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# Some Results On Strongly Edge Multiplicative Graphs

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**Abstract**. A graph $G(p,q)$ with $p$ vertices and $q$ edges is said to be *strongly edge multiplicative* if the edges of $G$ can be labeled with distinct integers from $\{1,2,3,\ldots,q\}$ such that labels induced on the vertices obtained by the product of the labels of incident edges are distinct. In this paper, we proved that some graphs are strongly edge multiplicative.

*Keywords:* Multiplicative labeling, Strongly multiplicative labeling, Strongly edge multiplicative labeling.

## 1. Introduction

Consider the graphs are all simple, finite and undirected. We refer [1] for standard notations and terminology. "A graph *labeling* is an assignment of integers to the vertices or edges or both subject to the specific conditions. We refer [3] for the latest update of graph labeling.A graph $G(p,q)$ with $p$ vertices is said to be *multiplicative* if the vertices of $G$ can be labeled with $p$ distinct positive integers such that label induced on the edges by the product of labels of end vertices are all distinct. Multiplicative labeling was introduced by Beineke and Hegde [2]. In this same paper, they proved that every graph admits a multiplicative labeling. They [2] defined the concept of *strongly multiplicative* graphs as follows: A graph $G(p,q)$ with $p$ vertices is said to be strongly multiplicative if the vertices of $G$ can be labeled with $p$ consecutive positive integers $1,2,3,\ldots,p$ such label induced on the edges by the product of labels of end vertices are all distinct. The concept of strongly edge multiplicative labeling was introduced by Jeyabalan et.al., [4]. A graph $G(p,q)$ with $p$ vertices and $q$ edges is said to be *strongly edge multiplicative* if the edges of $G$ can be labeled with distinct integers from $\{1,2,3,\ldots,q\}$ such that labels induced on the vertices obtained by the product of the labels of incident edges are distinct."

The following definitions are taken from [5],[6],[7] "A *fan graph* $f_n$ is obtained from a path $P_n$ by adding a new vertex and joining it to all the vertices of the path by an edge. A *wheel graph* $W_n$ is the graph formed by connecting single universal vertex to all vertices of a cycle. A *gear graph* $G_n$ is obtained from a wheel $W_n$ by adding a vertex between every pair of adjacent vertices of the $n$-cycle. The *helm* $H_n$ is the graph obtained from a wheel $W_n$ by attaching a pendant edge at each vertex of the $n$-cycle. The *flower graph* $Fl_n$ is the graph obtained from a helm $H_n$ by joining each pendant vertex to the apex of helm. The *star graph* is a complete bipartite graph $K_{1,n}$, where $n$ represents the number of vertices. The *triangular book* with $n$ pages is defined as $n$ copies of cycles $C_3$ sharing a common edge. The common edge is called the *spine* or *base* of the book. This graph is denoted by $B(3,n)$. The *rectangular book* with $n$ pages is defined as $n$ copies of cycles $C_4$ sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B(4,n)$. The *pentagonal book* with $n$ pages is defined as $n$ copies of cycles $C_5$ sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B(5,n)$. A *shell* $S_n$ is the graph obtained by taking $(n-3)$ concurrent chords in a cycle $C_n$. The vertex at which all the chords are concurrent is called the *apex*. The shell is also called fan $f_{n-1}$. The one vertex union of fan $f_m$, fan $f_n$ and star $K_{1,k}$ is called *butterfly graph* $B(m,n,k)$. A *coconut tree* $CT(m,n)$, $(m,n\geq 2)$ is the graph obtained from the path $P_n$, by appending $m$ new pendant edges at an end vertex of $P_n$. An *umbrella graph* $U(m,n)$, $(m,n\geq 2)$ obtained by joining a path $P_m$ with central vertex of a fan $f_n$. The one vertex union of 2 cycles $C_m$ and the star $K_{1,n}$ is called *butterfly graph* $B_{m,n}$. A *Mongolian tent* as a graph obtained from $P_m\times P_n$ by adding one extra vertex above the grid and joining every other vertex of the top row of $P_m\times P_n$ to the new vertex. The *triangular snake* $T_n$ is obtained from a path $P_n$ by replacing each edge of the path by a triangle $C_3$. A *double triangular snake* $D(T_n)$ consists of two triangular snakes that have a common path. The *Brush graph* $B_z$, $(z\geq 2)$ can be constructed by path graph $P_z$, $(z\geq 2)$ by joining the star graph $K_{1,1}$ at each vertex of the path. i.e, $B_z=P_z\odot zK_{1,1}$. A *Quadrilateral snake* $Q_n$ is obtained from a path $u_1,u_2,u_3,\ldots,u_n$ by joining $u_i$ and $u_{i+1}$ to the new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$. That is every edge of a path is replaced by a cycle $C_4$. The *double Quadrilateral snake* $D(Q_n)$ consists of two Quadrilateral snakes that have a common path. A *ladder* graph $L_n$ is defined by $P_n\times K_2$ where $P_n$ is a path with $n$ vertices and $K_2$ is a complete graph with two vertices. An *open ladder* $O(L_n)$, $n\geq 2$ is obtained from two paths of length $n-1$ with $V(O(L_n))=\{r_i,s_i : 1\leq i\leq n\}$ and $E(O(L_n))=\{r_ir_{i+1},s_is_{i+1} : 1\leq i\leq n-1\}\cup\{r_is_i : 2\leq i\leq n-1\}$. The $Z-P_n$ *graph* is obtained from the pair of paths $P'_n$ and $P''_n$ by joining $i^{th}$ vertex of path $P'_n$ with $(i+1)^{th}$ vertex of path $P''_n$ with new edges, for all $1\leq i\leq (n-1)$. The *prism graph* $D_n, n\geq 3$ is a cubic graph which can be represented as a Cartesian product $P_2\times C_n$ of a path on two vertices with a cycle on $n$ vertices. The *Jewel graph* $J_n$ is the graph with vertex set $V(J_n)=\{u,v,x,y,r_i : 1\leq i\leq n\}$ and $E(J_n)=\{ur_i,vr_i : 1\leq i\leq n\}\cup\{xu,xv,xy,yu,yv\}$. *Bistar* $B_{m,n}$ is the graph obtained by joining the center(apex) vertices of $K_{1,m}$ and $K_{1,n}$ by an edge. A class of planar graphs as graphs obtained by removing certain edges from the corresponding complete graphs. The classes of planar graphs so obtained are in $Pl_n$ and contained the maximum number of edges possible in a planar graph on $n$ vertices. The *splitting graph* $Spl(G)$ is obtained by adding a new vertex $v'$ corresponding to each vertex $v$ of $G$ such that $N(v)=N(v')$. The *total graph* $T(G)$

of the graph $G$ has the vertex set $V(G) \cup E(G)$ in which two vertices are adjacent whenever they are either adjacent or incident in $G$. For a simple connected graph $G$ the *square of graph G* is denoted by $G^2$ and defined as the graph with the same vertex set as of $G$ and two vertices are adjacent in $G^2$ if they are at a distance 1 or 2 apart in $G$. The middle graph $M(G)$ of a graph $G$ is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of $G$ or one is a vertex of $G$ and the other is an edge incident with it."

In section 2, we prove that some graphs are strongly edge multiplicative.

## 2. Strongly Edge Multiplicative Labeling of Some Graphs

**Theorem 2.1.** *The Fan $f_u$ is strongly edge multiplicative graph.*

*Proof.* Let $f_u$ be the fan graph. Then $V(f_u) = \{r, r_\alpha : 1 \le \alpha \le u\}$ and $E(f_u) = \{rr_\alpha : 1 \le \alpha \le u\} \cup \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\}$. Clearly $|V(f_u)| = u+1$ and $|V(f_u)| = 2u-1$. Define a labeling $g: E(f_u) \to \{1, 2, 3, \dots, 2u-1\}$ by $g(rr_\alpha) = \alpha$ for $1 \le \alpha \le u$, $g(r_\alpha r_{\alpha+1}) = u + \alpha$ for $1 \le \alpha \le u-1$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(f_u)$ are distinct. Hence the fan $f_u$ is strongly edge multiplicative graph.

**Theorem 2.2.** *The shell graph $S_u, u \ge 5$ is strongly edge multiplicative graph.*

*Proof.* Let $S_u, u \ge 5$ be the shell graph. $V(S_u) = \{r_\alpha : 1 \le \alpha \le u\}$ and $E(S_u) = \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\} \cup \{r_1 r_\alpha : 3 \le \alpha \le u-1\} \cup \{r_1 r_u\}$. Clearly, $|V(S_u)| = u$ and $|E(S_u)| = 2u-3$. Define a labeling $g: E(S_u) \to \{1, 2, 3, \dots, 2u-3\}$ by $g(r_\alpha r_{\alpha+1}) = \alpha$ for $1 \le \alpha \le u-1$, $g(r_1 r_u) = u$, $g(r_1 r_\alpha) = u + \alpha - 2$ for $3 \le \alpha \le u-1$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(S_u)$ are distinct. Hence $S_u$ is strongly edge multiplicative graph.

**Theorem 2.3.** *The gear $G_u$ is strongly edge multiplicative graph.*

*Proof.* Suppose $G_u$ is a gear graph. Then $V(G_u) = \{r, r_1, r_2, r_3, \dots, r_{2u}\}$ and $E(G_u) = \{rr_{2\alpha-1} : 1 \le \alpha \le u\} \cup \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le 2u-1\} \cup \{r_1 r_{2u}\}$. Here, $|V(G_u)| = 2u+1$ and $|E(G_u)| = 3u$.

Define a labeling $g: E(G_u) \to \{1, 2, 3, \dots, 3u\}$ by $g(rr_{2\alpha-1}) = \alpha$ for $1 \le \alpha \le u$, $g(r_\alpha r_{\alpha+1}) = 3u + 1 - \alpha$ for $1 \le \alpha \le 2u-1$, $g(r_1 r_{2u}) = u+1$.

It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(G_u)$ are distinct. Hence $G_u$ is strongly edge multiplicative graph.

**Theorem 2.4.** *The Flower graph $Fl_u$ is strongly edge multiplicative graph.*

*Proof.* Let $Fl_u$ be the Flower graph. Then $V(Fl_u) = \{r, r_\alpha, r'_\alpha : 1 \le \alpha \le u\}$ and $E(Fl_u) = \{rr_\alpha, rr'_\alpha, r_\alpha r'_\alpha : 1 \le \alpha \le u\} \cup \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\} \cup \{r_1 r_u\}$. Clearly $Fl_u$ has $2u+1$ vertices and $4u$ edges. Define a labeling $g: E(Fl_u) \to \{1, 2, 3, \dots, 4u\}$ by, for $1 \le \alpha \le u$, $g(rr_\alpha) = \alpha$, $g(r_\alpha r'_\alpha) = 2u + \alpha$, $g(rr'_\alpha) = 4u - (\alpha - 1)$. $g(r_\alpha r_{\alpha+1}) = 2u + 1 - \alpha$ for $1 \le \alpha \le u-1$, $g(r_1 r_u) = u+1$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(Fl_u)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct Hence the Flower $Fl_u$ is strongly edge multiplicative graph.

**Theorem 2.5.** *The Helm $H_u$ is strongly edge multiplicative graph.*

*Proof.* Let $H_u$ be the Helm graph. Then $V(H_u) = \{r, r_\alpha, r'_\alpha : 1 \le \alpha \le u\}$ and $E(H_u) = \{rr_\alpha, r_\alpha r'_\alpha : 1 \le \alpha \le u\} \cup \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\} \cup \{r_1 r_u\}$. Here, $V(H_u) = 2u+1$ and $E(H_u) = 3u$. Define a labeling $g: E(Fl_u) \to \{1, 2, 3, \dots, 3u\}$ by, for $1 \le \alpha \le u$, $g(rr_\alpha) = \alpha$, $g(r_\alpha r'_\alpha) = 2u + \alpha$. $g(r_\alpha r_{\alpha+1}) = 2u + 1 - \alpha$ for $1 \le \alpha \le u-1$, $g(r_1 r_u) = u+1$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(H_u)$ are distinct. Hence the Helm $H_u$ is strongly edge multiplicative graph.

**Theorem 2.6.** *The Umbrella $U(u, v)$ is strongly edge multiplicative graph.*

*Proof.* Let $U(u, v)$ be the Umbrella graph. Then $V(U(u, v)) = \{r_\alpha, s_\beta : 1 \le \alpha \le u, 1 \le \beta \le v\}$ and $E(U(u, v)) = \{s_1 r_\alpha : 1 \le \alpha \le u\} \cup \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\} \cup \{s_\beta s_{\beta+1} : 1 \le \beta \le v-1\}$. Note that, $|V(U(u, v))| = u + v$ and $|E(U(u, v))| = 2u + v - 2$. Define a labeling $g: E(U(u, v)) \to \{1, 2, 3, \dots, 2u + v - 2\}$ by $g(s_1 r_\alpha) = \alpha$ for $1 \le \alpha \le u$, $g(s_\beta s_{\beta+1}) = u + v - \beta$ for $1 \le \beta \le v-1$, $g(r_\alpha r_{\alpha+1}) = u + v + \alpha - 1$ for $1 \le \alpha \le u-1$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(U(u, v))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Umbrella $U(u,v)$ is strongly edge multiplicative graph.

**Theorem 2.7.** *The flower pot $fp(u,v), u \geq 3$ is strongly edge multiplicative graph.*

*Proof.* Let $G = fp(u,v), u \geq 3$ be the flower pot graph. Then $V(G) = \{r_\alpha, s_\beta : 1 \leq \alpha \leq u, 1 \leq \beta \leq v\}$ and $E(G) = \{r_1 s_\beta : 1 \leq \beta \leq v\} \cup \{r_\alpha r_{\alpha+1} : 1 \leq \alpha \leq u-1\} \cup \{r_1 r_u\}$. Note that, $G$ has $u+v$ vertices and $u+v$ edges. Define a labeling $g: E(G) \to \{1,2,3,\dots,u+v\}$ by $(r_1 s_\beta) = \beta$ for $1 \leq \beta \leq v$, $g(r_\alpha r_{\alpha+1}) = v + \alpha$ for $1 \leq \alpha \leq u-1$, $g(r_1 r_u) = u+v$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(G)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.8.** *The Coconut tree $CT(u,v)$ is strongly edge multiplicative graph.*

*Proof.* Let $CT(u,v)$ be the Coconut tree graph with $V(CT(u,v)) = \{r_\alpha, s_\beta : 1 \leq \alpha \leq u, 1 \leq \beta \leq v\}$ and $E(CT(u,v)) = \{s_1 r_\alpha : 1 \leq \alpha \leq u\} \cup \{s_\beta s_{\beta+1} : 1 \leq \beta \leq v-1\}$. Then $|V(CT(u,v))| = u+v$ and $|E(CT(u,v))| = u+v-1$. Define a labeling $g: E(CT(u,v)) \to \{1,2,3,\dots,u+v-1\}$ by $g(s_1 r_\alpha) = \alpha$ for $1 \leq \alpha \leq u$, $g(s_\beta s_{\beta+1}) = u+v-\beta$ for $1 \leq \beta \leq v-1$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(CT(u,v))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Coconut tree $CT(u,v)$ is strongly edge multiplicative graph.

**Theorem 2.9.** *The Butterfly $B(u,v,w), u,v \geq 3$ is strongly edge multiplicative graph.*

*Proof.* Let $G = B(u,v,w), u,v \geq 3$ be the Butterfly graph. Then $V(G) = \{r, r_\alpha, s_\beta, t_\gamma : 1 \leq \alpha \leq u, 1 \leq \beta \leq v, 1 \leq \gamma \leq w\}$ and $E(G) = \{rr_\alpha, rs_\beta, rt_\gamma : 1 \leq \alpha \leq u, 1 \leq \beta \leq v, 1 \leq \gamma \leq w\} \cup \{r_\alpha r_{\alpha+1}, s_\beta s_{\beta+1} : 1 \leq \alpha \leq u-1, 1 \leq \beta \leq v-1\}$. Clearly, $|V(G)| = u+v+w+1$ and $|E(G)| = 2(u+v-1)+w$. Without loss of generality we assume that $u \leq v$. Define a labeling $g: E(G) \to \{1,2,3,\dots,2(u+v-1)+w\}$ by $g(rr_\alpha) = \alpha$ for $1 \leq \alpha \leq u$, $g(rs_\beta) = u+\beta$ for $1 \leq \beta \leq v$, $g(rt_\gamma) = u+v+\gamma$ for $1 \leq \gamma \leq w$, $g(r_\alpha r_{\alpha+1}) = 2(u+v)+w-(\alpha+1)$ for $1 \leq \alpha \leq u-1$, $g(s_\beta s_{\beta+1}) = u+2v+w-\beta$ for $1 \leq \beta \leq v-1$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(G)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.10.** *The Butterfly $B_{u,v}$ is strongly edge multiplicative graph.*

*Proof.* Let $B_{u,v}$ be the Butterfly graph with $V(B_{u,v}) = \{r, r_\alpha : 1 \leq \alpha \leq 2u-2\} \cup \{t_\beta : 1 \leq \beta \leq v\}$ and $E(B_{u,v}) = \{rr_\alpha : \alpha = 1, u-1, u, 2u-2\} \cup \{rt_\beta : 1 \leq \beta \leq v\} \cup \{r_\alpha r_{\alpha+1} : 1 \leq \alpha \leq u-2, u \leq \alpha \leq 2u-3\}$. Then $|V(B_{u,v})| = 2u+v-1$ and $|E(B_{u,v})| = 2u+v$. Without loss of generality we assume that $u \leq v$. Define a labeling $g: E(B_{u,v}) \to \{1,2,3,\dots,2u+v\}$ by $g(rr_1) = 1$, $g(rr_{u-1}) = 2$, $g(rr_u) = 3$, $g(rr_{2u-2}) = 4$, $g(rt_\beta) = 4+\beta$ for $1 \leq \beta \leq v$, $g(r_\alpha r_{\alpha+1}) = v+4+\alpha$ for $1 \leq \alpha \leq u-2$, $g(r_\alpha r_{\alpha+1}) = v+4+\alpha-1$ for $u \leq \alpha \leq 2u-3$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(B_{u,v})$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Butterfly $B_{u,v}$ is strongly edge multiplicative graph.

**Theorem 2.11.** *The Jelly fish $J(u,v)$ is strongly edge multiplicative graph.*

*Proof.* Let $J(u,v)$ be the Jelly fish graph with $V(J(u,v)) = \{r, r_\alpha : 1 \leq \alpha \leq u\} \cup \{s, s_\beta : 1 \leq \beta \leq v\} \cup \{x,y\}$ and $E(J(u,v)) = \{rr_\alpha : 1 \leq \alpha \leq u\} \cup \{ss_\beta : 1 \leq \beta \leq v\} \cup \{xr, xs, yr, ys, xy\}$. Clearly, $|V(J(u,v))| = u+v+4$ and $|E(J(u,v))| = u+v+5$. Without loss of generality we assume that $u \leq v$. Define a labeling $g: E(J(u,v)) \to \{1,2,3,\dots,u+v+5\}$ by $g(rr_\alpha) = \alpha$ for $1 \leq \alpha \leq u$, $g(ss_\beta) = u+\beta$ for $1 \leq \beta \leq v$, $g(xr) = u+v+1$, $g(xs) = u+v+2$, $g(ys) = u+v+3$, $g(yr) = u+v+4$, $g(xy) = u+v+5$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(J(u,v))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the jelly fish $J(u,v)$ is strongly edge multiplicative graph.

**Theorem 2.12.** *The Bistar $B_{u,v}$ is strongly edge multiplicative graph.*

*Proof.* Let $B_{u,v}$ be the Bistar graph. Then $V(B_{u,v}) = \{r, r_\alpha, s, s_\beta : 1 \leq \alpha \leq u, 1 \leq \beta \leq v\}$ and $E(B_{u,v}) = \{rs, rr_\alpha, ss_\beta : 1 \leq \alpha \leq u, 1 \leq \beta \leq v\}$. Note that, $|V(B_{u,v})| = u+v+2$ and $|E(B_{u,v})| = u+v+1$. Without loss of generality we assume that $u \leq v$. Define a labeling $g: E(B_{u,v}) \to \{1,2,3,\dots,u+v+1\}$ by $g(rr_\alpha) = \alpha$ for $1 \leq \alpha \leq u$, $g(ss_\beta) = u+\beta$ for $1 \leq \beta \leq v$, $g(rs) =$

$u+v+1$. Observe that $g(r_\alpha)=\prod_{r_\beta\in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(B_{u,v})$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $B_{u,v}$ is strongly edge multiplicative graph.

**Theorem 2.13.** *The Triangular book $B(3,u)$ is strongly edge multiplicative graph.*

*Proof.* Let $B(3,u)$ be the triangular book graph. Then $V\big(B(3,u)\big)=\{r,s,t_\alpha : 1\le \alpha\le u\}$ and $E\big(B(3,u)\big)=\{rs, rt_\alpha, st_\alpha : 1\le \alpha\le u\}$. Note That, $|V(B(3,u))|=u+2$ and $|E(B(3,u))|=2u+1$. Define a labeling $g: E(B(3,u))\to\{1,2,3,\dots,2u+1\}$ by $g(rs)=1$ for each $1\le\alpha\le u$, $g(rt_\alpha)=2\alpha+1$, $g(st_\alpha)=2\alpha$. Observe that $g(r_\alpha)=\prod_{r_\beta\in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(B(3,u))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Triangular book $B(3,u)$ is strongly edge multiplicative graph.

**Theorem 2.14.** *The rectangular book $B_{4,u}$ is strongly edge multiplicative graph.*

*Proof.* Let $B_{4,u}$ be the book graph. Then $V\big(B_{4,u}\big)=\{r, r_\alpha, s, s_\alpha : 1\le\alpha\le u\}$ and $E(B_{4,u})=\{rs, rr_\alpha, ss_\alpha, r_\alpha s_\alpha : 1\le\alpha\le u\}$. Here, $B_{4,u}$ has $2u+2$ vertices and $3u+1$ edges.

Define a labeling $g: E\big(B_{4,u}\big)\to\{1,2,3,\dots,3u+1\}$ by $g(rs)=1$, for $1\le\alpha\le u$, $g(rr_\alpha)=2\alpha+1$ $g(ss_\alpha)=2\alpha$, $g(r_\alpha s_\alpha)=2u+1+\alpha$. Observe that $g(r_\alpha)=\prod_{r_\beta\in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha\in V(B_{4,u})$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the rectangular book $B_{4,u}$ is strongly edge multiplicative graph.

**Theorem 2.15.** *The Pentagonal Book $B_{5,u}$ is strongly edge multiplicative graph.*

*Proof.* Let $B_{5,u}$ be the Pentagonal book graph. Then $V\big(B_{5,u}\big)=\{r, s, r_\alpha, s_\alpha, t_\alpha : 1\le\alpha\le u\}$ and $E(B_{5,u})=\{rs, rr_\alpha, ss_\alpha, r_\alpha t_\alpha, s_\alpha t_\alpha : 1\le\alpha\le u\}$. Note that, $\left|V(B_{5,u})\right|=3u+2$ and $\left|E(B_{5,u})\right|=4u+1$. Define a labeling $g: E\big(B_{5,u}\big)\to\{1,2,3,\dots,4u+1\}$ as follows:

**Case (i).** If $u=2$, then $g(rs)=4u+1$, $g(rr_\alpha)=\alpha$ for $\alpha=1,2$, $g(r_\alpha t_\alpha)=\alpha+2$ for $\alpha=1,2$, $g(s_\alpha t_\alpha)=\alpha+4$ for $\alpha=1,2$, $g(ss_\alpha)=\alpha+6$ for $\alpha=1,2$.

**Case (ii).** If $u\neq 2$ then, $g(rs)=1$, for $1\le\alpha\le u$, $g(rr_\alpha)=4\alpha-2$, $g(r_\alpha t_\alpha)=4\alpha-1$, $g(s_\alpha t_\alpha)=4\alpha$, $g(ss_\alpha)=4\alpha+1$.

The labeling pattern defined above satisfies the vertex condition by $g(r_\alpha)=\prod_{r_\beta\in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha\in V(B_{5,u})$ are distinct. Hence the pentagonal book $B_{5,u}$ is strongly edge multiplicative graph.

**Theorem 2.16.** *The Jewel $J_p$ is strongly edge multiplicative graph.*

*Proof.* Let $J_p$ be the Jewel graph. Then $V\big(J_p\big)=\{u, v, x, y, r_\alpha : 1\le\alpha\le p\}$ and $E\big(J_p\big)=\{ur_\alpha, vr_\alpha : 1\le\alpha\le p\}\cup\{xu, xv, xy, yu, yv\}$. Then $\left|V(J_p)\right|=p+4$ and $\left|E(J_p)\right|=2p+5$. Define a labeling $g: E\big(J_p\big)\to\{1,2,3,\dots,2p+5\}$ by, for $1\le\alpha\le p$, $g(ur_\alpha)=2\alpha-1$, $g(vr_\alpha)=2\alpha$, $g(xu)=2p+1$, $g(xv)=2p+2$, $g(yu)=2p+3$, $g(yv)=2p+4$, $g(xy)=2p+5$. Observe that $g(r_\alpha)=\prod_{r_\beta\in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha\in V(J_p)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Jewel $J_p$ is strongly edge multiplicative graph.

**Theorem 2.17.** *The planar graph $Pl_u$ is strongly edge multiplicative graph.*

*Proof.* Let $Pl_u$ be the planar graph. Then $V(Pl_u)=\{r_\alpha : 1\le\alpha\le u\}$ and $E(Pl_u)=\{r_\alpha r_{\alpha+1} : 1\le\alpha\le u-3\}\cup\{r_\alpha r_u, r_\alpha r_{u-1} : 1\le\alpha\le u-2\}\cup\{r_{u-1}r_u\}$. Clearly, $|V(Pl_u)|=u$ and $|E(Pl_u)|=3(u-2)$. Define a labeling $g: E(Pl_u)\to\{1,2,3,\dots,3(u-2)\}$ by, for $1\le\alpha\le u-2$, $g(r_\alpha r_u)=3\alpha-2$, $g(r_\alpha r_{u-1})=3\alpha$. $g(r_\alpha r_{\alpha+1})=3\alpha-1$ for $1\le\alpha\le u-3$, $g(r_{u-1}r_u)=3u-7$. Observe that $g(r_\alpha)=\prod_{r_\beta\in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha\in V(Pl_u)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the planar graph $Pl_u$ is strongly edge multiplicative graph.

**Theorem 2.18.** *The Triangular snake $T_u, u\ge 2$ is strongly edge multiplicative graph.*

*Proof.* Let $T_u, u\ge 2$ be the Triangular snake graph. Then $V(T_u)=\{r_\alpha : 1\le\alpha\le u\}\cup\{s_\alpha : 1\le\alpha\le u-1\}$ and $E(T_u)=\{r_\alpha s_\alpha, s_\alpha r_{\alpha+1}, r_\alpha r_{\alpha+1} : 1\le\alpha\le u-1\}$. Here $|V(T_u)|=2u-1$ and $|E(T_u)|=3u-3$. Define a labeling $g: E(T_u)\to\{1,2,3,\dots,3u-3\}$ by, for each $1\le\alpha\le u-1$, $g(r_\alpha s_\alpha)=3\alpha-2$, $g(s_\alpha r_{\alpha+1})=3\alpha-1$, $g(r_\alpha r_{\alpha+1})=3\alpha$. It is easily to observe that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(T_\alpha)$ are distinct. Hence Triangular snake $T_u$ , $u \geq 2$ is strongly edge multiplicative graph.

**Theorem 2.19.** *The double Triangular snake $DT_u, u \geq 2$ is strongly edge multiplicative graph.*

*Proof.* Let $DT_u, u \geq 2$ be the double Triangular snake graph. Then $V(DT_u) = \{r_\alpha : 1 \leq \alpha \leq u\} \cup \{s_\alpha, t_\alpha : 1 \leq \alpha \leq u-1\}$ and $E(DT_u) = \{r_\alpha s_\alpha, r_\alpha t_\alpha, s_\alpha r_{\alpha+1}, t_\alpha r_{\alpha+1}, r_\alpha r_{\alpha+1} : 1 \leq \alpha \leq u-1\}$. Here, $|V(DT_u)| = 3u-2$ and $|E(DT_u)| = 5u-5$. Define a labeling $g: E(DT_u) \to \{1, 2, 3, \ldots, 5u-5\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha s_\alpha) = 5\alpha - 4$, $g(r_\alpha t_\alpha) = 5\alpha - 1$, $g(s_\alpha r_{\alpha+1}) = 5\alpha - 3$, $g(r_\alpha r_{\alpha+1}) = 5\alpha - 2$, $g(t_\alpha r_{\alpha+1}) = 5\alpha$. It is easily to observe that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(DT_u)$ are distinct. Hence the double Triangular snake $DT_u, u \geq 2$ is strongly edge multiplicative graph.

**Theorem 2.20.** *The Quadrilateral snake $Q(u)$ is strongly edge multiplicative graph.*

*Proof.* Let $Q(u)$ be the Quadrilateral snake graph. Then $V(Q(u)) = \{r_\alpha : 1 \leq \alpha \leq u+1\} \cup \{s_\alpha, t_\alpha : 1 \leq \alpha \leq u\}$ and $E(Q(u)) = \{r_\alpha s_\alpha, s_\alpha t_\alpha, t_\alpha r_{\alpha+1}, r_\alpha r_{\alpha+1} : 1 \leq \alpha \leq u\}$. Here, $|V(Q(u))| = 3u+1$ and $|E(Q(u))| = 4u$. Define a labeling $g: E(Q(u)) \to \{1, 2, 3, \ldots, 4u\}$ by, for each $1 \leq \alpha \leq u$, $g(r_\alpha s_\alpha) = 4\alpha - 3$, $g(s_\alpha t_\alpha) = 4\alpha - 2$, $g(t_\alpha r_{\alpha+1}) = 4\alpha - 1$, $g(r_\alpha r_{\alpha+1}) = 4\alpha$. It is easily to observe that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(Q(u))$ are distinct. Hence the Quadrilateral snake $Q(u)$ is strongly edge multiplicative graph.

**Theorem 2.21.** *The double Quadrilateral snake $D(Q(p))$ is strongly edge multiplicative graph.*

*Proof.* Let $G = D(Q(p))$ be the double Quadrilateral snake graph. Then $V(G) = \{r_\alpha : 1 \leq \alpha \leq u+1\} \cup \{s_\alpha, t_\alpha, u_\alpha, v_\alpha : 1 \leq \alpha \leq p\}$ and $E(G) = \{r_\alpha s_\alpha, s_\alpha t_\alpha, t_\alpha r_{\alpha+1}, r_\alpha r_{\alpha+1}, r_\alpha u_\alpha, u_\alpha v_\alpha, v_\alpha r_{\alpha+1} : 1 \leq \alpha \leq p\}$. Here $|V(G)| = 5p+1$ and $|E(G)| = 7p$. Define a labeling $g: E(G) \to \{1, 2, 3, \ldots, 7p\}$ by, for each $1 \leq \alpha \leq p$, $g(r_\alpha s_\alpha) = 7\alpha - 6$, $g(s_\alpha t_\alpha) = 7\alpha - 5$, $g(t_\alpha r_{\alpha+1}) = 7\alpha - 4$, $g(r_\alpha r_{\alpha+1}) = 7\alpha - 3$, $g(r_\alpha u_\alpha) = 7\alpha - 2$, $g(u_\alpha v_\alpha) = 7\alpha - 1$, $g(v_\alpha r_{\alpha+1}) = 7\alpha$. It is easily to observe that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(G)$ are distinct. Hence the double Quadrilateral snake $G$ is strongly edge multiplicative graph.

**Theorem 2.22.** *The Brush $B_u$ is strongly edge multiplicative graph.*

*Proof.* Let $B_u$ be the Brush graph. Then $V(B_u) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(B_u) = \{r_\alpha s_\alpha : 1 \leq \alpha \leq u\} \cup \{r_\alpha r_{\alpha+1} : 1 \leq \alpha \leq u-1\}$. Here, $B_u$ has $2u$ vertices and $2u-1$ edges. Define a labeling $g: E(B_u) \to \{1, 2, 3, \ldots, 2u-1\}$ by $g(r_\alpha s_\alpha) = \alpha$ for $1 \leq \alpha \leq u$, $g(r_\alpha r_{\alpha+1}) = u + \alpha$ for $1 \leq \alpha \leq u-1$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(B_u)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Brush $B_u$ is strongly edge multiplicative graph.

**Theorem 2.23.** *The Ladder $L_u$ is strongly edge multiplicative graph.*

*Proof.* Let $L_u$ be the Ladder graph. Then $V(L_u) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(L_u) = \{r_\alpha r_{\alpha+1}, s_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\} \cup \{r_\alpha s_\alpha : 1 \leq \alpha \leq u\}$. Clearly, $|V(L_u)| = 2u$ and $|E(L_u)| = 3u-2$. Define a labeling $g: E(L_u) \to \{1, 2, 3, \ldots, 3u-2\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha r_{\alpha+1}) = 2\alpha - 1$, $g(s_\alpha s_{\alpha+1}) = 2\alpha$. $g(r_\alpha s_\alpha) = 3u - 1 - \alpha$ for $1 \leq \alpha \leq u$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(L_u)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Ladder $L_u$ is strongly edge multiplicative graph.

**Theorem 2.24.** *The Open Ladder $O(L_u)$ is strongly edge multiplicative graph.*

*Proof.* Let $O(L_u)$ be the Open Ladder graph. Then $V(O(L_u)) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(O(L_u)) = \{r_\alpha r_{\alpha+1}, s_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\} \cup \{r_\alpha s_\alpha : 2 \leq \alpha \leq u-1\}$. Here, $|V(O(L_u))| = 2u$ and $|E(O(L_u))| = 3u-4$. Define a labeling $g: E(O(L_u)) \to \{1, 2, 3, \ldots, 3u-4\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha r_{\alpha+1}) = 2\alpha - 1$, $g(s_\alpha s_{\alpha+1}) = 2\alpha$. $g(r_\alpha s_\alpha) = 3u - 2 - \alpha$ for $2 \leq \alpha \leq u-1$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(O(L_u))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the open ladder $O(L_u)$ is strongly edge multiplicative graph.

**Theorem 2.25.** *The graph $Z - P_u$ , $u \geq 3$ is strongly edge multiplicative graph.*

*Proof.* Let $G = Z - P_u$ , $u \geq 3$ be the graph. Then $V(G) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(G) = \{r_\alpha s_{\alpha+1}, r_\alpha r_{\alpha+1}, s_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\}$. Clearly, $|V(G)| = 2u$ and $|E(G)| = 3u-3$. Define a labeling $g: E(G) \to \{1, 2, 3, \ldots, 3u-3\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha s_{\alpha+1}) = \alpha$, $g(s_\alpha s_{\alpha+1}) = u - 1 + \alpha$, $g(r_\alpha r_{\alpha+1}) = 2u - 2 + \alpha$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) =$

$\prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(G)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.26.** *The Prism $D_u$ , $u \geq 3$ is strongly edge multiplicative graph.*

*Proof.* Let $D_u$ , $u \geq 3$ be the Prism graph. Then $V(D_u) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(D_u) = \{r_\alpha r_{\alpha+1}, s_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\} \cup \{r_\alpha s_\alpha : 1 \leq \alpha \leq u\} \cup \{r_1 r_u, s_1 s_u\}$. Note that, $|V(D_u)| = 2u$ and $|E(D_u)| = 3u$. Define a labeling $g: E(D_u) \to \{1, 2, 3, \dots, 3u\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha r_{\alpha+1}) = 2\alpha - 1$, $g(s_\alpha s_{\alpha+1}) = 2\alpha$. $g(r_\alpha s_\alpha) = 3u - 1 - \alpha$ for $1 \leq \alpha \leq u$, $g(r_1 r_u) = 3u - 1$, $g(s_1 s_u) = 3u$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(D_u)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence the Prism $D_u$ is strongly edge multiplicative graph.

**Theorem 2.27.** *The Mongolian tent $M(2,u), u > 1$ is strongly edge multiplicative graph.*

*Proof.* Let $G = M(2,u), u > 1$ be the Mongolian tent graph. Then $V(G) = \{r, r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(G) = \{r r_\alpha, r_\alpha s_\alpha : 1 \leq \alpha \leq u\} \cup \{r_\alpha r_{\alpha+1}, s_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\}$. Here $|V(G)| = 2u+1$ and $|E(G)| = 4u-2$. Define a labeling $g: E(G) \to \{1, 2, 3, \dots, 4u-2\}$ by $g(r_\alpha s_\alpha) = \alpha$ for $1 \leq \alpha \leq u$, $g(r_\alpha r_{\alpha+1}) = u + \alpha$ for $1 \leq \alpha \leq u-1$, $g(s_\alpha s_{\alpha+1}) = 2u + \alpha - 1$ for $1 \leq \alpha \leq u-1$, $g(r r_\alpha) = 3u - 2 + \alpha$ for $1 \leq \alpha \leq u$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(G)$ are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.28.** *The splitting graph of star $spl(K_{1,u})$ is strongly edge multiplicative graph.*

*Proof.* Let $G = spl(k_{1,u})$ be the splitting graph of $K_{1,u}$. Then $V(G) = \{r, r_\alpha, s, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(G) = \{r r_\alpha, r s_\alpha, s r_\alpha : 1 \leq \alpha \leq u\}$. Here $|V(G)| = 2u + 2$ and $|E(G)| = 3u$. Define a labeling $g: E(G) \to \{1, 2, 3, \dots, 3u\}$ by, for $1 \leq \alpha \leq u$, $g(r s_\alpha) = \alpha$, $g(r r_\alpha) = u + \alpha$, $g(s r_\alpha) = 2u + \alpha$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(G)$ are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.29.** *The splitting graph of a path $spl(P_u)$ is strongly edge multiplicative graph.*

*Proof.* Let $G = spl(P_u)$ be the splitting graph of a path $P_u$. Then $V(G) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(G) = \{r_\alpha r_{\alpha+1}, s_\alpha r_{\alpha+1}, r_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\}$. Clearly $|V(G)| = 2u$ and $|E(G)| = 3u-3$. Define a labeling $g: E(G) \to \{1, 2, 3, \dots, 3u-3\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha s_{\alpha+1}) = 2\alpha - 1$, $g(s_\alpha r_{\alpha+1}) = 2\alpha$, $g(r_\alpha r_{\alpha+1}) = 2u - 2 + \alpha$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(G)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.30.** *The total graph of a path $T(P_u)$ is strongly edge multiplicative graph.*

*Proof.* Let $T(P_u)$ be the total graph of a path $P_u$. Then $V(T(P_u)) = \{r_\alpha : 1 \leq \alpha \leq u\} \cup \{s_\alpha : 1 \leq \alpha \leq u-1\}$ and $E(T(P_u)) = \{r_\alpha r_{\alpha+1}, r_\alpha s_\alpha, r_{\alpha+1} s_\alpha : 1 \leq i \leq n-1\} \cup \{s_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-2\}$. Clearly $|V(T(P_u))| = 2u-1$ and $|E(T(P_u))| = 4u-5$. Define a labeling $g: E(T(P_u)) \to \{1, 2, 3, \dots, 4u-5\}$ by, for $1 \leq \alpha \leq u-1$, $g(r_\alpha r_{\alpha+1}) = 4\alpha - 3$, $g(r_\alpha s_\alpha) = 4\alpha - 2$, $g(s_\alpha r_{\alpha+1}) = 4\alpha - 1$. $g(s_\alpha s_{\alpha+1}) = 4\alpha$ for $1 \leq \alpha \leq u-2$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(T(P_n))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $T(P_u)$ is strongly edge multiplicative graph.

**Theorem 2.31.** *The total graph of cycle $T(C_u), u \geq 3$ is strongly edge multiplicative graph.*

*Proof.* Let $T(C_u), u \geq 3$ be the total graph of cycle $C_u$. Then $V(T(C_u)) = \{r_\alpha, s_\alpha : 1 \leq \alpha \leq u\}$ and $E(T(C_u)) = \{r_\alpha r_{\alpha+1}, s_\alpha s_{\alpha+1}, r_\alpha s_{\alpha+1} : 1 \leq \alpha \leq u-1\} \cup \{r_\alpha s_\alpha : 1 \leq \alpha \leq u\} \cup \{r_1 r_u, s_1 s_u, s_1 r_u\}$. Clearly $|V(T(C_u))| = 2u$ and $|E(T(C_u))| = 4u$. Define a labeling $g: E(T(C_u)) \to \{1, 2, 3, \dots, 4u\}$ by $g(r_\alpha s_\alpha) = 4\alpha - 3$ for $1 \leq \alpha \leq u$. For $1 \leq \alpha \leq u-1$, $g(s_\alpha s_{\alpha+1}) = 4\alpha - 2$, $g(r_\alpha s_{\alpha+1}) = 4\alpha - 1$, $g(r_\alpha r_{\alpha+1}) = 4\alpha$, $g(s_1 s_u) = 4u - 2$, $g(s_1 r_u) = 4u - 1$, $g(r_1 r_u) = 4u$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g\left(r_\alpha r_\beta\right)$ for all $r_\alpha \in V(T(C_n))$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $G$ is strongly edge multiplicative graph.

**Theorem 2.32.** *The square graph of a path $P_u^2$ is strongly edge multiplicative graph.*

*Proof.* Let $P_u^2$ be the square graph of a path $P_u$. Then $V(P_u^2) = \{r_\alpha : 1 \leq \alpha \leq u\}$ and $E(P_u^2) = \{r_\alpha r_{\alpha+1} : 1 \leq \alpha \leq u-1\} \cup \{r_\alpha r_{\alpha+2} : 1 \leq \alpha \leq u-2\}$. Clearly $|V(P_u^2)| = u$ and $|E(P_u^2)| = 2u-3$. Define a labeling $g: E(P_u^2) \to \{1, 2, 3, \dots, 2u-3\}$ by

$g(r_\alpha r_{\alpha+1}) = \alpha$ for $1 \le \alpha \le u-1$, $g(r_\alpha r_{\alpha+2}) = 2u-2-\alpha$ for $1 \le \alpha \le u-2$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(P_u^2)$ are distinct. Hence $P_u^2$ is strongly edge multiplicative graph.

**Theorem 2.33.** *The square graph of cycle $C_u^2$ is strongly edge multiplicative graph.*

*Proof.* Let $C_u^2$ be the square graph of cycle $C_u$. Then $V(C_u^2) = \{r_\alpha : 1 \le \alpha \le u\}$ and $E(C_u^2) = \{r_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\} \cup \{r_\alpha r_{\alpha+2} : 1 \le \alpha \le u-2\} \cup \{r_1 r_u, r_u r_2, r_{u-1} r_1\}$. Clearly $|V(C_u^2)| = u$ and $|E(C_u^2)| = 2u$. Define a labeling $g: E(C_u^2) \to \{1, 2, 3, \dots, 2u\}$ by $g(r_\alpha r_{\alpha+1}) = \alpha$ for $1 \le \alpha \le u-1$. $g(r_1 r_u) = u$, $g(r_\alpha r_{\alpha+2}) = u + \alpha$ for $1 \le \alpha \le u-2$, $g(r_{u-1} r_1) = 2u-1$, $g(r_u r_2) = 2u$. Observe that $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(C_u^2)$ are distinct. It is straight forward to verify that the resulting labels on the vertices are distinct. Hence $C_u^2$ is strongly edge multiplicative graph.

**Theorem 2.34.** *The square graph of Bistar $B_{u,v}^2$ is strongly edge multiplicative graph.*

*Proof.* Let $B_{u,v}^2$ be the square graph of Bistar. Then $V(B_{u,v}^2) = \{r, r_\alpha, s, s_\beta : 1 \le \alpha \le u, 1 \le \beta \le v\}$ and $E(B_{u,v}^2) = \{rs, rr_\alpha, sr_\alpha, ss_\beta, rs_\beta : 1 \le \alpha \le u, 1 \le \beta \le v\}$. Then $|V(B_{u,v}^2)| = u + v + 2$ and $|E(B_{u,v}^2)| = 2(u+v) + 1$. Without loss of generality we assume that $u \le v$. Define a labeling $g: E(B_{u,v}^2) \to \{1, 2, 3, \dots, 2(u+v)+1\}$ by, for $1 \le \alpha \le u$, $g(rr_\alpha) = 2\alpha - 1$, $g(sr_\alpha) = 2\alpha$. For $1 \le \beta \le v$, $g(ss_\beta) = 2u - 1 + 2\beta$, $g(rs_\beta) = 2u + 2\beta$, $g(rs) = 2(u+v) + 1$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(B_{u,v}^2)$ are distinct. Hence $B_{u,v}^2$ is strongly edge multiplicative graph.

**Theorem 2.35.** *The middle graph of the path $M(P_u)$ is strongly edge multiplicative graph.*

*Proof.* Let $M(P_u)$ be the middle graph of the path $P_u$. Then $V(M(P_u)) = \{r_\alpha : 1 \le \alpha \le u\} \cup \{s_\alpha : 1 \le \alpha \le u-1\}$ and $E(M(P_u)) = \{r_\alpha s_\alpha, s_\alpha r_{\alpha+1} : 1 \le \alpha \le u-1\} \cup \{s_\alpha s_{\alpha+1} : 1 \le \alpha \le u-2\}$. Clearly $|V(M(P_u))| = 2u-1$ and $|E(M(P_u))| = 3u-4$. Define a labeling $g: E(M(P_u)) \to \{1, 2, 3, \dots, 3u-4\}$ by $g(s_\alpha s_{\alpha+1}) = \alpha$ for $1 \le \alpha \le u-2$. For $1 \le \alpha \le u-1$, $g(r_\alpha s_\alpha) = u + 2\alpha - 3$, $g(s_\alpha r_{\alpha+1}) = u - 2 + 2\alpha$. It is easily to verify that vertex labels are obtained by $g(r_\alpha) = \prod_{r_\beta \in N(r_\alpha)} g(r_\alpha r_\beta)$ for all $r_\alpha \in V(M(P_u))$ are distinct. Hence $M(P_u)$ is strongly edge multiplicative graph.

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