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ODD PRIME GRAPH FOR UNION AND GRAPH SUPERIMPOSING OF GRAPHS.

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Abstract. An *Odd prime labeling* if its vertices can be labeled distinctly from the vertex set $\{1, 3, 5, \ldots, 2l - 1\}$, where l is the number of vertices such that uv of $E(G)$ the labels assigned to the vertices u and v are relatively prime. A graph that admits an odd prime labeling is called an *odd prime graph*. An odd prime graphs for one vertex union of graphs, graph with identification of some graphs and graphs have been described in this paper.

Keywords: prime labeling, relatively prime, odd prime labeling, one vertex union of graphs.

1.Introduction.

Consider connected graphs are finite, simple and undirected. For more details the reader is referred to [1]. The symbol $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G respectively.

In 1980, Entringer originated the prime labeling, it was first studied by Tout, Dabboucy and Howalla [8] in 1982. Defined this labeling as follows. A graph has a *prime labeling* if its vertices of G can be labeled distinctly with first n positive integers such that each pair of adjacent vertices are relatively prime. Such a prime labeling is said to be *prime graph.* An odd prime labeling is a variation of prime labeling. The notion of an odd prime graph have introduced by Prajapati et. al [3].

A bijection g from vertex set $V(G)$ to $\{1, 3, 5, \ldots, 2l - 1\}$ of a graph G is called an *odd prime labeling* of G if for each edge $e = uv \in E(G)$, such that $\gcd\big(g(u), g(v)\big) = 1$. A graph which admits that labeling is said to be an *odd prime graph.*

We will show that few families of one vertex union of graphs, graph with identification of some graphs and graphs are odd prime.

Here we are listing some preliminary definitions. The following definitions are taken from [2], [5], [6], [7] "The *helm* H_n is a graph obtained from a wheel by attaching a pendent edge ateach vertex of the n – cycle. The *flower graph* Fl_n is the graph obtained from a helm H_n by joining each pendent vertex to the apex of helm. The *star graph* is a complete bipartite graph $K_{1,m}$, where m represents the number of vertices and S_m has m -1 edges. A set of graph vertices partitioned into three independent sets, such that no two graph vertices within the same set are adjacent such a graph is said to be *complete tripartite graph*. A *fan graph* f_n is defined to be the join of the complete graph K_1 and path P_n . i.e, $f_n = P_n \odot K_1$. An *udukkai graph* $A(m,n)$, (m, n ≥ 2) can be constructed by two fan graphs $2f_m$, (m ≥ 2) joining two path graphs $2P_n$, (n ≥ 2) with sharing a common vertex. i.e., $A(m,n) = 2f_m \odot 2P_n$. An *Octopus graph* O(m, n), (m, n ≥ 2) can be constructed by a fan graph f_m , (m ≥ 2) joining a star graph $K_{1,n}$ with sharing a common vertex. i.e., O(m, n)= $f_m \odot K_{1,n}$. The *drums graph* D(m, n), (m ≥ 3, n ≥ 2) can be constructed by two cycle graphs 2 C_m, m ≥ 3 joining two path graphs 2 P_n, n ≥ 2 with sharing common vertex. i.e., D(m, n) = 2 C_m+ 2 P_n.A *triangular snake* is the graph obtained from a path $x_1, x_2, \ldots, x_p$ by joining x_r and x_{r+1} to a new vertex y_r for $r = 1, 2, \ldots, p - 1$. The *planter graph* R(m, n), (m, n ≥3) can be constructed by joining a fan graph f_m ,(m ≥3) and a cycle graph C_n , (n ≥3) with sharing a common vertex . i.e., R(m, n) = $f_m \odot C_n$. A *circular ladder graph* CL_n is a 3-regular simple graph consists of two concentric n-cycle in which each of the n corresponding vertices joining by an edge. *Barycentric subdivision* is the graph obtained by inserting a vertex of degree two into every edge of original graph .Consider *barycentric subdivision of cycle* and join each newly inserted vertices of incident edges by an edge. Denote the new graph by $C_n(C_n)$ as it look like C_n inscribed in C_n. The *sunlet graph* is a graph of 2n vertices is obtained by attaching n pendent edges to the cycle C_n and it is denoted by S_n. An *umbrella graph* U(m, n) is the graph obtained by joining a path P_n with the central vertex of a fan f_m . The *ladder graph* L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with n vertices and × denotes the Cartesian product and K_2 is a complete graph with two – vertices. A *whell graph* is the graph formed by connecting single universal vertex to all vertices of a cycle.The *gear graph* G_n is obtained from the wheel by subdividing each of its rim edge. The *splitting graph* of the graph G is obtained by adding to each vertex v a new vertex v' is adjacent to every vertex which is adjacent to v in G, that is N(v) = N(v'). It is denoted by S(G). A graph G in which a vertex is distinguished from other vertices is called a rooted graph and the vertex is called the *root* of G. Let G be *rooted graph*. The graph $G^{(n)}$ obtained by identifying the roots of n copies of G is called a *one vertex union* of the n copies of G. A coconut tree CT(m, n) (m, n ≥ 2) is the graph obtained from the path P_n, by appending n new pendant edges at an end vertex of P_n. The *H graph* of a path P_n is the graph obtained from two copies of P_n with vertices $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$ if n is even. The Vanessa graph $V(m,n)$ (m, n ≥2) can be constructed by two fan graphs f_m , f_n (m, n ≥2) of the different order sharing a common vertex .i.e., V(m, n, l) one vertex union of different fans f_m , f_n and star $K_{1,l}$ which is nothing but called butterfly graph $B_{m,n,l}$ with shell orders m and n. The *switching of a vertex* v in a graph G means removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G. The graph obtained by switching of a vertex v in a graph G is denoted by G_v ".

In this paper, we determine some graphs, one vertex union of graphs and graph with superimposing of some graphs are odd prime graphs.

2. ONE VERTEX UNION OF GRAPHS AND SOME GRAPHS.

Theorem 2.1. *The one vertex union of q copies of the splitting graph of star graph admits odd prime.*

Proof. Let u be the apex vertex and u_1 , u_2 , u_3, . . . , u_r be the pendant vertices of star graph $K_{1,r}$. Construct splitting graph of star graph $K_{1,r}$ and v is new apex vertex and v_1 , v_2 , v_3, . . . , v_r be the newly added vertices of $S^q(K_{1,r})$. Let $S^q(K_{1,r})$ be the one vertex union of k copies of the splitting graph of star graph. Then vertex set V_1 ($S^q(K_{1,r})$) = $\{u,\ v^q, u_s^q,\ v_s^q : 1 \le s \le r\}$ and edge set E_1 ($S^q(K_{1,r})$) = $\{uu_s^q, uv_s^q, v^q u_s^q : 1 \le s \le r\}$. Here $|V(S^q(K_{1,r}))|$ = $q(2r+1)$+1. Define odd prime labeling g_1: V_1 ($S^q(K_{1,r})$) → $\{1, 3, 5,\ .\ .\ .\ ,\ 2q(2r+1)+1\}$ by g_1 (u) = 1, g_1 (v^q) = p_q, where p_q is the highest prime prime number such that (q -1) (4 r +2) + 2 r + 3 ≤ p_q ≤ $2q(2r+1)+1$, g_1 (v_s^q) = (q -1) (4 r +2) + 2 s + 1 for $1 \le s \le r$. Now label the remaining vertices u_s^q, $1 \le s \le r$, consecutively odd number from the set $\{(q-1)(4r+2) + 2r + 3, (q-1)(4r+2) + 2r + 5,$ . . . , $2q(2r+1)+1\}/\{p_q\}$. It is easily to verify that g_1 is odd prime labeling . Hence $S^q(K_{1,r})$ is odd prime graph.

Theorem 2.2. *The one vertex union of d copies of the barycentric subdivision of cycle $C_a(\ C_a)$, a ≥3 is odd prime graph.*

Proof. Let H = $[C_a(\ C_a)]^d$, a ≥3 be the one vertex union of d copies of the barycentric subdivision of cycle $C_a(\ C_a)$. Then vertex set V_2 (H) = $\{v_1, {v_1'}^d, v_e^d, {v_e'}^d : 2 \le e \le a\}$ and edge set E_2 (H) = $\{v_1 v_2^d,\ {v_1'}^d v_2^d, v_e^d v_{e+1}^d, {v_e'}^d v_{e+1}^d : 2 \le e \le a-1\}$ ∪ $\{v_1 v_a^d\}$ ∪ $\{v_1 {v_a'}^d\}$ ∪ $\{v_1 {v_1'}^d,\ v_e^d {v_e'}^d : 2 \le e \le a\}$. Also |V($H$)| = d (2 a -1)+1. Define odd prime labeling g_2: V_2 (H) → $\{1, 3, 5,\ .\ .\ .\ , 2(d-1)(2a-1)+4a-1\}$ by g_2 (v_1) = 1, g_2 (v_e^d) = $2(d-1)(2a-1)+4e-3$ for $2 \le e \le a$, g_2 (${v_e'}^d$) = $2(d-1)(2a-1)+4e-1$ for $1 \le e \le a$. Clearly, for any edge uv ∈ E(H), gcd (g_2 (u), g_2 (v)) = 1. Hence H = $[C_a(\ C_a)]^d$, a ≥3 is odd prime graph.

Theorem 2.3. *The one vertex union of k copies of Fl_d, $d \ge 3$ is an odd prime graph.*

Proof. Let Fl_d^k, $d \ge 3$ be the one vertex union of k copies of Flower graph Fl_d. Then vertex set V_3 $(Fl_d^k) = \{a, a_c^k, b_c^k : 1 \le c \le d\}$ and edge set $E_3(Fl_d^k) = \{aa_c^k, ab_c^k : 1 \le c \le d\} \cup \{a_c a_{c+1} : 1 \le c \le d-1\} \cup \{a_1 a_d\} \cup \{a_c b_c : 1 \le c \le d\}$. Also, $|V(Fl_d^k)| = 2kd + 1$. Define odd prime graph g_3: V_3 (Fl_d^k) → $\{1,3,5, \dots, 4kd+1\}$ by $g(a) = 1$. We have two cases to consider.
Case (1). $d \not\equiv 1(mod\ 3)$. $g_3(a_c^k) = (k-1)4d + 4c-1$ for $1 \le c \le d$, $g_3(b_c^k) = (k-1)4d + 4c + 1$ for $1 \le c \le d$. Next in case (2), we consider two subcases.
Subcase (2a). $d \equiv 1(mod\ 3)$ and $k \not\equiv 1(mod\ 3)$. Labels are in case (1).
Subcase (2b). $d \equiv 1(mod\ 3)$ and $k \equiv 1(mod\ 3)$. $g_3(a_c^k) = (k-1)4d + 4c - 1$ for $1 \le c \le d-1$, $g_3(b_c^k) = (k-1)4d + 4c + 1$ for $1 \le c \le d-1$, $g_3(a_d^k) = (k-1)4d + 4d + 1$, $g_3(b_d^k) = (k-1)4d + 4d - 1$. Clearly, for any edge $ab \in E(Fl_d^k)$, $\gcd(g_3(a), g_3(b)) = 1$. Hence Fl_d^k, $d \ge 3$, is odd prime graph.
Delete the edges ab_c^k, $1 \le c \le d$, from the above theorem, thus we get the following corollary.

Corollary 2.3 (a). *The H_d^k one vertex union of k copies of H_d is an odd prime graph.*

Theorem 2.4. *The one vertex union of b copies of circular ladder CL_c, $c \ge 3$ is odd prime graph.*

Proof. Let CL_c^b, $c \ge 3$ be the one vertex union of b copies of circular ladder CL_c. Then vertex set $V_4(CL_c^b) = \{u_1, v_1^b, u_a^b, v_a^b : 2 \le a \le c\}$ and edge set $E_4(CL_c^b) = \{u_1 u_c^b,\ u_a^b u_{a+1}^b : 1 \le a \le c-1\} \cup \{v_1^b v_c^b, v_a^b v_{a+1}^b : 1 \le a \le c-1\} \cup \{u_1 v_1^b, u_a^b v_a^b : 2 \le a \le c\}$. Also, $|V(CL_c^b)| = b(2c-1)+1$. Define odd prime labeling $g_4 : V_4(CL_c^b) \to \{1, 3, 5, \dots , 2b(2c-1)+1\}$ by $g_4(u_1) = 1$, $g_4(u_a^b) = 2(b-1)(2c-1) + 4a - 3$ for 2≤ $a \le c$, $g_4(v_a^b) = 2(b-1)(2c-1) + 4a - 1$ for $1 \le a \le c$. Clearly, for any edge $uv \in E(CL_c^b)$, $\gcd(g_4(u), g_4(v)) = 1$. Hence CL_c^b, $c \ge 3$ is odd prime graph.
Delete the edges $v_1^b v_c^b, v_a^b v_{a+1}^b$, $1 \le a \le c$, from the above theorem, we get the following corollary

Corollary 2.4 (a). *The SL_n^k one vertex union of k copies of sunlet graphs SL_n, $n \ge 3$ is odd prime.*

Theorem 2.5. *The complete tripartite graph $K_{1,1,a}$ is odd prime for every a ≥1.*

Proof. Let $K_{1,1,a}$ be the complete tripartite graph. Then vertex set V_5 ($K_{1,1,a}$) = $\{x, y,\ z_s : 1 \le s \le a\}$ and edge set E_5 ($K_{1,1,a}$) = $\{xy,\ xz_s, yz_s : 1 \le s \le a\}$. Also $|V(K_{1,1,a})|$ = $a + 2$. Define odd prime labeling g_5: V_5 ($K_{1,1,a}$) → $\{1, 3, 5,\ .\ .\ .\ ,\ 2a+3\}$ by g_5 (x) = 1. We have two cases to consider.
Case 1. $a \equiv 0$ (mod 3). g_5 (y) = 2 a + 1, g_5 (z_s) = 2 s + 1 for $1 \le s \le a-1$, g_5 (z_a) = 2 a + 3.

Case 2. $a \equiv 1, 2$ (mod 3). g_5 (y) = 2 a + 3, g_5 (z_s) = 2 s + 1 for $1 \leq s \leq a$. It is easy to satisfy that g is odd prime labeling. Hence $K_{1,1,a}$ is odd prime graph for every $a \geq 1$.

Theorem 2.6. *Every generalized star $K_{1,\ t_1,\ t_2,\ t_3,\ ...\ ,\ t_m}$ admits odd prime graph.*

Proof. The generalized star graph $K_{1,\ t_1,\ t_2,\ t_3,\ ...\ ,\ t_m}$ is obtained by joining one of the pendant vertices of each paths P_{t_1}, P_{t_2}, P_{t_3}, ..., P_{t_m} by an edge to a common vertex. Then vertex set $V_6 = \{v_r : r = 1, 2, 3,\ ...\ ,\ t_1 + t_2 + t_3 + ... + t_m + 1\}$ and edge set $E_6 = \{v_r v_{r+1} : 2 \leq r \leq t_1,\ t_1 + 2 \leq r \leq t_1 + t_2, ..., t_1 + t_2 + t_3 + ... + t_{m-1} + 2 \leq r \leq t_1 + t_2 + t_3 + ... + t_m\} \cup \{v_1 v_2\} \cup \{v_1 v_{t_1+2}\} \cup \{v_1 v_{t_1+t_2+2}\} \cup ... \cup \{v_1 v_{t_1+t_2+t_3+...+t_{m-1}+2}\}$. This graph has $t_1 + t_2 + t_3 + ... + t_m + 1$ vertices and $t_1 + t_2 + t_3 + ... + t_m$ edges. Let us define odd prime labeling $g_6 : V_6 \rightarrow \{1, 3, 5,\ ...\ ,\ 2(t_1 + t_2 + t_3 + ... + t_m + 1) - 1\}$ by $g_6(v_r) = 2r$ -1 for $1 \leq r \leq t_1 + t_2 + t_3 + ... + t_m$+1. Clearly, for any edge uv ∈ E($K_{1,\ t_1,\ t_2,\ t_3,\ ...\ ,\ t_m}$), the numbers g_6(u) and g_6(v) are relatively prime. Hence, $K_{1,\ t_1,\ t_2,\ t_3,\ ...\ ,\ t_m}$ has odd prime graph.

Adding the edges $v_1\ V_{t_r}$, $1 \leq r \leq m$, from the above theorem, we get the following corollary.

Corollary 2.6 (a). *One vertex union of m copies of C_{t_i} is odd prime graph.*

Theorem 2.7. *The union of u copies of ladder $L_e \cup L_e \cup ... \cup L_e$ is odd prime graph for all $e \geq 2$.*

Proof. Let H = $L_e \cup L_e \cup ... \cup L_e$ be the union of u copies of ladder L_e. It has $2u\ e$ vertices and u (3 e - 2) edges. Then the vertex set V_7 (H) = $\{x_t^u, y_t^u : 1 \leq t \leq e\}$ and edge set E_7 (H) = $\{x_t^u x_{t+1}^u, y_t^u y_{t+1}^u : 1 \leq t \leq e - 1\} \cup \{x_t^u y_t^u : 1 \leq t \leq e\}$. Define odd prime labeling $g_7 : V_7$ (H) → $\{1, 3, 5,\ ...\ , 4ue - 1\}$ by g_7 (x_t^u) = $(u - 1)4e + 4t$-3 for $1 \leq t \leq e$, g_7 (y_t^u) = $(u - 1)4e + 4t$-1 for $1 \leq t \leq e$. An easy check proves that g_7 is the required odd prime labeling of H. Thus H = $L_e \cup L_e \cup ... \cup L_e$ is odd prime graph for all $e \geq 2$.

GRAPH IDENTIFICATION OF SOME GRAPHS.

Theorem 3.1. *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘf_c^k that admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define a bijective function $g_8 : V_8 \rightarrow \{1, 3, 5,\ ...\ , 2l - 1\}$ with satisfied the condition of odd prime. Consider f_c^k the one vertex union of k copies of fan graph f_c with vertex set V_8 (f_c^k) = $\{e, e_t^k : 1 \leq t \leq c\}$ and edge E_8 (f_c^k) = set $\{e\ e_t^k : 1 \leq t \leq c\} \cup \{e_t^k\ e_{t+1}^k : 1 \leq t \leq c - 1\}$. We superimpose one of the vertex say u of f_c on selected vertex v_1 in G with g_8 (v_1) = 1. Now we define a new graph H = G ʘf_c^k with vertex set $V_8(H) = V_8$ (G) ∪ V_8 (f_c^k) and $E_8(H) = E_8$ (G) ∪ E_8 (f_c^k). Note that, $|V(H)| = l$ + k c. Define a bijective function $h_8 : V_8(H) \rightarrow \{1, 3, 5,\ ...\ , 2l - 1 + 2\ kc\}$ by h_8 (v) = g_8 (v) for all v∈ V(G), h_8 (e) = g_8 (v_1) =1, h_8 (e_t^k) = $2l - 1 + 2c(k - 1) + 2t$ for $1 \leq t \leq c$. We have to show that H is odd prime graph. Earlier G is odd prime graph, it is enough to prove that pq in H which is not in G, such that the numbers h(p) and h(q) are relatively prime. It is easily verified that for any edge pq ∈ E(H), gcd(h_8 (p), h_8 (q)) = 1. Thus H = G ʘf_c^k admits odd prime graph.

Theorem 3.2. *If G(l, m) has odd prime graph , then there exists a graph from the class G ʘ O^k(r, s), r, s ≥2, admits odd prime graph*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define a bijective function $g_9 : V_9$ (G) → $\{1, 3, 5,\ ...\ , 2l - 1\}$ with satisfied the property that given any two adjacent vertices have relatively prime labels. Consider H = O^k(r, s), r, s ≥2, the one vertex union of k copies of Octopus graph with vertex set V_9 ($O^k(r, s)$) = $\{d,\ d_u^k, e_v^k : 1 \leq u \leq r, 1 \leq v \leq s\}$ and edge set E_9 ($O^k(r, s)$) = $\{d\ d_u^k : 1 \leq u \leq r\} \cup \{d_u^k d_{u+1}^k : 1 \leq u \leq r - 1\} \cup \{d\ e_v^k : 1 \leq v \leq s\}$. We overlay one of the vertex say u of $O^k(r$, s) on selected vertex p_1 in G with g_9 (p_1) =1. Define a new graph G^* = G ʘ H with $V(G^*)$ = V_9 (G) ∪ V_9 (H) and $E(G^*)$ = E_9 (G) ∪ E_9 (H). Clearly, $|V(G^*)| = l$ + k (r + s) + 1. Define a bijective function h_9: $V(G^*) \rightarrow \{1, 3, 5,\ ...\ , 2l - 1,\ 2l + 1,\ ...\ , 2l + 2k(r + s) - 1\}$ $by\ h_9\ (v) = g_9\ (v)\ for$ all v ∈ V(G), h_9 (d) = g_9 (p_1) =1, h_9 (d_u^k) = $2l - 1 + 2(k - 1)(r + s) + 2u$ for $1 \leq u \leq r$, h_9 (e_v^k) = $2l - 1 + 2(k - 1)(r + s) + 2r + 2v$ for $1 \leq v \leq s$. We have to show that G^* is odd prime graph. Already G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv in G^*, which is not in G, such that the numbers h_9 (u) and h_9 (v) are relatively prime. Clearly, for any edge uv ∈ E(G^*), gcd(h_9 (u), h_9 (v)) = 1. Thus, G^* = G ʘ $O^k(r, s)$ is odd prime graph.

Theorem 3.3. *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘ $(B_{r,s,t}^k)$, (r, s ≥3 and t ≥1) admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define odd prime labeling g_{10}: $V_{10} \rightarrow \{1, 3, 5,\ ...\ , 2l - 1\}$ with satisfied the property of odd prime. Consider $B_{r,s,t}^k$, (r, s ≥3 and t ≥1) the one vertex union of k copies of butterfly graph with shell orders and r and s. Then vertex set V_{10} ($B^k_{r,s,t}$) = $\{c, c^k_a, d^k_b, e^k_o : 1 \le a \le r, 1 \le b \le s, 1 \le o \le t\}$ and edge set E_{10} ($B^k_{r,s,t}$) = $\{cc^k_a, cd^k_b, ce^k_o : 1 \le a \le r, 1 \le b \le s, 1 \le o \le t\}$ ∪ $\{c^k_a c^k_{a+1} : 1 \le a \le r-1\}$ ∪ $\{d^k_b d^k_{b+1} : 1 \le b \le s-1\}$. We superimpose one of the vertex say u of $B^k_{r,s,t}$ on selected vertex p_1 in G with g_{10} (p_1) =1. Now let us consider a new graph H = G ʘ $B^k_{r,s,t}$ with vertex set $V_{10}(H)$ = V_{10} (G) ∪ V_{10} ($B^k_{r,s,t}$) and edge set E_{10} (H) = E_{10} (G) ∪ E_{10} ($B^k_{r,s,t}$). Also, |V(H)| = l + k (r + s+ t). Define a bijective function h_{10}: V_{10} (H) → $\{1, 3, 5, \ldots, 2l-1+2\,k\,(r+s+t)\}$ by g_{10} (d) = h_{10} (d) for all v ∈ V(G), h_{10} (c) = 1, h_{10} (c^k_a) = $2l-1+2(k-1)(r+s+t)+2a$ for $1 \le a \le r$, h_{10} (d^k_a) = $2l-1+2(k-1)(r+s+t)+2r+2a$ – 2 for $1 \le a \le s$, h_{10} (e^k_a) = $2l-1+2(k-1)(r+s+t)+2r+2s+2a$ – 2 for $1 \le a \le t$. In order to show that H is odd prime graph. Clearly G is odd prime graph, it is enough to prove that for any two adjacent vertices uv ∈ H, which is not in G, gcd (h_{10}(u), h_{10} (v)) = 1. For any edge uv ∈ E(H), the numbers h_{10}(u) and h_{10}(v) are relatively prime. Thus, H = G ʘ $B^k_{r,s,t}$ is odd prime graph.

In Prajapati et. al., [3] proved that flower graph Fl_n and deduce that helm graph H_n is odd prime. From this, we observe that the following corollary.

Corollary 3.3(a). *The graph obtained by switching of an apex in helm H_n is odd prime.*

Theorem 3.4. *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘ D^k(m, n), that admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define odd prime labeling g_{11}: V_{11} → $\{1, 3, 5, \ldots, 2l-1\}$ with satisfy the property of odd prime graph. Consider D^k(m, n) , m ≥3, n ≥2, the one vertex union of k copies of Drum graph with vertex set V_{11} (D^k(m, n)) = $\{r, r^k_b, s^k_c : 1 \le b \le 2m-2, 1 \le c \le 2n-2\}$ and edge set E_{11} (D^k(m, n)) = $\{rr^k_b : b = 1, m-1, m, 2m-2\}$ ∪ $\{r^k_b r^k_{b+1} : 1 \le b \le m-2,\ m \le b \le 2m-3\}$ ∪ $\{rs^k_b : b = 1, n\}$ ∪ $\{s^k_b s^k_{b+1} : 1 \le b \le n-2,\ n \le b \le 2n-3\}$. We identify one of the vertex say u of D^k(m, n) on selected vertex p_1 in G with g_{11} (p_1) =1. Now let us construct a new graph H = G ʘ D^k(m, n) with vertex set $V_{11}(H)$ = V_{11} (G) ∪ V_{11} (D^k(m, n)) and edge set E_{11} (H) = E_{11} (G) ∪ E_{11} (D^k(m, n)). Note that, |V(H)| = l + k (2(m-1) + 2(n-1)). Define odd prime labeling h_{11}: V_{11} (H) → $\{1, 3, 5, \ldots, 2l-1+2k(2(m-1)+2(n-1))\}$ by g_{11} (v) = h_{11} (v) for all v∈ V(G), h_{11} (r^k_b) = $2l-1+2(k-1)(2(m-1)+2(n-1))+2b$ for $1 \le b \le 2m$-2 , h_{11} (s^k_c) = $2l-1+2(k-1)(2(m-1)+2(n-1))+4(m-1)+2c$ for $1 \le c \le 2n-2$. We have to show that H is odd prime graph. Already, G is odd prime graph, it is sufficient to prove that pq in H, which is not in G, such that the numbers h_{11} (p) and h_{11}(q) are relatively prime. It is easily verify that for any edge pq ∈ E(H), gcd(h_{11}(p), h_{11}(q)) = 1. Thus, H = G ʘ D^k(m, n), m ≥3, n ≥2 is odd prime graph.

Theorem 3.5. *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘ A^k(s, t), s, t ≥2, that admits odd prime.*

*Proof***.** Let G(l, m) be odd prime graph with l vertices and m edges. Define odd prime labeling g_{12} : V_{12} → $\{1, 3, 5, \ldots, 2l-1\}$ with satisfied the property of odd prime graph. Let A^k(s, t) , s, t ≥2, be the one vertex union of k copies of Udukkai graph with vertex set V_{12} (A^k(s, t)) = $\{d,\ d^k_p, e^k_q : 1 \le p \le 2s, 1 \le q \le 2t\}$ and edge set E_{12} (A^k(s, t)) = $\{dd^k_p : 1 \le p \le 2s\}$ ∪ $\{d^k_p d^k_{p+1} : 1 \le p \le s-1,\ s+1 \le p \le 2s-1\}$ ∪ $\{de^k_q : q = 1, t\}$ ∪ $\{e^k_q e^k_{q+1} : 1 \le q \le t-2,\ t \le q \le 2t-3\}$. We overlap one of the vertex say u of A^k(s, t) on selected vertex p_1 in G with g_{12} (p_1) =1. Now let us construct a new graph H = G ʘ A^k(s, t) with vertex set $V_{12}(H)$ = V_{12} (G) ∪ V_{12}(A^k(s, t)) and edge set E_{12} (H) = E_{12} (G) ∪ E_{12} (A^k(s, t)). Also, |V(H)| = l + k (2 s + 2(t -1)). Define odd prime labeling h_{12}: V_{12} (H) → $\{1, 3, 5, \ldots, 2l-1+2k(2s+2(t-1))\}$ by g_{12} (v) = h_{12} (v) for all v ∈ V(G), h_{12} (d^k_p) = $2l-1+2(k-1)(2s+2(t-1))+2p$ for $1 \le p \le 2s$, h_{12} (e^k_q) = $2l-1+2(k-1)(2s+2(t-1))+4s+2q$ for $1 \le q \le 2t-3$. We have to show that H is odd prime graph. Earlier, G is odd prime graph, it is enough to prove that uv in H, which is not in G, such that the numbers h_{12}(u) and h_{12}(v) are relatively prime. It is easily verify that for any edge uv ∈ E(H), gcd (h_{12} (u), h_{12} (v)) = 1. Thus, H = G ʘ A^k(s, t) is odd prime graph.

Theorem 3.6. *G(l, m) has odd prime graph, then there exists a graph from the class G ʘ R^k(a, b), a ≥2, b ≥3, that admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define odd prime labeling g_{13}: V_{13}(G) → $\{1, 3, 5, \ldots, 2l-1\}$ with satisfied the condition of odd prime graph. Consider H = R^k(a, b) , a≥2, b ≥3, the one vertex union of k copies of planter graph with vertex set V_{13} (R^k(a, b)) = $\{c,\ c^k_q, d^k_r : 1 \le q \le a, 1 \le r \le b\}$ and edge set E_{13} (R^k(a, b)) = $\{c\,c^k_q : 1 \le q \le a\}$ ∪ $\{c^k_q c^k_{q+1} : 1 \le q \le a-1\}$ ∪ $\{c\,d^k_1\}$ ∪ $\{d^k_r d^k_{r+1} : 1 \le r \le b-2\}$ ∪ $\{c\,d^k_{b-1}\}$. We overlay one of the vertex say u of R^k(a, b) on selected vertex p_1 in G with g_{13} (p_1) =1. Now define a new graph G^* = G ʘ H with V(G^*) = V_{13} (G) ∪ V_{13} (H) and E(G^*) =E_{13} (G) ∪ E_{13} (H). Also, |V(G^*)| = l + k ($a+b$ - 1). Define odd prime labeling h_{13}: V(G^*) → $\{1, 3, 5, \ldots, 2l-1,\ 2l+1, \ldots, 2l+2k(a+b-1)-1\}$ by h_{13} (v) = g_{13} (v) for all v ∈ V(G), h_{13} (c) = g_{13} (p_1) =1, h_{13} (c^k_q) = $2l-1+2(k-1)(a+b-1)+2q$ for $1 \le q \le a$, h_{13} (d^k_r) = $2l-1+2(k-1)(a+b-1)+2a+2r$ for $1 \le r \le b-1$. We have to show that G^* is odd prime graph. Already G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv in G^*, which is not in G, such that the numbers h_{13} (u) and h_{13} (v) are relatively prime. It is easily verify that for any edge uv ∈ E(G^*), gcd(h_{13} (u), h_{13} (v)) = 1. Thus, G^* = G ʘ H = R^k(a, b), a ≥2, b ≥3, is odd prime graph.

From the above theorem, delete the edges $c\ d_b^k$, we get the following corollary,

Corollary 3.6(a). *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘ U^k(a, b), a, b ≥2 that admits odd prime.* From the above corollary, delete the edges $c_q^k\ c_{q+1}^k$ $(1 \le q \le m)$, thus we have the following corollary.

Corollary 3.6(b). *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘ C T^k(r, s), r, s ≥2 that admits odd prime.*

Theorem 3.7. *If G(l, m) has odd prime graph, then there exists a graph from the class G ʘH_p^r that admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define a bijective function g_{14}: $V_{14} \rightarrow \{1,3,5,\ ...\ ,2l-1\}$ with satisfy the property of odd prime graph. Let H_p^k be the one vertex union of r copies of H- graph with vertex set $V_{14}(H_p^r)$ = $\{s_e^r : 1 \le e \le p\} \cup \left\{t_e^r : 1 \le e \le \left\lceil\frac{p}{2}\right\rceil - 1\right\} \cup \left\{t_e^r : \left\lceil\frac{p}{2}\right\rceil + 1 \le e \le p\right\} \cup \left\{t_{\left\lceil\frac{p}{2}\right\rceil}\right\}$ and edge set E_{14} (H_p^r) = $\{s_e^r\ s_{e+1}^r : 1 \le e \le p - 1\} \cup \left\{t_e^r\ t_{e+1}^r : 1 \le e \le \left\lceil\frac{p}{2}\right\rceil - 1\right\} \cup \left\{t_e^r t_{e+1}^r : \left\lceil\frac{p}{2}\right\rceil + 1 \le e \le p - 1\right\} \cup \left\{t^r_{\left\lceil\frac{p}{2}\right\rceil-1} t_{\left\lceil\frac{p}{2}\right\rceil}\right\} \cup \left\{t_{\left\lceil\frac{p}{2}\right\rceil} t_{\left\lceil\frac{p}{2}\right\rceil+1}\right\} \cup \left\{s^r_{\frac{p+1}{2}} t_{\left\lceil\frac{p}{2}\right\rceil}\ for\ p\ is\ odd\right\}$ or $\left\{s^r_{\frac{p}{2}+1} t_{\left\lceil\frac{p}{2}\right\rceil}\ for\ p\ is\ even\right\}$. We overlap one of the vertex say $t_{\left\lceil\frac{p}{2}\right\rceil}$ of H_p^r on selected vertex of p_1 in G with $g_{14}(p_1)$ = 1. Now we define a new graph H = G ʘH_p^r with vertex set $V_{14}(H) = V_{14}$ (G) ∪ V_{14} (H_p^r) and $E_{14}(H) = E_{14}$ (G) ∪ E_{14} (H_p^r). Also, $|V(H)| = l + r$ $(2p$ - 1). Define a bijective function h_{14}: $V_{14}(H) \rightarrow \{1,3,5,\ ...\ ,2l\ -\ 1 + 2\ r\ (4p\ -\ 1)\}$ by g_{14}(v) = h_{14} (v) for all v ∈ V(G), h_{14} $(t_{\left\lceil\frac{p}{2}\right\rceil})$ = g_{14} $(t_{\left\lceil\frac{p}{2}\right\rceil})$ = 1, $h_{14}(s_e^r) = 2l - 1 + (r-1)(4p-1) + e$ for $1 \le e \le p$, $h_{14}(t_e^r) = 2l - 1 + (r-1)(4p-1) + 2p + e$ for $1 \le e \le \left\lceil\frac{p}{2}\right\rceil - 1$, $h_{14}(t_e^r) = 2l - 1 + (r-1)(4p-1) + 2p + e$ - 1 for $\left\lceil\frac{p}{2}\right\rceil + 1 \le e \le p$. We have to prove that H is odd prime graph. Earlier G is odd prime graph, it is sufficient to show that for any adjacent vertices uv ∈ H, which are not in G, the numbers h_{14}(u) and h_{14}(v) are relatively prime. It is easily satisfied that for any edge uv ∈ H, gcd(h_{14}(u), h_{14}(v)) = 1. Hence H = G ʘH_p^r admits odd prime graph.

Theorem 3.8. *G(l, m) has odd prime graph, then there exists a graph from the class G ʘ $[C_a(\ C_a)]^s$, a ≥3 that admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define a bijective function g_{15}:$V_{15} \rightarrow \{1,3,5,\ ...\ ,2l-1\}$ with satisfy the property of odd prime graph. Consider H = $[C_a(\ C_a)]^s$, a ≥3 the one vertex union of s copies of the barycentric subdivision of cycle $C_a(\ C_a)$ with vertex set $V_{15}(H) = \{v_1, {v_1}'^s, v_e^s, {v_e}'^s : 2 \le e \le a\}$ and edge set $E_{15}(H) = \{v_1 v_2^s, {v_1}'^s v_2^s, v_e^s v_{e+1}^s, {v_e}'^s v_{e+1}^s : 2 \le e \le a-1\} \cup \{v_1 v_a^s\} \cup \{v_1 {v_a}'^s\} \cup \{v_1 {v_1}'^s,\ v_e^s {v_e}'^s : 2 \le e \le a\}$. We superimpose one of the vertex say v_1 of H on selected vertex of p_1 in G with g_{15} (p_1) = 1. Let us construct a new graph G^* = G ʘ H with vertex set V(G^*) = V_{15}(G) ∪ $V_{15}(H)$ and edge set E(G^*) = E_{15}(G) ∪ $E_{15}(H)$. Also, |V(G^*)| = l + s (2 a -1)+1. Define odd prime labeling h_{15}: V(G^*) → $\{1,3,5,\ ...\ ,\ 2l-1,\ 2l+1, ...\ ,\ 2l-1+2(s-1)(2a-1)+4a-1\}$ by g_{15}(v) = h_{15} (v) for all v∈ V(G), $g_{15}(p_1) = h_{15}(v_1)$ =1, $h_{15}(v_e^s) = 2l - 1 + 2(s-1)(2a-1) + 4e - 4$ for $2 \le e \le a$, h_{15} $({v_e}'^s) = 2l - 1 + 2(s-1)(2a-1) + 4e - 2$ for $1 \le e \le a$. We have to prove that G^* is odd prime graph. Already, G is odd prime graph, it is sufficient to show that for any two adjacent vertices uv ∈ G^*, which is not in G, gcd (h_{15}(u), h_{15}(v)) = 1. It is easily verify that for any edge uv ∈ G^* such that the numbers h_{15}(u) and h_{15}(v) are relatively prime. Hence G^* = G ʘ H = $[C_a(\ C_a)]^s$, a ≥3 admits odd prime graph.

Theorem 3.9. *If G has odd prime graph, then there exists graph from the class $G \odot Fl_a^s$, $a \ge 3$ that admits odd prime.*

Proof. Let $G(l,m)$ be odd prime graph with l vertices and m edges. Define odd prime labeling $g_{16}: V_{16} \rightarrow \{1,3,5,...,2l-1\}$ with satisfy the property of odd prime graph. Consider the one vertex union of s copies of the Flower graph Fl_a^s, $a \ge 3$ with vertex set $V_{16}(Fl_a^s) = \{d, d_c, e_c : 1 \le c \le a\}$ and edge set $E_{16}(Fl_a^s) = \{dd_c, de_c:\ 1 \le c \le a\} \cup \{d_c d_{c+1} : 1 \le c \le a-1\} \cup \{d_1 d_a\} \cup \{d_c e_c : 1 \le c \le a\}$. We identify one of the vertex say u of Fl_a^s on selected vertex p_1 in G with $g_{16}(p_1) = 1$. Now let us construct a new graph $G^* = G \odot Fl_a^s$ with vertex set $V(G^*) = V_{16}(G) \cup V_{16}(Fl_a^s)$ and edge set $E(G^*) = E_{16}(G) \cup E_{16}(Fl_a^s)$. Note that, $|V(G^*)| = l + 2sa$. Define odd prime graph labeling $h_{16}: V(G^*) \rightarrow \{1,3,5,...,2l-1+4sa\}$ by $g_{16}(v) = h_{16}(v)$ for all $v \in V(G)$, $h_{16}(d) = 1$. We have two cases to consider.
Case (1). $2l - 1 \not\equiv 1 (mod\ 3)$ and for every a, that is $a \equiv 0,1,2 (mod\ 3)$ and for every s, $s \equiv 1 (mod\ 3)$ $h_{16}(d_c^s) = 2l - 1 + (s-1)4a + 4c - 1$ for $1 \le c \le a$, $h_{16}(e_c^s) = 2l - 1 + (s-1)4a + 4c + 1$ for $1 \le c \le a$. Next in **case (2)**, we consider two subcases.
Subcase (2a). $2l - 1 \equiv 1 (mod\ 3)$, $a \not\equiv 1 (mod\ 3)$ and $s \not\equiv 1 (mod\ 3)$. Labels are in case (1).
Subcase (2b). $2l - 1 \equiv 1 (mod\ 3)$, $a \equiv 1 (mod\ 3)$, and $s \equiv 1 (mod\ 3)$. $h_{16}(d_c^s) = 2l - 1 + (s-1)4a + 4c - 1$ for $1 \le c \le a-1$, $h_{16}(e_c^s) = 2l - 1 + (k-1)4a + 4c + 1$ for $1 \le c \le a-1, h_{16}(d_a^s) = 2l - 1 + (s-1)4a + 4a + 1, h_{16}(e_a^s) = 2l - 1 + (s-1)4a + 4a - 1$. To prove that G^* is odd prime graph. Earlier, G is odd prime graph, it is enough to prove that $pq \in G^*$, which are not in G, $\gcd(h_{16}(p), h_{16}(q)) = 1$. We can easily check that for any edge $pq \in E(G^*)$ such that the numbers $h_{16}(p)$ and $h_{16}(q)$ are relatively prime. Hence $G \odot Fl_a^s$, $a \ge 3$, admits odd prime graph.
Delete the edges de_q^s, $1 \le q \le a$, from the above theorem, thus we get the following corollary.

Corollary 3.9(a). *If G has odd prime graph, then there exists graph from the class $G \odot H_a$, that admits odd prime.*

Theorem 3.10. *If G has odd prime graph, then there exists graph from the class $G \odot G_r$ that admits odd prime.*

Proof. Let $G(l,m)$ be odd prime graph with l vertices and m edges. Define an odd prime labeling $g_{17}: V_{17} \to \{1,3,5,\ldots,2l-1\}$ with satisfied the property of odd prime graph. Consider the gear graph G_r with vertex set $V_{17}(G_r) = \{e, e_t: 1 \le t \le 2r\}$ and edge set $E_{17}(G_r) = \{ee_{2t}, 1 \le t \le r\} \cup \{e_t e_{t+1}: 1 \le t \le 2r-1\} \cup \{e_1 e_{2r-1}\}$. We identify one of the vertex say u of G_r on selected vertex p_1 in G with $g_{17}(p_1) = 1$. Now let us construct a new graph $G^* = G \odot G_r$ with vertex set $V(G^*) = V_{17}(G) \cup V_{17}(G_r)$ and edge set $E(G^*) = E_{17}(G) \cup E_{17}(G_r)$. Note that, $|V(G^*)| = l + 2r$. Define odd prime graph labeling $h_{17}: V(G^*) \to \{1,3,5, \ldots, 2l+4r-1\}$ by $g_{17}(v) = h_{17}(v)$ for all $v \in V(G)$, $h_{17}(e) = 1$. We have two cases to consider. **Case (1)**. $2l-1 \not\equiv 1(mod\ 3)$, i.e., $2l-1 \equiv 0,2(mod\ 3)$. $h_{17}(e_t) = 2l-1+2t$ for $1 \le t \le 2r$. Next in **case (2)**, we consider two subcases. **Subcase (2a)**. $2l-1 \equiv 1(mod\ 3)$, and $r \not\equiv 0(mod\ 4)$. $h_{17}(e_t) = 2l-1+2$ for $1 \le t \le 2r-2$, $h_{17}(e_{2r-1}) = 2l+4r-1$, $h_{17}(e_{2r}) = 2l+4r-3$. **Subcase (2b)**. $2l-1 \equiv 1(mod\ 3)$, and $r \equiv 0(mod\ 4)$. $h_{17}(e_t) = 2l-1+2t$ for $1 \le t \le 2r-3$, $h_{17}(e_{2r-2}) = 2l+4r-1, h_{17}(e_{2r-1}) = 2l+4r-3$, $h_{17}(e_{2r}) = 2l+4r-5$. To prove that G^* is odd prime graph. Earlier, G is odd prime graph, it is enough to prove that $pq \in G^*$, which are not in G, $\gcd(h_{17}(p), h_{17}(q)) = 1$. We can easily check that for any edge $pq \in E(G^*)$ such that the numbers $h_{17}(p)$ and $h_{17}(q)$ are relatively prime. Hence $G \odot G_r$ admits odd prime graph.

Theorem 3.11. *If G(l, m) has odd prime graph, then there exists a disconnected graph from the class G ∪ ($L_a \cup L_a \cup \ldots \cup L_a$) admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges and bijective function g_{18}: $V_{18} \to \{1, 3, 5, \ldots, 2l-1\}$ satisfied the condition of odd prime graph. Consider the union of c copies of ladder graph $L_a \cup L_a \cup \ldots \cup L_a$, $a \ge 2$ with vertex set $V_{18}(L_a \cup L_a \cup \ldots \cup L_a) = \{x_t^c, y_t^c: 1 \le t \le a\}$ and edge set $E_{18}(L_a \cup L_a \cup \ldots \cup L_a) = \{x_t^c x_{t+1}^c, y_t^c y_{t+1}^c : 1 \le t \le a-1\} \cup \{x_t^c y_t^c: 1 \le t \le a\}$. Let us construct a new graph G^* = G ∪ ($L_a \cup L_a \cup \ldots \cup L_a$) with vertex set V($G^*$) = V_{18} (G) ∪ $V_{18}(L_a \cup L_a \cup \ldots \cup L_a)$ and edge set E(G^*) = E_{18} (G) ∪ $E_{18}(L_a \cup L_a \cup \ldots \cup L_a)$. Note that, |V($G^*$)| = l + $2c\,a$. Define odd prime labeling h_{18}: V(G^*) → $\{1, 3, 5, \ldots, 2l-1, 2l+1, \ldots, 2l-1+4ca\}$ by g_{18} (v) = h_{18}(v) for all v ∈ V(G), for each $1 \le t \le a$, $h_{18}(x_t)$ = $2l+(c-1)4a+4t$- 3, h_{18} (y_t) = $2l+(c-1)4a+4t$- 1. In order to show that G^* is odd prime graph. Clearly, G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv ∈ E^*, which is not in G, numbers h_{18}(u) and h_{18}(v) are relatively prime. Hence G^* = G ∪ ($L_a \cup L_a \cup \ldots \cup L_a$), $a \ge 2$ is odd prime graph.

Theorem 3.12. *If G(l, m) has odd prime graph, then there exists a disconnected graph from the class G ∪ ($T_a \cup T_a \cup \ldots \cup T_a$) admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges and bijective function g_{19}: $V_{19} \to \{1, 3, 5, \ldots, 2l-1\}$ with satisfied the condition of odd prime graph. Consider the union of k copies of triangular snake H = $T_a \cup T_a \cup \ldots \cup T_a$, $a \ge 3$, with vertex set $V_{19}(H)$ = $\{c_r^k, d_s^k: 1 \le r \le a, 1 \le s \le a-1\}$ and edge set $E_{19}(H)$ = $\{c_r^k c_{r+1}^k, c_r^k d_r^k, c_{r+1}^k d_r^k : 1 \le r \le a-1\}$. Let us construct a new graph G^* = G ∪ H with vertex set V(G^*) = V_{19}(G) ∪ $V_{19}(H)$ and edge set E(G^*) = E_{19} (G) ∪ $E_{19}(H)$. Note that, |V(G^*)| = l + k($2a$ -1). Define odd prime labeling h_{19}: V(G^*) → $\{1, 3, 5, \ldots, 2l-1, 2l+1, \ldots, 2(l+k(2a-1))-1\}$ by g_{19} (v) = h_{19}(v) for all v ∈ V(G), h_{19} (c_r) = $2l+2(k-1)(2a-1)+4r$- 3 for $1 \le r \le a$, $h_{19}(d_r)$ = $2l+2(k-1)(2a-1)+4r$- 1 for $1 \le r \le a-1$. In order to show that G^* is odd prime graph. Clearly, G is odd prime graph, it is sufficient to prove that for any two adjacent vertices uv ∈ E^*, which is not in G, the numbers h_{19}(u) and h_{19}(v) are relatively prime. Hence G^* = G ∪ H.where H = $T_a \cup T_a \cup \ldots \cup T_a$ is odd prime graph.

Theorem 3.13. *If G has odd prime graph, then there exists a graph from the class G Ө $K_{1,\ t_1,\ t_2,\ t_3,\ \ldots,\ t_m}$ that admits odd prime.*

Proof. Let G(l, m) be odd prime graph with l vertices and m edges. Define odd prime labeling g_{20}: $V_{20} \to \{1, 3, 5, \ldots, 2l-1\}$ with satisfied the property of odd prime graph. The generalized star graph $K_{1,\ t_1,\ t_2,\ t_3,\ \ldots,\ t_m}$ is obtained by joining one of the pendant vertices of each $P_{t_1}, P_{t_2}, P_{t_3}, \ldots, P_{t_m}$ by an edge to a common vertex, which is nothing but one vertex union of m copies of paths, with vertex set V_{20} = $\{v_e: e = 1, 2, 3, \ldots, t_1+t_2+t_3+\ldots+t_m+1\}$ and edge set E_{20} = $\{v_e v_{e+1}: 2 \le e \le t_1,\ t_1+2 \le e \le t_1+t_2, \ldots, t_1+t_2+t_3+\ldots+t_{m-1}+2 \le e \le t_1+t_2+t_3+\ldots+t_m\}$ ∪ $\{v_1 v_2\}$ ∪ $\{v_1 v_{t_1+2}\}$ ∪ $\{v_1 v_{t_1+t_2+2}\}$ ∪ . . . ∪ $\{v_1 v_{t_1+t_2+t_3+\ldots+t_{m-1}+2}\}$. We superimpose one of the vertex say v_1 of $K_{1,\ t_1,\ t_2,\ t_3,\ \ldots,\ t_m}$ on selected vertex p_1 in G with $g_{20}(p_1)$ = 1. Now let us construct a new graph G^* = G Ө $K_{1,\ t_1,\ t_2,\ t_3,\ \ldots,\ t_m}$ with vertex set V(G^*) = V_{20} (G) ∪ V_{20} ($K_{1,\ t_1,\ t_2,\ t_3,\ \ldots,\ t_m}$) and edge set E($G^*$) = E_{20} (G) ∪ $E_{20}(K_{1,\ t_1,\ t_2,\ t_3,\ \ldots,\ t_m})$. Note that, |V($G^*$)| = l +$t_1+t_2+t_3+\ldots+t_m$. Define odd prime labeling h_{20}: V(G^*) → $\{1, 3, 5, \ldots, 2l-1+2(t_1+t_2+t_3+\ldots+t_m)\}$ by g_{20} (v) = h_{20} (v) for all v ∈ V(G^*), h_{20} (v_1) = 1, h_{20} (v_e) = $2l-1+2e$ - 2 for $2 \le e \le t_1+t_2+t_3+\ldots+t_m$+ 1 We have to prove that G^* is odd prime graph. Earlier, G is odd prime graph, it is

enough to prove that pq in G^*, which is not in G, such that the numbers h_{20} (p) and h_{20}(q) are relatively prime. It is easily verify that for any edge pq ∈ E(G^*), gcd(h_{20}(p), h_{20}(q)) = 1. Thus, G^* = G ʘ $K_{1,\ t_1\ ,\ t_{2,}\ t_3\ ,\ ...\ ,\ t_m}$ is odd prime graph.
Adding the edges $v_1\ v_{t_e}$, $1 \leq e \leq m$ from the above theorem, we get the following result.

Corollary 3.13(a). *If G has odd prime graph, then there exists a graph from the class G ʘ C_{t_e}, $1 \leq e \leq m$ that admits odd prime graph.*

Theorem 3.14. *If $G(l,m)$ has odd prime graph, then there exists graph from the class $G \odot CL_t^s$, $t \geq 3$ that admits odd prime.*

Proof. Let $G(l,m)$ be odd prime graph with l vertices and m edges. Define a bijection function $g_{21}: V_{21} \to \{1,3,5,...\ ,2l-1\}$ with satisfied the condition of odd prime graph. Consider $CL_t^s, t \geq 3$, the one vertex union of s copies of the circular ladder CL_t with vertex set $V_{21}(CL_t^s) = \{u_1, v_1^s, u_r^s, v_r^s : 2 \leq r \leq t\}$ and edge set $E_{21}(CL_t^s) = \{u_1u_t^s,\ u_r^s u_{r+1}^s:\ 1 \leq r \leq t-1\} \cup \{v_1^s v_t^s, v_r^s v_{r+1}^s : 1 \leq r \leq t-1\} \cup \{u_1 v_{1,}^s, u_r^s v_r^s : 2 \leq r \leq t\}$. We identify one of the vertex say u_1 of CL_t^s on selected vertex p_1 in G with $g(p_1) = 1$. Let us construct a new graph $G^* = G \odot CL_t^s$ with vertex set $V(G^*) = V_{21}(G) \cup V_{21}(CL_t^s)$ and edge set $E(G^*) = E_{21}(G) \cup E_{21}(CL_t^s)$. Also, $|V(CL_t^s)| = l + s(2t-1)$. Define odd prime graph $h_{21}: V(G^*) \to \{1,3,5,...\ ,2l-1, 2l+1,...\ ,\ 2l-1+2s(2t-1)\}$ by $g_{21}(v) = h_{21}(v)$, for all $v \in V(G)$, g_{21} (p_1) = $h_{21}(u_1) = 1$, $h_{21}(u_r^s) = 2l-1+2(s-1)(2t-1)+4r-4$ for 2≤ $r \leq t$, $h_{21}(v_r^s) = 2l-1+2(s-1)(2t-1)+4r-2$ for $1 \leq r \leq t$. We have to prove that G^* is odd prime graph. Already, G is odd prime graph, it is sufficient to show that for any adjacent vertices $uv \in G^*$, which are not in G, $\gcd\big(h_{21}(u), h_{21}(v)\big) = 1$. An easy to check that the numbers $h_{21}(u)$ and $h_{21}(v)$ are relatively prime. Hence $G \odot CL_t^s, t \geq 3$, admits odd prime graph.
Delete the edges $v_1^s v_t^s, v_r^s v_{r+1}^s, 1 \leq r \leq t$, from the above theorem, we get the following corollary

Corollary 3.14(a). *If $G(l,m)$ has odd prime graph, then there exists a graph from the class $G \odot SL_t^s$, $t \geq 3$ that admits odd prime, where SL_t is sunlet graph.*

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