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FEKETE SZEGö INEQUALITY FOR NEW SUBCLASS OF HOLOMORPHIC MAPPING ASSOCIATED WITH LEMNISCATE OF BERNOULLI

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Abstract:

In this paper, we consider new subclass of holomorphic mapping connected to lemniscate of Bernoulli. Also, an approach to the estimates of Fekete szegö inequality for new subclass.

Keywords: Subordination, Fekete-Szegö Inequality, Analytic functions, Lemniscate of Bernoulli.

1 Introduction

Let A represent a class of entire holomorphic mapping f, defined in the unit disc $D = \{z: z \in C: |z| < 1\}$. Thus the function $f \in A$ has the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, (z \in D) \tag{1.1}$$

Let S denote the subclass of A consist of functions of the form (1.1) which are univalent.

For two functions $f, g \in A$, the function $f(z)$ is said to be subordinate to $g(z)$ in D, written $f(z) \prec g(z)$, if there exists a schwarz function w with satisfying the properties $w(0)$=0 and $|w(z)| < 1$ such that $f(z)$=$g(w(z))$. In particular, if the function g is univalent in D, then we have the following equivalence

$$f(z) \prec g(z) \Leftrightarrow f(0)=g(0) \text{ and } f(D) \subset g(D)$$

It is well known that the Fekete-Szegö inequality is an inequality for the coefficients of univalent analytic functions found by Fekete and Szegö [5], related to the Bieberbach conjecture.

Let P indicate the family of members consisting of p, where p is the Caratheodory function [4] of the form

$$p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n, (z \in D) \tag{1.2}$$

which are regular with $R(p(z)) > 0$ in D. Several authors created the bounds for first two coefficient $|a_2|$ and $|a_3|$. Indeed, for many results on Fekete-Szegö problems see [1],[2],[9],[10],[11],[12],[13].

2 Definition

A member f of A is in the family $f \in Sl*$, consisting of regular mappings connected to lemniscate of Bernoulli

$$\Leftrightarrow \left\{\frac{zF'(z)}{F(z)}\right\} \prec \{1+z\}^{1/2} = q(z), z \in D \tag{2.1}$$

It is noted from definition that the set q(D) lies in the region bounded by the right loop of the lemniscate of Bernoulli γ_1: $(x^2+y^2)^2-2(x^2-y^2) = 0$.

3 Preliminary Lemmas

We need the following lemmas to prove our results.

Lemma 1. Let $p \in P$ have the series expansion of the form (1.2). Then

$$2c_2 = c_1^2 + x(4-c_1^2) \tag{3.1}$$

for some x, $|x| \leq 1$ and

$$4c_3 = c_1^3 + 2(4-c_1^2)c_1x - c_1(4-c_1^2)x^2 + 2(4-c_1^2)(1-|x|^2)z \tag{3.2}$$

for some x, $|z| \leq 1$.

Remark 1.In lemma 1, for the formula for c_2, see [7],[8]. The formula for c_3 is due to Libera and Zlotkiewicz [7].

Lemma 2. If $p \in P$ and has the series of the form (1.2), then

$$|c_n+_k - \mu c_n c_k| \leq 2{,}0 \leq \mu \leq 1, \tag{3.3}$$

$$|c_n| \leq 2, n \geq 1, \tag{3.4}$$

$$|c_2 - \varsigma c_1^2| \leq 2max\{1, |2\varsigma - 1|\}, \varsigma \in D \tag{3.5}$$

Remark 2. Inequalities (3.3) and (3.4) in the above can be found in [3],[8] and (3.5) is given by [6].

4 Main Results

Theorem 1. If the function $f \in Sl^*$. Then,

$$|a_3 - \varsigma a_2^2| \leq \frac{1}{4} max\left\{1, \left|\frac{4\varsigma-1}{4}\right|\right\} \tag{4.1}$$

where

$$a_2 = \frac{c_1}{4} \tag{4.2}$$

and

$$a_3 = \frac{1}{8}\left(c_2 - \frac{3}{8}c_1^2\right) \qquad (4.3)$$

Proof: If $f \in Sl^*$ then,

$$|a_3 - \varsigma a_2^2| = \left|\frac{c_2}{8} - \frac{3}{64}c_1^2 - \varsigma\frac{c_1^2}{16}\right| \qquad (4.4)$$

By simple calculation we get

$$|a_3 - \varsigma a_2^2| = \frac{1}{8}\left|c_2 - \left(\frac{3+4\varsigma}{8}\right)c_1^2\right|$$

Using (3.5) to the above equation and after simple calculations, we get

$$|a_3 - \varsigma a_2^2| \leq \frac{1}{4} max\left\{1, \left|\frac{4\varsigma - 1}{4}\right|\right\}$$

Thus, we proved the result.

5 Conclusion

In the present study, we have introduced and studied a new subclass of holomorphic mapping in the unit disc D, which involves lemniscate of bernoulli. Moreover, we have derived the Fekete-szegö inequality for the subclass given. In future, we can come up with the third and fourth hankel determinant.

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