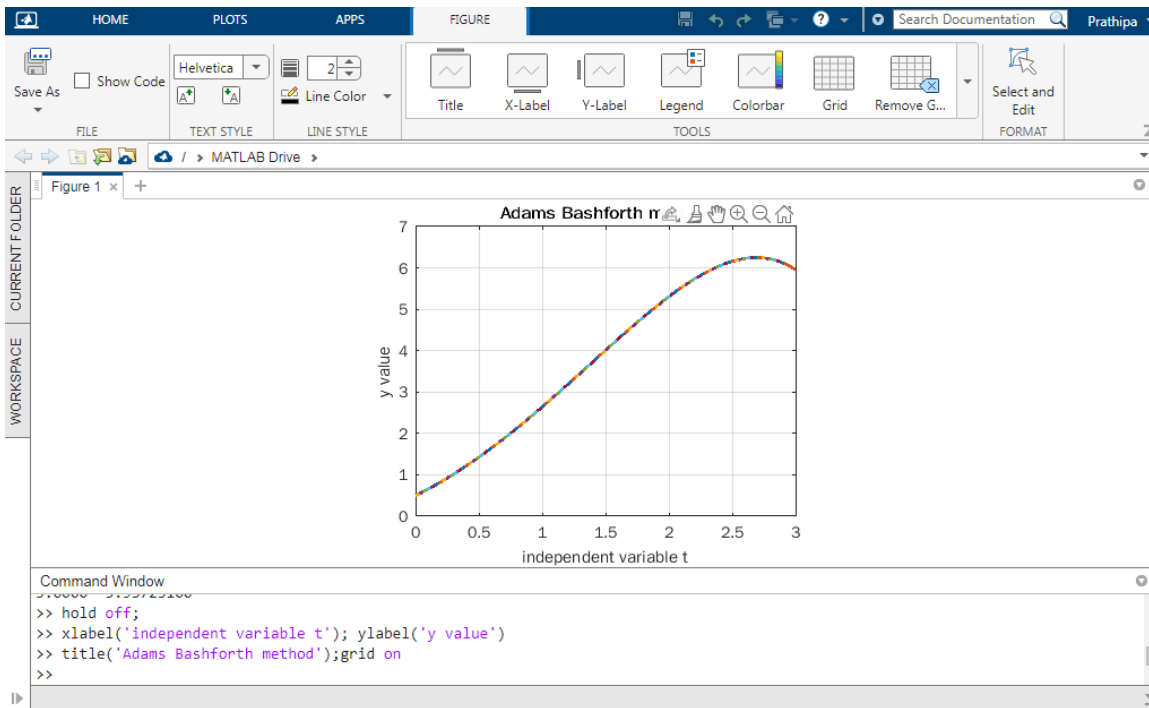
$$\text{Hence, } y_4 = y(0.8) = 0.3046.$$

Proposed algorithm using MATLAB

```
% dy/dt =y-t^2+1;0<=t<=2;y(0)=0.5;
f=@(t,y) (y-t^2+1);
a=0;
b=3;
n=100;
alpha=0.5;
h=(b-a)/n;
t(1)=a;
w(1)=alpha;
fprintf('t w\n');
fprintf('%5.4f %11.8f\n',t(1),w(1));
for i=1:3
t(i+1)=t(i)+h;
k1=h*f(t(i),w(i));
k2=h*f(t(i)+0.5*h,w(i)+0.5*k1);
k3=h*f(t(i)+0.5*h,w(i)+0.5*k2);
k4=h*f(t(i+1),w(i)+k3);
w(i+1)=w(i)+(k1+2.0*(k2+k3)+k4)/6.0;
fprintf('%5.4f %11.8f\n',t(i+1),w(i+1));
plot(t,w,'linewidth',2);hold on;
end
for i=4:n
t0=a+i*h;
part1=55.0*f(t(4),w(4))-59.0*f(t(3),w(3))+37.0*f(t(2),w(2));
part2=-9.0*f(t(1),w(1));
w0=w(4)+h*(part1+part2)/24.0;
part1=9.0*f(t0,w0)+19.0*f(t(4),w(4))-5.0*f(t(3),w(3))+f(t(2),w(2));
w0=w(4)+h*(part1)/24.0;
fprintf('%5.4f %11.8f\n',t0,w0);
for j=1:3
t(j)=t(j+1);
w(j)=w(j+1);
end
t(4)=t0;
w(4)=w0;
plot(t,w,'linewidth',2);
end
hold off;
xlabel('Independent variable t');
ylabel('y value');
title('Adams-Bashforth method');grid on
```

Output





Conclusion

The examples presented in this paper clearly show that the interval predictor-corrector methods can be applied successfully to both nonstiff and stiff problems, and to both one- and more than one-dimensional problems. Comparing the results with other interval methods of the same or similar order, we see the high compatibility of the interval solutions obtained. Although our methods are of low order, in the case when the integration intervals are not too big they give as good results as the methods based on high-order Taylor series. Undoubtedly, an advantage of our methods consists in their simplicities—they can be easy programmed in any computer language. We hope that an implementation of variable step size controls would benefit in speed for these methods. We plan to study this problem in our next works.

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