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METHOD FOR SOLVING ASSIGNMENT PROBLEM IN HEXAGONAL FUZZY NUMBER

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Abstract

In this paper, fuzzy assignment problem in which the values of assignment costs are represented as Hexagonal fuzzy number is considered.The hexagonal fuzzy number are converted into crisp using Graded Mean Integration Representation Method and Ranking Technique based on Centroid method.Then the optimum assignment schedule of fuzzy assignment problem is obtained by usual Hungarian Method.We find the optimal minimum total cost required for the problem.Finally,examples are given to explain the method.

Keywords: Fuzzy Assignment Problem, Hexagonal Fuzzy Number, Optimal Solution Hexagonal, Graded Men Integration Method, Ranking Technique based on Centroid Method.

Introduction

The assignment method is used to solve real-world problems all over the globe. The fuzzy assignment problem(AP) is a special type of linear programming problem. Assignment problem is a well.It is one fundamental optimization problem in the branch of optimization(or) the operation research. This is important in the theory of decision making in the normal case of assignment problem where the objective is to assign the available resources to the activity going on to get the minimum cost (or) maximum total benefits of allocation.AP has many applications in healthcare, shipping, education, and sports. Using assignment problems we minimize time, minimize cost, minimize length path route, and maximize profit, etc.Six salesmen are to be performed by six products sales are depending on their salesmen. In this problem, C_{ij} denotes the cost of assigning the six salesmen to the six products. We find the optimal assignment and it gives us the required optimal total cost for the Minimum.

Mathematical Model

The mathematical model of the assignment problem is, associated to each assignment problem there is a matrix called cost or effectiveness matrix $[C_{ij}]$ where C_{ij} is the cost of assigning i^{th} products to j^{th} salesmen. In this paper we call it assignment matrix and represent it as follows:

$$\begin{pmatrix} C_{i1} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{pmatrix}$$

The general fuzzy assignment problem can be mathematically stated as follows:

Minimize $\sum_{j=1}^{n} \bar{C}_{ij} X_{ij}$

Subject to $\sum_{i=1}^{n} X_{ij} = 1$, for i = 1,2,…,n

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Preliminaries

Definition:

A fuzzy is characterized by a membership function mapping the element of domain, space or universe of discourse X to the unit interval0,1. A fuzzy set A is a universe of discourse X is defined as the following set of pairs A=$\{(x,\mu_A(x));x\in X\}$. Here μ_A:X→[0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A(x) is called the membership value of x∈X on the fuzzy set A. These membership grade are often represented by real numbers ranging from [0,1].

Definition:

A fuzzy set $\bar{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

(1) $\bar{A}$ is normal

(2) $\bar{A}$ is convex

(3) The support of $\bar{A}$ is closed and bounded then $\bar{A}$ is called fuzzy number.

Definition:

A fuzzy number $\bar{A}_H$ is a hexagonal fuzzy number denoted by $\bar{A}_H$ $(a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real number and its membership function $\mu\bar{A}$ (x) is given below.

$$\mu\bar{A}_H = \begin{cases} 0 \\ 1/2(\frac{x-a_1}{a_2-a_1}) \\ \frac{1}{2} + 1/2(\frac{x-a_2}{a_3-a_2}) \\ 1 \\ 1 - 1/2(\frac{x-a_4}{a_5-a_4}) \\ 1/2(\frac{a_6-x}{a_6-a_5}) \\ 0 \end{cases}$$

Graded Mean Integration Representation Method (GMI)

Graded Mean Integration Representation Method is another method used to defuzzify Hexagonal fuzzy number. The defuzzified value

R (GMI_H) of the Graded Mean Integration Representation method is given by the following formula:

$$R(GMI_H) = \int_0^w y\left[\frac{g_A^L(y)+g_A^R(y)}{2}\right]dy / \int_0^w y dy$$

where $0 < h \leq w \ and \ 0 < w \leq 1.$

The inverse function of Hexagonal fuzzy number is $g_A^{L1}(y), g_A^{L2}(y), g_A^{R1}(Y), g_A^{R2}(y)$ and the value of these defined as above. If $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is a HFN, then the Graded Mean Integration of A_H from the above formula is calculated as follows:

$$R(GMI_H) = \int_0^1 y\left[\frac{g_A^{L1}(y)+g_A^{L2}(y)+g_A^{R1}(y)-g_A^{R2}(y)}{2}\right]dy / \int_0^1 ydy$$

$$R(GMI_H) = \frac{7_{a2} - 2_{a1} + a_3 + a_4 + 6_{a5} - a_6}{6}$$

Ranking Technique based on Centroid Method (CM)

The magnitude of a HFN A_H =$(a_1, a_2, a_3, a_4, a_5, a_6)$using Ranking techniques based on CM is defined as

$$R(CM_H) = \frac{7a_2 - 2a_1 + a_3 + a_6 6a_5 - a_6}{6}$$

Algorithm

First we defuzzify the fuzzy assignment problem into general assignment problem and then go to the steps.

Step:1 Subtract the smallest element in each row from every element is its row.

Step:2 Subtract the smallest element in each column of the assignment matrix obtained in step-1 from every element in that column.

Step:3 Cover all zeros with minimum number of straight lines.As the number of lines required to cover all zeros is less than the number of rows/columns,optimal solution has not reached.

Step:4 Select the smallest element not covered by the lines,subtract it from each uncovered element and add it to elements which are at the intersection of the lines.

Step:5 Cover all zeros by minimum number of straight lines.Since the number of lines covering all zeros is equal to the number of rows/columns.Hence the optimal solution has reached.

Numerical Example

Let us consider a fuzzy assignment problem with rows representing 6 salesmen A,B,C,D,E,F and columns representing 6 products. The problem is to find the optimal assignment of salesmen to products that will minimize total cost.

Salesmen products	A	B	C	D	E	F
1	(1,2,3,4,5,6)	(7,8,9,10,11,12)	(6,1,2,3,5,4)	(7,4,2,3,8,9)	(5,4,2,1,6,7)	(11,12,13,14,3,2)
2	(2,4,6,8,10,12)	(9,10,11,15,5,6)	(5,8,10,11,3,7)	(3,5,6,7,11,13)	(5,8,3,7,1,2)	(5,8,10,11,1,7)
3	(7,8,10,12,4,5)	(3,5,6,7,8,9)	(6,4,2,8,10,12)	(5,7,10,11,14,12)	(8,11,13,15,10,12)	(6,8,10,12,4,1)
4	(6,8,1,4,10,2)	(2,5,6,7,1,13)	(4,6,7,9,1,3)	(11,12,14,1,2,4)	(6,7,1,6,2,5)	(9,7,1,3,4,6)
5	(12,8,7,15,4,7)	(9,1,14,10,6,3)	(12,6,7,1,2,4)	(9,6,12,10,3,1)	(4,5,11,10,12,14)	(15,11,13,10,1,2)
6	(6,14,4,11,7,9)	(2,1,4,3,10,11)	(1,3,5,7,9,11)	(6,10,2,14,8,7)	(4,1,3,11,10,12)	(10,1,7,6,3,4)

Solution:

i. Defuzzify the above fuzzy assignment problem by using graded mean integration representation method,

$$R(GMI_H) = \frac{7a_2 - 2a_1 + a_3 + a_4 + 6a_5 - a_6}{6}$$

$a_{11} = 7$ $a_{12} = 19$ $a_{13} = 4$ $a_{14} = 10$ $a_{15} = 8$ $a_{16} = 17.5$

$a_{21} = 14$ $a_{22} = 17$ $a_{23} = 13$ $a_{24} = 16$ $a_{25} = 10$ $a_{26} = 8$

$a_{31} = 14$ $a_{32} = 13.5$ $a_{33} = 12$ $a_{34} = 22$ $a_{35} = 23$ $a_{36} = 15$

$a_{41} = 18$ $a_{42} = 6$ $a_{43} = 9$ $a_{44} = 14$ $a_{45} = 8.5$ $a_{46} = 9$

$a_{51} = 12$ $a_{52} = 8$ $a_{53} = 6$ $a_{54} = 10.5$ $a_{55} = 18$ $a_{56} = 12$

$a_{61} = 22$ $a_{62} = 10$ $a_{63} = 12$ $a_{64} = 17.5$ $a_{65} = 10$ $a_{66} = 2$

$$\begin{bmatrix} 7 & 19 & 4 & 10 & 8 & 17.5 \\ 14 & 17 & 13 & 16 & 10 & 8 \\ 14 & 13.5 & 12 & 22 & 23 & 15 \\ 18 & 6 & 9 & 14 & 8.5 & 9 \\ 12 & 8 & 6 & 10.5 & 18 & 12 \\ 22 & 10 & 12 & 17.5 & 10 & 2 \end{bmatrix}$$

Applying the Hungarian Method the solution is

$$\begin{bmatrix} 1 & 15 & (0) & 1.5 & 2 & 13.5 \\ 4 & 9 & 5 & 3.5 & (0) & 0 \\ (0) & 1.5 & 0 & 5.5 & 9 & 3 \\ 10 & (0) & 3 & 3.5 & 0.5 & 3 \\ 4 & 2 & 0 & (0) & 10 & 6 \\ 18 & 8 & 10 & 11 & 6 & (0) \end{bmatrix}$$

$$\begin{bmatrix} 7 & 19 & 4^* & 10 & 8 & 17.5 \\ 14 & 17 & 13 & 16 & 10^* & 8 \\ 14^* & 13.5 & 12 & 22 & 23 & 15 \\ 18 & 6^* & 9 & 14 & 8.5 & 9 \\ 12 & 8 & 6 & 10.5^* & 18 & 12 \\ 22 & 10 & 12 & 17.5 & 10 & 2^* \end{bmatrix}$$

*Optimal allocation.

The minimum cost = 4+10+14+6+10.5+2

= 46.5.

ii. **Ranking Technique based on Centroid Method**

$$R(CMH) = \frac{7_{a_2} - 2_{a_1} + a_3 + a_4 6_{a_5} - a_6}{6}$$

By defuzzification the given problem is reduced to assignment table as follows,

$$\begin{bmatrix} 21.5 & 116 & 14 & 25 & 8 & 54 \\ 83 & 84.5 & 41 & 81 & 15 & 19 \\ 56 & 60 & 81 & 160 & 160 & 19 \\ 47 & 11 & 15 & 14 & 17.5 & 16 \\ 65 & 60 & 5.5 & 36 & 124 & 18 \\ 90.5 & 29 & 65 & 121 & 108 & 16 \end{bmatrix}$$

Applying the Hungarian Method the solution is

$$\begin{bmatrix} (0) & 95.5 & 6 & 1 & 0 & 46 \\ 54.5 & 56.5 & 26 & 50 & (0) & 4 \\ 23.5 & 28 & 62 & 125 & 141 & (0) \\ 33.5 & 0 & 17 & (0) & 19.5 & 18 \\ 46 & 41.5 & (0) & 14.5 & 118.5 & 12.5 \\ 61 & (0) & 49 & 89 & 92 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 21.5^* & 116 & 14 & 25 & 8 & 54 \\ 83 & 84.5 & 41 & 81 & 15^* & 19 \\ 56 & 60 & 81 & 160 & 160 & 19^* \\ 47 & 11 & 15 & 14^* & 17.5 & 16 \\ 65 & 60 & 5.5^* & 36 & 124 & 18 \\ 90.5 & 29^* & 65 & 121 & 108 & 16 \end{bmatrix}$$

*Optimal allocation.

Therefore, the minimum cost =21.5+15+19+14+5.5+29

=104

Conclusion

In this paper, To determine the minimum or maximum objective function using hexagonal fuzzy number the assignment method is utilised to solve the fuzzy assignment problem.We also utilised the graded mean integration and ranking technique based on centroid method approach in this sort of assignment problem, we calculate the fuzzy optimal minimize cost or maximize cost using another fuzzy number and solve it using the Hungarian approach.

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