ISSN: 0974-5823 Vol. 7 No. 4 April, 2022

*International Journal of Mechanical Engineering*

# Graph superimposing of vertex edge neighborhood prime

**M. Simaringa[1], K.Vijayalakshmi[2]**

[1,2]PG and Research Department of Mathematics,

Thiru Kolanjiappar Govt. Arts College, Virudhachalam,Tamilnadu, India.

**Abstract:** In the present paper, we deal with few collection of union and duplicating of graphs are vertex edge neighborhood prime.

**Key Words and Phrases**: Vertex edge neighborhood prime graphs, union of graphs, duplicating of graphs.

## 1 Introduction

Consider the graphs are simple, finite and undirected. We refer [1] for standard terminology and notations. $V(G)$ and $E(G)$ denote the vertex and edge set of $G$. Let $p$ and $q$ be the cardinality of vertex and edge set is called the order and size of a graph $G$. We refer [2] the latest update of dynamic survey of graph labeling by Gallian. The following definitions are taken from [4] "prime graph, neighborhood prime graph, total neighborhood prime graph. Motivated by neighborhood prime graph and total neighborhood prime graph, Pandya and Shrimali [3] defined the concept of vertex edge neighborhood prime labeling.

Vertex edge neighborhood prime labeling is a function $f: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,p+q\}$ with the property that if degree of vertex is exactly one, then that neighborhood vertex and its incident edges are relatively prime and if the degree of vertex is at least two then their neighborhood vertices are relatively prime and its incident edges are also relatively prime. A graph which admits vertex edge neighborhood prime labeling is called *vertex edge neighborhood prime graph*."

The following definitions are taken from [3], [4], [5], [6]. "The sunlet $S_r$, wheel $W_k$, Petersen graph $P(n,2)$, quadrilateral snake $Q_n$, double triangular snake $DT_n$, prism graph $C_l \times K_2$, $k$ −polygonal book $B_{k,n}$, convex polytope $R_n$, Shell graph $S_n$, butterfly graph $BF(m,n)$, octopus graph $O_s$, planter graph $R_z$, lotus inside a circle $LC_n$, helm $H_n$, closed helm $CH_n$, udukkai graph $A_z$, barycentric cycle $BC_z$. $G^* = G * \overline{K_1}$ is obtained by joining a single pendant edge to each outer node of $G$. The Mycielskian graph $\mu(G)$ of $G$ is defined as follows: The vertex set $V(\mu(G))$ of $\mu(G)$ is the disjoint union $V \cup V' \cup u$, where $V' = \{x' : x \in V\}$ and the edge set of $\mu(G)$ is $E(\mu(G)) = E \cup \{x'y : xy \in E\} \cup \{x'u : x' \in V'\}$. If $G_1$ and $G_2$ are two connected graphs, then the graph acquired by superimposing any selected vertex of $G_2$ on any selected vertex of $G_1$ is denoted by $G_1 \odot G_2$. Duplication of a vertex $v_k$ of a graph $G$ produces a new graph $G_1$ by adding a vertex $v'_k$ with $N(v'_k) = N(v_k)$." •

In section 2,3, we prove that union of graphs and duplicating of graphs are vertex edge neighborhood prime.

## 2 Graph superimposing of union of graphs

The union $G = G_1 \cup G_2$ of graphs $G_1$ and $G_2$ with disjoint point sets $V_1$ and $V_2$ , edge sets $E_1$ and $E_2$ is the graph with $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. Here, we discuss about graph superimposing of union of graphs.

**Theorem 2.1.** If $G_1(p_1,q_1)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_1 \odot [K_{n_1} \odot \overline{K_2} \cup K_{n_2} \odot \overline{K_2} \cup \ldots \cup K_{n_m} \odot \overline{K_2}]$ that admits vertex edge neighborhood prime graph

Proof. Let $G_1(p_1,q_1)$ be vertex edge neighborhood prime graph with bijection $g_1: V(G_1) \cup E(G_1) \rightarrow \{1,2,\ldots,|V(G_1) \cup E(G_1)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider the graph $H_1 = K_{n_1} \odot \overline{K_2} \cup K_{n_2} \odot \overline{K_2} \cup \ldots \cup K_{n_m} \odot \overline{K_2}$ with

$V(H_1) = \{u_{ij} : 1 \le i \le m, 1 \le j \le n_i\} \cup \{v_{ij} : 1 \le i \le m, 1 \le j \le 2n_i\}$ and

$E(H_1) = \{u_{ij}v_{i(2j-1)}, u_{ij}v_{i(2j)} : 1 \le i \le m, 1 \le j \le n_i\} \cup \{u_{ij}u_{i(n_i+1-k)} : 1 \le i \le m, 1 \le j \le n_i - 1, 1 \le k \le n_i - j\}$.

We identify one of the vertex say $u_{11}$ of $H_1$ on selected vertex of $s_1$ in $G_1$ with $g_1(s_1) = 1$.

Let us construct a new graph $G_1^* = G_1 \odot H_1$ with $V(G_1^*) = V(G_1) \cup V(H_1)$ and $E(G_1^*) = E(G_1) \cup E(H_1)$.

$|V(G_1^*)| = p_1 + 3(n_1 + n_2 + \ldots + n_m) - 1$ and $|E(G_1^*)| = q_1 + 2(n_1 + n_2 + \ldots + n_m) + \sum_{i=1}^{m} \frac{n_i(n_i-1)}{2}$.

Define $h_1: V(G_1^*) \cup E(G_1^*) \to \left\{1,2,\ldots,p_1+q_1+5(n_1+n_2+\ldots+n_m)+\sum_{i=1}^{m} \frac{n_i(n_i-1)}{2}-1\right\}$ by

$g_1(z_1)=h_1(z_1)$ for all $z_1 \in V(G_1)$ and $g_1(e_1)=h_1(e_1)$ for all $e_1 \in E(G_1)$.

$h_1(u_{11})=h_1(s_1)=1, h_1(u_{11}v_{11})=p_1+q_1+1$.

$h_1(u_{ij}v_{i(2j)})=p_1+q_1+5\sum_{m=1}^{i-1} n_m+\sum_{m=1}^{i} \frac{n_{m-1}(n_{m-1}-1)}{2}+3j-1$ for $1 \le i \le m$ and $1 \le j \le n_i$.

For each $2 \le j \le n_1, h_1(u_{1j}v_{1(2j-1)})=p_1+q_1+3j-3, h_1(u_{1j})=p_1+q_1+3j-2$.

For each $2 \le i \le m$ and $1 \le j \le n_i$ , $h_1(u_{ij}v_{i(2j-1)})=p_1+q_1+5\sum_{m=1}^{i-1} n_m+\sum_{m=1}^{i} \frac{n_{m-1}(n_{m-1}-1)}{2}+3j-3, h_1(u_{ij})=p_1+q_1+5\sum_{m=1}^{i-1} n_m+\sum_{m=1}^{i} \frac{n_{m-1}(n_{m-1}-1)}{2}+3j-2$.

$h_1(u_{i1}u_{i(n_i+1-k)})=p_1+q_1+5\sum_{m=1}^{i-1} n_m+[\sum_{m=1}^{n_1-1} m+\sum_{m=1}^{n_2-1} m+\ldots+\sum_{m=1}^{n_{i-1}-1} m]+3n_i+k-1$ for $1 \le i \le m$ and $1 \le k \le n_i-1$.

$h_1(v_{ij})=p_1+q_1+5\sum_{m=1}^{i-1} n_m+[\sum_{m=1}^{n_1-1} m+\sum_{m=1}^{n_2-1} m+\ldots+\sum_{m=1}^{n_i-1} m]+3n_i+j-1$ for $1 \le i \le m$ and $1 \le j \le 2n_i$.

$h_1(u_{ij}u_{i(n_i+1-k)})=p_1+q_1+5\sum_{m=1}^{i-1} n_m+[\sum_{m=1}^{n_1-1} m+\sum_{m=1}^{n_2-1} m+\ldots+\sum_{m=1}^{n_{i-1}-1} m]+3n_i+\sum_{m=1}^{j-1}(n_i-m)+k-1$ for $1 \le i \le m$ and $2 \le j \le n_i-1$ and $1 \le k \le n_i-j$.

We claim that $G_1^*$ is vertex edge neighborhood prime graph. Clearly, $G_1$ is vertex edge neighborhood prime graph. We have to prove $H_1$ is vertex edge neighborhood prime graph. Let $x_1$ be any vertex of $H_1$. Consider the following two cases.

**Case 1.** If $x_1=v_{ij}$, where $1 \le i \le m$ and $1 \le j \le 2n_i$ with $deg(x_1)=1$, then $h_1(u_{ij})$, $h_1(x_1u_{ij})$ are consecutive integers.

**Case 2.** If $x_1=u_{ij}$, where $1 \le i \le m$ and $1 \le j \le n_i$ with $deg(x_1) \ge 2$, then $\{h_1(w_1): w_1 \in N_V(x_1)\}$ and $\{h_1(e_1): e_1 \in N_E(x_1)\}$ are consecutive integers.

Hence $G_1^*=G_1 \odot H_1$ is vertex edge neighborhood prime graph. ■

**Theorem 2.2.** If $G_2(p_2,q_2)$ has vertex edge neighborhood prime graph, then there exists a graph from the class $G_2 \odot [C_{r_1} \odot P_2 \odot C_{s_1} \cup C_{r_2} \odot P_2 \odot C_{s_2} \cup\ldots\cup C_{r_u} \odot P_2 \odot C_{s_v}]$, where $s_a(1 \le a \le v)$ is odd, that admits vertex edge neighborhood prime graph.

Proof. Let $G_2(p_2,q_2)$ be vertex edge neighborhood prime graph with bijection $g_2: V(G_2) \cup E(G_2) \to \{1,2,\ldots,|V(G_2) \cup E(G_2)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_2=C_{r_1} \odot P_2 \odot C_{s_1} \cup C_{r_2} \odot P_2 \odot C_{s_2} \cup\ldots\cup C_{r_u} \odot P_2 \odot C_{s_v}$, where $s_a(1 \le a \le v)$ is odd, with

$V(H_2)=\{k_{yz}, l_{yz}: 1 \le y \le i, 1 \le z \le r_y\} \cup \{m_{xyz}: 1 \le x \le i, 1 \le y \le r_x, 1 \le z \le s_x\}$ and

$E(H_2)=\{k_{yz}k_{y(z+1)}: 1 \le y \le i, 1 \le z \le r_y-1\} \cup \{k_{y1}k_{yr_y}: 1 \le y \le i\} \cup \{k_{yz}l_{yz}: 1 \le y \le i, 1 \le z \le r_y\} \cup \{m_{xyz}m_{xy(z+1)}: 1 \le x \le i, 1 \le y \le r_x, 1 \le z \le s_x-1\} \cup \{m_{xy1}m_{xys_x}: 1 \le x \le i, 1 \le y \le r_x\} \cup \{l_{xy}m_{xyz}: 1 \le x \le i, 1 \le y \le r_x, 1 \le z \le s_x\}$.

We superimposing one of the vertex say $k_{11}$ of $H_2$ on selected vertex of $s_1$ in $G_2$ with $g_2(s_1)=1$.

Let us construct a new graph $G_2^*=G_2 \odot H_2$ with $V(G_2^*)=V(G_2) \cup V(H_2)$ and $E(G_2^*)=E(G_2) \cup E(H_2)$.

$|V(G_2^*)|=p_2+2\sum_{c=1}^{i} r_c+\sum_{c=1}^{i} r_cs_c-1$ and $|E(G_2^*)|=q_2+2\sum_{c=1}^{i}(r_c+r_cs_c)$.

Define $h_2: V(G_2^*) \cup E(G_2^*) \to \{1,2,\ldots,p_2+q_2+4\sum_{c=1}^{i} r_c+3\sum_{c=1}^{i} r_cs_c-1\}$ by

$g_2(z_2)=h_2(z_2)$ for all $z_2 \in V(G_2)$ and $g_2(e_2)=h_2(e_2)$ for all $e_2 \in E(G_2)$.

$h_2(k_{11})=h_2(s_1)=1$.

$h_2(k_{y1})=p_2+q_2+\sum_{c=1}^{y} r_cs_c+2\sum_{c=1}^{y} r_c-1$ for $2 \le y \le i$.

$h_2(k_{y1}k_{y(r_y)})=p_2+q_2+\sum_{c=1}^{i} r_cs_c+2\sum_{c=1}^{i} r_c+2\sum_{c=1}^{y-1} r_c+r_y-1$ for $1 \le y \le i$.

$h_2(k_{1z})=p_2+q_2+(z-1)(s_1+2)$ for $2 \le z \le r_1$.

$h_2(k_{yz})=p_2+q_2+\sum_{c=1}^{y-1} r_cs_c+2\sum_{c=1}^{y-1} r_c+(z-1)(s_y+2)-1$ for $2 \le y \le i$ and $2 \le z \le r_y$.

$h_2(l_{1z})=p_2+q_2+zs_1+2z-1$ for $1 \le z \le r_1$.

$h_2(l_{yz})=p_2+q_2+\sum_{c=1}^{y-1} r_cs_c+2\sum_{c=1}^{y-1} r_c+zs_y+2z-2$ for $2 \le y \le i$ and $1 \le z \le r_y$.

$h_2(k_{yz}l_{yz})=p_2+q_2+\sum_{c=1}^{i} r_cs_c+2\sum_{c=1}^{i} r_c+2\sum_{c=1}^{y-1} r_c+r_y+z-1$ for $1 \le y \le i$ and $1 \le z \le r_y$.

$h_2(m_{1y(2z-1)})=p_2+q_2+(y-1)(s_1+2)+z$ for $1 \le y \le r_1$ and $1 \le z \le \left\lceil\frac{s_1}{2}\right\rceil$.

$h_2(m_{xy(2z-1)})=p_2+q_2+\sum_{c=1}^{x-1} r_cs_c+2\sum_{c=1}^{x-1} r_c+(y-1)(s_x+2)+z-1$ for $2 \le x \le i, 1 \le y \le r_x$ and $1 \le z \le \left\lceil\frac{s_x}{2}\right\rceil$.

$h_2(m_{1y(2z)}) = p_2 + q_2 + (y-1)(s_1+2) + \left\lceil \frac{s_1}{2} \right\rceil + z$ for $1 \le y \le r_1$ and $1 \le z \le \left\lfloor \frac{s_1}{2} \right\rfloor$.

$h_2(m_{xy(2z)}) = p_2 + q_2 + \sum_{c=1}^{x-1} r_c s_c + 2\sum_{c=1}^{x-1} r_c + (y-1)(s_x+2) + \left\lceil \frac{s_x}{2} \right\rceil + z - 1$ for $2 \le x \le i, 1 \le y \le r_x$ and $1 \le z \le \left\lfloor \frac{s_x}{2} \right\rfloor$.

$h_2(k_{yz}k_{y(z+1)}) = p_2 + q_2 + \sum_{c=1}^{i} r_c s_c + 2\sum_{c=1}^{i} r_c + 2\sum_{c=1}^{y-1} r_y + z - 1$ for $1 \le y \le i$ and $1 \le z \le r_y - 1$.

$h_2(m_{xyz}m_{xy(z+1)}) = p_2 + q_2 + 2\sum_{c=1}^{i} r_c s_c + 4\sum_{c=1}^{i} r_c + 2\sum_{c=1}^{x-1} r_c s_c + 2(y-1)s_x + z - 1$ for $1 \le x \le i, 1 \le y \le r_x$ and $1 \le z \le s_x - 1$.

$h_2(m_{xy1}m_{xys_x}) = p_2 + q_2 + \sum_{c=1}^{i} r_c s_c + 4\sum_{c=1}^{i} r_c + 2\sum_{c=1}^{x-1} r_c s_c + (2y-1)s_x - 1$ for $1 \le x \le i$ and $1 \le y \le r_x$.

$h_2(l_{xy}m_{xyz}) = p_2 + q_2 + \sum_{c=1}^{i} r_c s_c + 4\sum_{c=1}^{i} r_c + 2\sum_{c=1}^{x-1} r_c s_c + (2y-1)s_x + z - 1$ for $1 \le x \le i, 1 \le y \le r_x$ and $1 \le z \le s_x$.

Hence $G_2^* = G_2 \odot H_2$ admits vertex edge neighborhood prime graph. ■

**Theorem 2.3.** If $G_3(p_3, q_3)$ has vertex edge neighborhood prime graph, then $G_3 \odot [LC_{j_1} * \overline{K_1} \cup LC_{j_2} * \overline{K_1} \cup \ldots \cup LC_{j_i} * \overline{K_1}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_3(p_3, q_3)$ be vertex edge neighborhood prime graph with bijection

$g_3: V(G_3) \cup E(G_3) \to \{1,2,\ldots,|V(G_3) \cup E(G_3)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_3 = LC_{j_1} * \overline{K_1} \cup LC_{j_2} * \overline{K_1} \cup \ldots \cup LC_{j_i} * \overline{K_1}$ with

$V(H_3) = \{x_r : 1 \le r \le i\} \cup \{y_{rs}, z_{rs}, z'_{rs} : 1 \le r \le i, 1 \le s \le j_r\}$ and

$E(H_3) = \{x_r y_{rs}, y_{rs} z_{rs}, z_{rs} z'_{rs} : 1 \le r \le i, 1 \le s \le j_r\} \cup \{y_{r1} z_{rj_r}, z_{rj_r} z_{r1} : 1 \le r \le i\} \cup \{y_{r(s+1)} z_{rs}, z_{rs} z_{r(s+1)} : 1 \le r \le i, 1 \le s \le j_r - 1\}$.

We superimposing one of the vertex say $x_1$ of $H_3$ on selected vertex of $a_1$ in $G_3$ with $g_3(a_1) = 1$.

Let us construct a new graph $G_3^* = G_3 \odot H_3$ with $V(G_3^*) = V(G_3) \cup V(H_3)$ and $E(G_3^*) = E(G_3) \cup E(H_3)$

$|V(G_3^*)| = p_3 + 3(j_1 + j_2 + \ldots + j_i) + i - 1$ and $|E(G_3^*)| = q_3 + 5(j_1 + j_2 + \ldots + j_i)$.

Define $h_3: V(G_3^*) \cup E(G_3^*) \to \{1,2,\ldots,p_3 + q_3 + i + 8(j_1 + j_2 + \ldots + j_i) - 1\}$ by

$g_3(z_3) = h_3(z_3)$ for all $z_3 \in V(G_3)$ and $g_3(e_3) = h_3(e_3)$ for all $e_3 \in E(G_3)$.

$h_3(x_1) = h_3(a_1) = 1$.

For each $1 \le r \le i$ and $1 \le s \le j_r, h_3(y_{rs}) = p_3 + q_3 + 2(j_1 + j_2 + \ldots + j_i) + 2\sum_{t=1}^{r-1} j_t + j_r + s, h_3(z'_{rs}) = p_3 + q_3 + 2(j_1 + j_2 + \ldots + j_i) + 2\sum_{t=1}^{r-1} j_t + s, h_3(y_{rs} z_{rs}) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + i + 4\sum_{t=1}^{r-1} j_t + j_r + 2s - 2, h_3(x_r y_{rs}) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + i + 4\sum_{t=1}^{r-1} j_t + 3j_r + s - 1$.

For each $1 \le r \le i$ and $1 \le s \le j_r - 1, h_3(z_{rs} z_{r(s+1)}) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + i + 4\sum_{t=1}^{r-1} j_t + s - 1, h_3(y_{r(s+1)} z_{rs}) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + i + 4\sum_{t=1}^{r-1} j_t + j_r + 2s - 1$.

$h_3(x_r) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + r - 1$ for $2 \le r \le i$.

For each $1 \le r \le i,, h_3(z_{r1} z_{rj_r}) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + i + 4\sum_{t=1}^{r-1} j_t + j_r - 1, h_3(x_{r1} z_{rj_r}) = p_3 + q_3 + 4(j_1 + j_2 + \ldots + j_i) + i + 4\sum_{t=1}^{r-1} j_t + 3j_r - 1$.

Let us consider the following cases.

**Case 1.** $p_3 + q_3$ is odd.

For each $1 \le r \le i$ and $1 \le s \le j_r, h_3(z_{rs}) = p_3 + q_3 + 2\sum_{t=1}^{r-1} j_t + 2s, h_3(z_{rs} z'_{rs}) = p_3 + q_3 + 2\sum_{t=1}^{r-1} j_t + 2s - 1$.

**Case 2.** $p_3 + q_3$ is even.

For each $1 \le r \le i$ and $1 \le s \le j_r, h_3(z_{rs}) = p_3 + q_3 + 2\sum_{t=1}^{r-1} j_t + 2s - 1, h_3(z_{rs} z'_{rs}) = p_3 + q_3 + 2\sum_{t=1}^{r-1} j_t + 2s$.

Hence $G_3^*$ is vertex edge neighborhood prime graph. ■

**Theorem 2.4.** If $G_4(p_4, q_4)$ has vertex edge neighborhood prime graph, then $G_4 \odot [R_{t_1} * \overline{K_1} \cup R_{t_2} * \overline{K_1} \cup \ldots \cup R_{t_s} * \overline{K_1}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_4(p_4, q_4)$ be vertex edge neighborhood prime graph with bijection $g_4: V(G_4) \cup E(G_4) \to \{1,2,\ldots,|V(G_4) \cup E(G_4)|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_4 = R_{t_1} * \overline{K_1} \cup R_{t_2} * \overline{K_1} \cup \ldots \cup R_{t_s} * \overline{K_1}$ with

$V(H_4) = \{u'_{ab}, v'_{ab}, w'_{ab}, x'_{ab} : 1 \le a \le s, 1 \le b \le t_a\}$ and

$E(H_4) = \{u'_{ab} u'_{a(b+1)}, u'_{ab} v'_{a(b+1)}, v'_{ab} v'_{a(b+1)}, w'_{ab} w'_{a(b+1)} : 1 \le a \le s, 1 \le b \le t_a - 1\} \cup$

$\{u'_{ab}v'_{ab}, v'_{ab}w'_{ab}, w'_{ab}x'_{ab}: 1 \le a \le s, 1 \le b \le t_a\} \cup \{u'_{a1}u'_{at_a}, u'_{at_a}v'_{a1}, v'_{a1}v'_{at_a}, w'_{a1}w'_{at_a}: 1 \le a \le s\}$.

We overlay one of the vertex say $x_{11}$ of $H_4$ on selected vertex of $t_1$ in $G_4$ with $g_4(t_1) = 1$.

Let us construct a new graph $G_4^* = G_4 \odot H_4$ with $V(G_4^*) = V(G_4) \cup V(H_4)$ and $E(G_4^*) = E(G_4) \cup E(H_4)$.

$|V(G_4^*)| = p_4 + 4(t_1 + t_2 + \ldots + t_s) - 1$ and $|E(G_4^*)| = q_4 + 7(t_1 + t_2 + \ldots + t_s)$.

Define $h_4: V(G_4^*) \cup E(G_4^*) \to \{1,2,\ldots,p_4 + q_4 + 11(t_1 + t_2 + \ldots + t_s) - 1\}$ by

$g_4(z_4) = h_4(z_4)$ for all $z_4 \in V(G_4)$ and $g_4(e_4) = h_4(e_4)$ for all $e_4 \in E(G_4)$.

$h_4(x_{11}) = h_4(t_1) = 1$.

$h_4(x'_{1b}) = p_4 + q_4 + 4(t_1 + t_2 + \ldots + t_s) + b - 1$ for $2 \le b \le t_a$.

For each $1 \le a \le s$ and $1 \le b \le t_a, h_4(x'_{ab}) = p_4 + q_4 + 4(t_1 + t_2 + \ldots + t_s) + \sum_{e=1}^{a-1} t_e + b - 1, h_4(u'_{ab}v'_{ab}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 3t_a + 2b - 1, h_4(v'_{ab}w'_{ab}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + t_a + b - 1$.

For each $1 \le a \le s$ and $1 \le b \le t_a - 1, h_4(u'_{ab}v'_{a(b+1)}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 3t_a + 2b, h_4(v'_{ab}v'_{a(b+1)}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 2t_a + b - 1, h_4(w'_{ab}w'_{a(b+1)}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + b - 1, h_4(u'_{ab}u'_{a(b+1)}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 5t_a + b$.

For each $1 \le a \le s, h_4(u'_{at_a}v'_{a1}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 3t_a, h_4(u'_{a1}u'_{at_a}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 5t_a, h_4(v'_{a1}v'_{at_a}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + 3t_a - 1, h_4(w'_{a1}w'_{at_a}) = p_4 + q_4 + 5(t_1 + t_2 + \ldots + t_s) + 6\sum_{e=1}^{a-1} t_e + t_a - 1$.

Consider the following cases.

**Case 1.** $p_4 + q_4$ is odd.

For each $1 \le a \le s$ and $1 \le b \le t_a, h_4(u'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2b - 1, h_4(v'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2b, h_4(w'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2t_a + 2b, h_4(w'_{ab}x'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2t_a + 2b - 1$.

**Case 2.** $p_4 + q_4$ is even.

For each $1 \le a \le s$ and $1 \le b \le t_a, h_4(u'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2b, h_4(v'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2b - 1, h_4(w'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2t_a + 2b - 1, h_4(w'_{ab}x'_{ab}) = p_4 + q_4 + 4\sum_{e=1}^{a-1} t_e + 2t_a + 2b$.

Hence $G_4^* = G_4 \odot H_4$ admits vertex edge neighborhood prime graph. ■

**Theorem 2.5.** If $G_5$ has vertex edge neighborhood prime graph, then $G_5 \odot [S_{r_1} \cup S_{r_2} \cup \ldots \cup S_{r_i}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_5(p_5, q_5)$ be vertex edge neighborhood prime graph with bijection $g_5: V(G_5) \cup E(G_5) \to \{1,2,\ldots,|V(G_5) \cup E(G_5)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_5 = S_{r_1} \cup S_{r_2} \cup \ldots \cup S_{r_i}$ with

$V(H_5) = \{w_{kl}, x_{kl}: 1 \le k \le i, 1 \le l \le r_i\}$ and

$E(H_5) = \{w_{kl}x_{kl}: 1 \le k \le i, 1 \le l \le r_k\} \cup \{w_{k1}w_{kr_k}: 1 \le k \le i\} \cup \{w_{kl}w_{k(l+1)}: 1 \le k \le i, 1 \le l \le r_k - 1\}$.

We identify one of the vertex say $w_{11}$ of $H_5$ on selected vertex of $u_1$ in $G_5$ with $g_5(u_1) = 1$.

Let us construct a new graph $G_5^* = G_5 \odot H_5$ with $V(G_5^*) = V(G_5) \cup V(H_5)$ and $E(G_5^*) = E(G_5) \cup E(H_5)$.

$|V(G_5^*)| = p_5 + 2(r_1 + r_2 + \ldots + r_i) - 1$ and $|E(G_5^*)| = q_5 + 2(r_1 + r_2 + \ldots + r_i)$.

Define $h_5: V(G_5^*) \cup E(G_5^*) \to \{1,2,\ldots,p_5 + q_5 + 4(r_1 + r_2 + \ldots + r_i) - 1\}$ by

$g_5(z_5) = h_5(z_5)$ for all $z_5 \in V(G_5)$ and $g_5(d_5) = h_5(d_5)$ for all $d_5 \in E(G_5)$.

$h_5(w_{11}) = h_5(u_1) = 1$.

$h_5(w_{1l}) = p_5 + q_5 + 3l - 3$ for $2 \le l \le r_1$.

$h_5(w_{kl}) = p_5 + q_5 + 3\sum_{t=1}^{k-1} r_t + 3l - 3$ for $2 \le k \le i$ and $1 \le l \le r_i$.

For each $1 \le k \le i$ and $1 \le l \le r_i, h_5(x_{kl}) = p_5 + q_5 + 3\sum_{t=1}^{k-1} r_t + 3l - 1, h_5(w_{kl}x_{kl}) = p_5 + q_5 + 3\sum_{t=1}^{k-1} r_t + 3l - 2$.

Consider the following cases.

**Case 1.** $p_5 + q_5$ is odd.

$h_5(w_{kl}w_{k(l+1)}) = p_5 + q_5 + 3(r_1 + r_2 + \ldots + r_i) + \sum_{t=1}^{k-1} r_t + l - 1$ for $1 \le k \le i$ and $1 \le l \le r_k - 1$.

$h_5(w_{k1}w_{kr_k}) = p_5 + q_5 + 3(r_1 + r_2 + \ldots + r_i) + \sum_{t=1}^{k} r_t - 1$ for $1 \le k \le i$.

**Case 2.** $p_5 + q_5$ is even.

For each $1 \leq k \leq i, h_5(w_{k1}w_{k2}) = p_5 + q_5 + 3(r_1 + r_2 + \ldots + r_i) + \sum_{t=1}^{k} r_t - 1, h_5(w_{k1}w_{kr_k}) = p_5 + q_5 + 3(r_1 + r_2 + \ldots + r_i) + \sum_{t=1}^{k} r_t - 2$.

$h_5(w_{kl}w_{k(l+1)}) = p_5 + q_5 + 3(r_1 + r_2 + \ldots + r_i) + \sum_{t=1}^{k-1} r_t + l - 2$ for $1 \leq k \leq i$ and $2 \leq l \leq r_k - 1$.

Hence $G_5^*$ is vertex edge neighborhood prime graph. ■

**Theorem 2.6.** If $G_6$ has vertex edge neighborhood prime graph, then $G_6 \odot [W_{k_1} \cup W_{k_2} \cup \ldots \cup W_{k_l}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_6(p_6, q_6)$ be vertex edge neighborhood prime graph with bijection $g_6: V(G_6) \cup E(G_6) \to \{1,2,\ldots,|V(G_6) \cup E(G_6)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_6 = W_{k_1} \cup W_{k_2} \cup \ldots \cup W_{k_l}$ with

$V(H_6) = \{y_{w1}: 1 \leq w \leq l\} \cup \{z_{wx}: 1 \leq w \leq l, 1 \leq x \leq k_w\}$ and

$E(H_6) = \{y_{w1}z_{wx}: 1 \leq w \leq l, 1 \leq x \leq k_w\} \cup \{z_{w1}z_{wk_w}: 1 \leq w \leq l\} \cup \{z_{wx}z_{w(x+1)}: 1 \leq w \leq l, 1 \leq x \leq k_w - 1\}$.

We superimposing one of the vertex say $z_{11}$ of $H_6$ on selected vertex of $r_1$ in $G_6$ with $g_6(r_1) = 1$.

Let us construct a new graph $G_6^* = G_6 \odot H_6$ with $V(G_6^*) = V(G_6) \cup V(H_6)$ and $E(G_6^*) = E(G_6) \cup E(H_6)$

$|V(G_6^*)| = p_6 + l + (k_1 + k_2 + \ldots + k_l) - 1$ and $|E(G_6^*)| = q_6 + 2(k_1 + k_2 + \ldots + k_l)$.

Define $h_6: V(G_6^*) \cup E(G_6^*) \to \{1,2,\ldots,p_6 + q_6 + l + 3(k_1 + k_2 + \ldots + k_l) - 1\}$ by

$g_6(z_6) = h_6(z_6)$ for all $z_6 \in V(G_6)$ and $g_6(d_6) = h_6(d_6)$ for all $d_6 \in E(G_6)$.

$h_6(z_{11}) = h_6(r_1) = 1$.

$h_6(z_{1(2x-1)}) = p_6 + q_6 + x - 1$ for $2 \leq x \leq \left\lceil \frac{k_1}{2} \right\rceil$.

$h_6(z_{w(2x-1)}) = p_6 + q_6 + \sum_{u=1}^{w-1} k_u + (w-1) + x - 1$ for $2 \leq w \leq l$ and $1 \leq x \leq \left\lceil \frac{k_w}{2} \right\rceil$.

$h_6(z_{w(2x)}) = p_6 + q_6 + \sum_{u=1}^{w-1} k_u + \left\lceil \frac{k_w}{2} \right\rceil + w + x - 1$ for $1 \leq w \leq l$ and $1 \leq x \leq \left\lfloor \frac{k_w}{2} \right\rfloor$.

$h_6(y_{w1}z_{wx}) = p_6 + q_6 + l + (k_1 + k_2 + \ldots + k_l) + 2\sum_{u=1}^{w-1} k_u + k_w + x - 1$ for $1 \leq w \leq l$ and $1 \leq x \leq k_w$.

$h_6(z_{wx}z_{w(x+1)}) = p_6 + q_6 + l + (k_1 + k_2 + \ldots + k_l) + 2\sum_{u=1}^{w-1} k_u + x - 1$ for $1 \leq w \leq l$ and $1 \leq x \leq k_w - 1$.

For each $1 \leq w \leq l, h_6(y_{w1}) = p_6 + q_6 + \sum_{u=1}^{w-1} k_u + \left\lceil \frac{k_w}{2} \right\rceil + w - 1, h_6(z_{w1}z_{wk_w}) = p_6 + q_6 + l + (k_1 + k_2 + \ldots + k_l) + 2\sum_{u=1}^{w-1} k_u + k_w - 1$.

Hence $G_6^*$ is vertex edge neighborhood prime graph. ■

**Theorem 2.7.** If $G_7(p_7, q_7)$ has vertex edge neighborhood prime graph, then $G_7 \odot [H_{t_1} \cup H_{t_2} \cup \ldots \cup H_{t_s}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_7(p_7, q_7)$ be vertex edge neighborhood prime graph with bijection $g_7: V(G_7) \cup E(G_7) \to \{1,2,\ldots,|V(G_7) \cup E(G_7)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_7 = H_{t_1} \cup H_{t_2} \cup \ldots \cup H_{t_s}$ with

$V(H_7) = \{a_{x1}: 1 \leq x \leq s\} \cup \{b_{xy}, c_{xy}: 1 \leq x \leq s, 1 \leq y \leq t_x\}$ and

$E(H_7) = \{a_{x1}b_{xy}, b_{xy}c_{xy}: 1 \leq x \leq s, 1 \leq y \leq t_x\} \cup \{b_{x1}b_{xt_x}: 1 \leq x \leq s\} \cup \{b_{xy}b_{x(y+1)}: 1 \leq x \leq s, 1 \leq y \leq t_x - 1\}$.

We overlay one of the vertex say $a_{11}$ of $H_7$ on selected vertex of $s_1$ in $G_7$ with $g_7(s_1) = 1$.

Let us construct a new graph $G_7^* = G_7 \odot H_7$ with $V(G_7^*) = V(G_7) \cup V(H_7)$ and $E(G_7^*) = E(G_7) \cup E(H_7)$

$|V(G_7^*)| = p_7 + s + 2(t_1 + t_2 + \ldots + t_s) - 1$ and $|E(G_7^*)| = q_7 + 3(t_1 + t_2 + \ldots + t_s)$.

Define $h_7: V(G_7^*) \cup E(G_7^*) \to \{1,2,\ldots,p_7 + q_7 + s + 5(t_1 + t_2 + \ldots + t_s) - 1\}$ by

$g_7(z_7) = h_7(z_7)$ for all $z_7 \in V(G_7)$ and $g_7(d_7) = h_7(d_7)$ for all $d_7 \in E(G_7)$.

$h_7(a_{11}) = h_7(s_1) = 1$.

$h_7(c_{1y}) = p_7 + q_7 + 2(t_1 + t_2 + \ldots + t_s) + y$ for $1 \leq y \leq t_1$.

$h_7(c_{xy}) = p_7 + q_7 + 2(t_1 + t_2 + \ldots + t_s) + \sum_{r=1}^{x-1} t_r + (x-1) + y - 1$ for $2 \leq x \leq s$ and $1 \leq y \leq t_x$.

$h_7(a_{x1}b_{xy}) = p_7 + q_7 + 3(t_1 + t_2 + \ldots + t_s) + s + 2\sum_{r=1}^{x-1} t_r + t_x + y - 1$ for $1 \leq x \leq s$ and $1 \leq y \leq t_x$.

$h_7(a_{x1}) = p_7 + q_7 + 2(t_1 + t_2 + \ldots + t_s) + \sum_{r=1}^{x} t_r + x - 1$ for $2 \leq x \leq s$.

$h_7(b_{x1}b_{xt_x}) = p_7 + q_7 + 3(t_1 + t_2 + \ldots + t_s) + s + 2\sum_{r=1}^{x-1} t_r + t_x - 1$ for $1 \le x \le s$.

$h_7(b_{xy}b_{x(y+1)}) = p_7 + q_7 + 3(t_1 + t_2 + \ldots + t_s) + s + 2\sum_{r=1}^{x-1} t_r + y - 1$ for $1 \le x \le s$ and $1 \le y \le t_x - 1$.

We consider the following two cases.

**Case 1.** $p_7 + q_7$ is odd.

For each $1 \le x \le s$ and $1 \le y \le t_x, h_7(b_{xy}) = p_7 + q_7 + 2\sum_{r=1}^{x-1} t_r + 2y, h_7(b_{xy}c_{xy}) = p_7 + q_7 + 2\sum_{r=1}^{x-1} t_r + 2y - 1$.

**Case 2.** $p_7 + q_7$ is even.

For each $1 \le x \le s$ and $1 \le y \le t_x, h_7(b_{xy}) = p_7 + q_7 + 2\sum_{r=1}^{x-1} t_r + 2y - 1, h_7(b_{xy}c_{xy}) = p_7 + q_7 + 2\sum_{r=1}^{x-1} t_r + 2y$.

Hence $G_7^* = G_7 \odot H_7$ is vertex edge neighborhood prime graph. ■

**Theorem 2.8.** If $G_8(p_8, q_8)$ has vertex edge neighborhood prime graph, then $G_8 \odot [CH_{b_1} \cup CH_{b_2} \cup \ldots \cup CH_{b_a}]$, where $b_r(1 \le r \le a)$ is odd, admits vertex edge neighborhood prime graph.

Proof. Let $G_8(p_8, q_8)$ be vertex edge neighborhood prime graph with bijection $g_8: V(G_8) \cup E(G_8) \to \{1,2,\ldots,|V(G_8) \cup E(G_8)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_8 = CH_{b_1} \cup CH_{b_2} \cup \ldots \cup CH_{b_a}$, where $b_r(1 \le r \le a)$ is odd with

$V(H_8) = \{x_{r1}: 1 \le r \le a\} \cup \{y_{rs}, z_{rs}: 1 \le r \le a, 1 \le s \le b_r\}$ and

$E(H_8) = \{x_{r1}y_{rs}, y_{rs}z_{rs}: 1 \le r \le a, 1 \le s \le b_r\} \cup \{y_{r1}y_{rb_r}, z_{r1}z_{rb_r}: 1 \le r \le a\} \cup \{y_{rs}y_{r(s+1)}, z_{rs}z_{r(s+1)}: 1 \le r \le a, 1 \le s \le b_r - 1\}$.

We identify one of the vertex say $z_{11}$ of $H_8$ on selected vertex of $s_1$ in $G_8$ with $g_8(s_1) = 1$.

Let us construct a new graph $G_8^* = G_8 \odot H_8$ with $V(G_8^*) = V(G_8) \cup V(H_8)$ and $E(G_8^*) = E(G_8) \cup E(H_8)$.

$|V(G_8^*)| = p_8 + a + 2(b_1 + b_2 + \ldots + b_a) - 1$ and $|E(G_8^*)| = q_8 + 4(b_1 + b_2 + \ldots + b_a)$.

Define $h_8: V(G_8^*) \cup E(G_8^*) \to \{1,2,\ldots,p_8 + q_8 + a + 6(b_1 + b_2 + \ldots + b_a) - 1\}$ by

$g_8(z_8) = h_8(z_8)$ for all $z_8 \in V(G_8)$ and $g_8(e_8) = h_8(e_8)$ for all $e_8 \in E(G_8)$.

$h_8(z_{11}) = h_8(s_1) = 1$.

$h_8(z_{1(2s-1)}) = p_8 + q_8 + s - 1$ for $1 \le s \le \left\lceil \frac{b_1}{2} \right\rceil$.

$h_8(z_{r(2s-1)}) = p_8 + q_8 + 2\sum_{t=1}^{r-1} b_t + (r-1) + s - 1$ for $2 \le r \le a$ and $1 \le s \le \left\lceil \frac{b_r}{2} \right\rceil$.

For each $1 \le r \le a$ and $1 \le s \le \left\lfloor \frac{b_r}{2} \right\rfloor, h_8(z_{r(2s)}) = p_8 + q_8 + 2\sum_{t=1}^{r-1} b_t + (r-1) + \frac{b_r}{2} + s - 1, h_8(y_{r(2s-1)}) = p_8 + q_8 + 2\sum_{t=1}^{r-1} b_t + (r-1) + b_r + \left\lceil \frac{b_r}{2} \right\rceil + s - 1, h_8(y_{r(2s)}) = p_8 + q_8 + 2\sum_{t=1}^{r-1} b_t + r + b_r + s - 1$.

For each $1 \le r \le a, h_8(x_{r1}) = p_8 + q_8 + 2\sum_{t=1}^{r} b_t + r - 1, h_8(y_{rb_r}) = p_8 + q_8 + 2\sum_{t=1}^{r-1} b_t + r + b_r - 1, h_8(y_{r1}y_{rb_r}) = p_8 + q_8 + a + 2(b_1 + b_2 + \ldots + b_a) + 4\sum_{t=1}^{r-1} b_t + 3b_r - 1, h_8(z_{r1}z_{rb_r}) = p_8 + q_8 + a + 2(b_1 + b_2 + \ldots + b_a) + 4\sum_{t=1}^{r-1} b_t + b_r - 1$.

For each $1 \le r \le a$ and $1 \le s \le b_r - 1, h_8(y_{rs}y_{r(s+1)}) = p_8 + q_8 + a + 2(b_1 + b_2 + \ldots + b_a) + 4\sum_{t=1}^{r-1} b_t + 2b_r + s - 1, h_8(z_{rs}z_{r(s+1)}) = p_8 + q_8 + 2(b_1 + b_2 + \ldots + b_a) + 4\sum_{t=1}^{r-1} b_t + s - 1$.

For each $1 \le r \le a$ and $1 \le s \le b_r, h_8(y_{rs}z_{rs}) = p_8 + q_8 + a + 2(b_1 + b_2 + \ldots + b_a) + 4\sum_{t=1}^{r-1} b_t + b_r + s - 1, h_8(x_{r1}y_{rs}) = p_8 + q_8 + a + 2(b_1 + b_2 + \ldots + b_a) + 4\sum_{t=1}^{r-1} b_t + 3b_r + s - 1$.

Hence $G_8^* = G_8 \odot H_8$ admits vertex edge neighborhood prime graph. ■

**Theorem 2.9.** If $G_9(p_9, q_9)$ has vertex edge neighborhood prime graph, then $G_9 \odot [CH_{b_1} \cup CH_{b_2} \cup \ldots \cup CH_{b_a}]$, where $b_r(1 \le r \le a)$ is even, that admits vertex edge neighborhood prime graph.

Proof. Let $G_9(p_9, q_9)$ be vertex edge neighborhood prime graph with bijection $g_9: V(G_9) \cup E(G_9) \to \{1,2,\ldots,|V(G_9) \cup E(G_9)|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_9 = CH_{b_1} \cup CH_{b_2} \cup \ldots \cup CH_{b_a}$, where $b_r(1 \le r \le a)$ is even with

$V(H_9) = \{c_{r1}: 1 \le r \le a\} \cup \{d_{rs}, e_{rs}: 1 \le r \le a, 1 \le s \le b_r\}$ and

$E(H_9) = \{c_{r1}d_{rs}, d_{rs}e_{rs}: 1 \le r \le a, 1 \le s \le b_r\} \cup \{d_{r1}d_{rb_r}, e_{r1}e_{rb_r}: 1 \le r \le a\} \cup \{d_{rs}d_{r(s+1)}, e_{rs}e_{r(s+1)}: 1 \le r \le a, 1 \le s \le b_r - 1\}$.

We overlay one of the vertex say $e_{11}$ of $H_9$ on selected vertex of $z_1$ in $G_9$ with $g_9(z_1) = 1$.

Let us construct a new graph $G_9^* = G_9 \odot H_9$ with $V(G_9^*) = V(G_9) \cup V(H_9)$ and $E(G_9^*) = E(G_9) \cup E(H_9)$.

$|V(G_9^*)| = p_9 + a + 2(b_1 + b_2 + \ldots + b_a) - 1$ and $|E(G_9^*)| = q_9 + 4(b_1 + b_2 + \ldots + b_a)$.

Define $h_9: V(G_9^*) \cup E(G_9^*) \to \{1,2,\dots,p_9+q_9+a+6(b_1+b_2+\dots+b_a)-1\}$ by

$g_9(z_9) = h_9(z_9)$ for all $z_9 \in V(G_9)$ and $g_9(e_9) = h_9(e_9)$ for all $e_9 \in E(G_9)$.

$h_9(e_{11}) = h_9(z_1) = 1$.

$h_9(e_{1(2s-1)}) = p_9+q_9+s-1$ for $1 \le s \le \frac{b_1}{2}$.

$h_9(e_{r(2s-1)}) = p_9+q_9+2\sum_{h=1}^{r-1} b_h+(r-1)+s-1$ for $2 \le r \le a$ and $1 \le s \le \frac{b_r}{2}$.

For each $1 \le r \le a$ and $1 \le s \le \frac{b_r}{2}, h_9(e_{r(2s)}) = p_9+q_9+2\sum_{h=1}^{r-1} b_h+(r-1)+\frac{b_r}{2}+s-1, h_9(d_{r(2s-1)}) = p_9+q_9+2\sum_{h=1}^{r-1} b_h+(r-1)+b_r+s-1, h_9(d_{r(2s)}) = p_9+q_9+2\sum_{h=1}^{r-1} b_h+(r-1)+\frac{3b_r}{2}+s-1$.

For each $1 \le r \le a, h_9(c_{r1}) = p_9+q_9+2\sum_{h=1}^{r} b_h+r-1, h_9(e_{r1}e_{rb_r}) = p_9+q_9+a+2(b_1+b_2+\dots+b_a)+4\sum_{h=1}^{r-1} b_h+b_r-1, h_9(d_{r1}d_{rb_r}) = p_9+q_9+a+2(b_1+b_2+\dots+b_a)+4\sum_{h=1}^{r-1} b_h+3b_r-1$.

For each $1 \le r \le a$ and $1 \le s \le b_r-1$, $h_9(e_{rs}e_{r(s+1)}) = p_9+q_9+a+2(b_1+b_2+\dots+b_a)+4\sum_{h=1}^{r-1} b_h+s-1, h_9(d_{rs}d_{r(s+1)}) = p_9+q_9+a+2(b_1+b_2+\dots+b_a)+4\sum_{h=1}^{r-1} b_h+2b_r+s-1$.

For each $1 \le r \le a$ and $1 \le s \le b_r, h_9(d_{rs}e_{rs}) = p_9+q_9+a+2(b_1+b_2+\dots+b_a)+4\sum_{h=1}^{r-1} b_h+b_r+s-1, h_9(c_{r1}d_{rs}) = p_9+q_9+a+2(b_1+b_2+\dots+b_a)+4\sum_{h=1}^{r-1} b_h+3b_r+s-1$.

Hence $G_9^* = G_9 \odot H_9$ admits vertex edge neighborhood prime graph. ∎

**Theorem 2.10.** If $G_{10}(p_{10},q_{10})$ has vertex edge neighborhood prime graph, then $G_{10} \odot [R_{e_1} \cup R_{e_2} \cup \dots \cup R_{e_d}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_{10}(p_{10},q_{10})$ be vertex edge neighborhood prime graph with labeling

$g_{10}: V(G_{10}) \cup E(G_{10}) \to \{1,2,\dots,|V(G_{10}) \cup E(G_{10})|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_{10} = R_{e_1} \cup R_{e_2} \cup \dots \cup R_{e_d}$ with

$V(H_{10}) = \{a_{rs}, b_{rs}, c_{rs}: 1 \le r \le d, 1 \le s \le e_r\}$ and

$E(H_{10}) = \{a_{rs}b_{rs}, b_{rs}c_{rs}: 1 \le r \le d, 1 \le s \le e_r\} \cup \{a_{r1}a_{re_r}, a_{r1}b_{re_r}, b_{r1}b_{re_r}, c_{r1}c_{re_r}: 1 \le r \le d\} \cup \{a_{rs}a_{r(s+1)}, a_{r(s+1)}b_{rs}, b_{rs}b_{r(s+1)}, c_{rs}c_{r(s+1)}: 1 \le r \le d, 1 \le s \le e_r-1\}$.

We identify one of the vertex say $c_{11}$ of $H_{10}$ on selected vertex of $z_1$ in $G_{10}$ with $g_{10}(z_1) = 1$.

Let us construct a new graph $G_{10}^* = G_{10} \odot H_{10}$ with $V(G_{10}^*) = V(G_{10}) \cup V(H_{10})$ and $E(G_{10}^*) = E(G_{10}) \cup E(H_{10})$.

$|V(G_{10}^*)| = p_{10}+3(e_1+e_2+\dots+e_d)-1$ and $|E(G_{10}^*)| = q_{10}+6(e_1+e_2+\dots+e_d)$.

Define $h_{10}: V(G_{10}^*) \cup E(G_{10}^*) \to \{1,2,\dots,p_{10}+q_{10}+9(e_1+e_2+\dots+e_d)-1\}$ by

$g_{10}(z_{10}) = h_{10}(z_{10})$ for all $z_{10} \in V(G_{10})$ and $g_{10}(e_{10}) = h_{10}(e_{10})$ for all $e_{10} \in E(G_{10})$.

$h_{10}(c_{11}) = h_{10}(z_1) = 1$.

For each $1 \le r \le d$ and $1 \le s \le e_r-1, h_{10}(c_{rs}c_{r(s+1)}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+s-1, h_{10}(a_{r(s+1)}b_{rs}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+2e_r+2s-1, h_{10}(a_{rs}a_{r(s+1)}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+4e_r+s-1$.

For each $1 \le r \le d, h_{10}(a_{r1}a_{re_r}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+5e_r-1, h_{10}(a_{r1}b_{re_r}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+4e_r-1, h_{10}(c_{r1}c_{re_r}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+e_r-1$.

For each $1 \le r \le d$ and $1 \le s \le e_r, h_{10}(b_{rs}c_{rs}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+e_r+s-1, h_{10}(a_{rs}b_{rs}) = p_{10}+q_{10}+4(e_1+e_2+\dots+e_d)+5\sum_{t=1}^{r-1} e_t+2e_r+2s-2$.

Let us consider the following two cases.

**Case 1.** $p_{10}+q_{10}$ is odd.

$h_{10}(c_{1s}) = p_{10}+q_{10}+2(e_1+e_2+\dots+e_d)+2s-2$ for $2 \le s \le e_1$.

$h_{10}(c_{rs}) = p_{10}+q_{10}+2(e_1+e_2+\dots+e_d)+2\sum_{t=1}^{r-1} e_t+2s-2$ for $2 \le r \le d$ and $1 \le s \le e_r$.

For each $1 \le r \le d$ and $1 \le s \le e_r, h_{10}(b_{rs}) = p_{10}+q_{10}+2\sum_{t=1}^{r-1} e_t+2s, h_{10}(a_{rs}) = p_{10}+q_{10}+2\sum_{t=1}^{r-1} e_t+2s-1$.

$h_{10}(b_{rs}b_{r(s+1)}) = p_{10}+q_{10}+2(e_1+e_2+\dots+e_d)+2\sum_{t=1}^{r-1} e_t+2s-1$ for $1 \le r \le d$ and $1 \le s \le e_r-1$.

$h_{10}(b_{r1}b_{re_r}) = p_{10}+q_{10}+2(e_1+e_2+\dots+e_d)+2\sum_{t=1}^{r-1} e_t+2e_r-1$ for $1 \le r \le d$.

**Case 2.** $p_{10}+q_{10}$ is even.

$h_{10}(c_{1s}) = p_{10}+q_{10}+2(e_1+e_2+\dots+e_d)+2s-3$ for $2 \le s \le e_1$.

$h_{10}(c_{rs}) = p_{10} + q_{10} + 2(e_1 + e_2 + \ldots + e_d) + 2\sum_{t=1}^{r-1} e_t + 2s - 1$ for $2 \le r \le d$ and $1 \le s \le e_r$.

For each $1 \le r \le d$ and $1 \le s \le e_r, h_{10}(b_{rs}) = p_{10} + q_{10} + 2\sum_{t=1}^{r-1} e_t + 2s - 1, h_{10}(a_{rs}) = p_{10} + q_{10} + 2\sum_{t=1}^{r-1} e_t + 2s$.

$h_{10}(b_{1s}b_{1(s+1)}) = p_{10} + q_{10} + 2(e_1 + e_2 + \ldots + e_d) + 2s$ for $1 \le s \le e_1 - 1$.

$h_{10}(b_{rs}b_{r(s+1)}) = p_{10} + q_{10} + 2(e_1 + e_2 + \ldots + e_d) + 2\sum_{t=1}^{r-1} e_t + 2s - 2$ for $2 \le r \le d$ and $1 \le s \le e_r - 1$.

$h_{10}(b_{11}b_{1e_1}) = p_{10} + q_{10} + 2(e_1 + e_2 + \ldots + e_d) + 2e_1 - 1$.

$h_{10}(b_{r1}b_{re_r}) = p_{10} + q_{10} + 2(e_1 + e_2 + \ldots + e_d) + 2\sum_{t=1}^{r-1} e_t + 2e_r - 2$ for $2 \le r \le d$.

Hence $G_{10}^* = G_{10} \odot H_{10}$ is vertex edge neighborhood prime graph. ■

**Theorem 2.11.** If $G_{11}(p_{11}, q_{11})$ has vertex edge neighborhood prime graph, then $G_{11} \odot [B_{3,k} \cup C_l \times K_2 \cup S_m \cup W_n \cup Q_p]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_{11}(p_{11}, q_{11})$ is vertex edge neighborhood prime graph with bijection $g_{11}: V(G_{11}) \cup E(G_{11}) \to \{1,2,\ldots,|V(G_{11}) \cup E(G_{11})|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_{11} = B_{3,k} \cup C_l \times K_2 \cup S_m \cup W_n \cup Q_p$ with

$V(H_{11}) = \{u_i: 1 \le i \le k+2\} \cup \{u'_i, u''_i: 1 \le i \le l\} \cup \{v_i, v'_i: 1 \le i \le m\} \cup \{v_0\} \cup \{v''_i: 1 \le i \le n\} \cup \{w_i: 1 \le i \le p\} \cup \{w'_i, w''_i: 1 \le i \le p-1\}$ and

$E(H_{11}) = \{u_1u_2\} \cup \{u_1u_{i+2}, u_2u_{i+2}: 1 \le i \le k\} \cup \{u'_iu'_{i+1}, u''_iu''_{i+1}: 1 \le i \le l-1\} \cup \{u'_1u'_l\} \cup \{u''_1u''_l\} \cup \{u'_iu''_i: 1 \le i \le l\} \cup \{v_iv_{i+1}: 1 \le i \le m-1\} \cup \{v_1v_m\} \cup \{v_iv'_i: 1 \le i \le m\} \cup \{v''_iv''_{i+1}: 1 \le i \le n-1\} \cup \{v''_1v''_n\} \cup \{v_0v'_i: 1 \le i \le n\} \cup \{w_iw_{i+1}, w'_iw''_i, w_iw'_i, w_{i+1}w''_i: 1 \le i \le p-1\}$.

We superimposing one of the vertex say $u_1$ of $H_{11}$ on selected vertex of $r_1$ in $G_{11}$ with $g_{11}(r_1) = 1$.

Let us construct a new graph $G_{11}^* = G_{11} \odot H_{11}$ with $V(G_{11}^*) = V(G_{11}) \cup V(H_{11})$ and $E(G_{11}^*) = E(G_{11}) \cup E(H_{11})$.

$|V(G_{11}^*)| = p_{11} + k + 2l + 2m + n + 3(p-1) + 3$ and $|E(G_{11}^*)| = q_{11} + 2k + 3l + 2m + 2n + 4(p-1) + 1$.

Define $h_{11}: V(G_{11}^*) \cup E(G_{11}^*) \to \{1,2,\ldots,p_{11} + q_{11} + 3k + 5l + 4m + 3n + 7(p-1) + 4\}$ by

$g_{11}(u_{11}) = h_{11}(u_{11})$ for all $u_{11} \in V(G_{11})$ and $g_{11}(e_{11}) = h_{11}(e_{11})$ for all $e_{11} \in E(G_{11})$.

$h_{11}(u_1) = h_{11}(r_1) = 1$.

$h_{11}(u_i) = p_{11} + q_{11} + i - 1$ for $2 \le i \le k+2$.

$h_{11}(u''_i) = p_{11} + q_{11} + k + 2i$ for $1 \le i \le l$.

$h_{11}(u'_i) = p_{11} + q_{11} + k + 2i - 1$ for $2 \le i \le l$.

$h_{11}(u'_1) = p_{11} + q_{11} + k + 2l + 1$.

For each $1 \le i \le m, h_{11}(v_i) = p_{11} + q_{11} + k + 2l + 3i - 1, h_{11}(v_iv'_i) = p_{11} + q_{11} + k + 2l + 3i, h_{11}(v'_i) = p_{11} + q_{11} + k + 2l + 3i + 1$.

$h_{11}(v_0) = p_{11} + q_{11} + k + 2l + 3m + \left\lceil \frac{n}{2} \right\rceil + 2$.

$h_{11}(v''_{2i}) = p_{11} + q_{11} + k + 2l + 3m + \left\lceil \frac{n}{2} \right\rceil + 2 + i$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$.

$h_{11}(v''_{2i-1}) = p_{11} + q_{11} + k + 2l + 3m + 1 + i$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$.

$h_{11}(w_i) = p_{11} + q_{11} + k + 2l + 3m + n + 3i$ for $1 \le i \le p$.

For each $1 \le i \le p-1, h_{11}(w'_i) = p_{11} + q_{11} + k + 2l + 3m + n + 3i + 2, h_{11}(w''_i) = p_{11} + q_{11} + k + 2l + 3m + n + 3i + 1$.

$h_{11}(u_1u_2) = p_{11} + q_{11} + k + 2l + 3m + n + 3p + 1$.

For each $1 \le i \le \left\lceil \frac{k}{2} \right\rceil, h_{11}(u_1u_{2i+1}) = p_{11} + q_{11} + k + 2l + 3m + n + 3p + 4i - 2, h_{11}(u_2u_{2i+1}) = p_{11} + q_{11} + k + 2l + 3m + n + 3p + 4i - 1$.

For each $1 \le i \le \left\lfloor \frac{k}{2} \right\rfloor, h_{11}(u_1u_{2i+2}) = p_{11} + q_{11} + k + 2l + 3m + n + 3p + 4i + 1, h_{11}(u_2u_{2i+2}) = p_{11} + q_{11} + k + 2l + 3m + n + 3p + 4i$.

For each $1 \le i \le l-1, h_{11}(u'_iu'_{i+1}) = p_{11} + q_{11} + 3k + 4l + 3m + n + 3p + i + 2, h_{11}(u''_iu''_{i+1}) = p_{11} + q_{11} + 3k + 2l + 3m + n + 3p + i + 1$.

$h_{11}(u'_iu''_i) = p_{11} + q_{11} + 3k + 3l + 3m + n + 3p + i + 1$ for $1 \le i \le l$.

$h_{11}(u''_1u''_n) = p_{11} + q_{11} + 3k + 3l + 3m + n + 3p + 1, h_{11}(u'_1u'_n) = p_{11} + q_{11} + 3k + 4l + 3m + n + 3p + 2$.

$h_{11}(v_iv_{i+1}) = p_{11} + q_{11} + 3k + 5l + 3m + n + 3p + i + 1$ for $1 \le i \le m-1$. $h_{11}(v_1v_m) = p_{11} + q_{11} + 3k + 5l + 4m + n + 3p + 1$.

$h_{11}(v''_iv''_{i+1}) = p_{11} + q_{11} + 3k + 5l + 4m + n + 3p + i + 1$ for $1 \le i \le n-1$. $h_{11}(v''_1v''_n) = p_{11} + q_{11} + 3k + 5l + 4m + 2n + 3p + 1$.

$h_{11}(v_0v''_i) = p_{11} + q_{11} + 3k + 5l + 4m + 2n + 3p + i + 1$ for $1 \le i \le n$.

For each $1 \le i \le p-1, h_{11}(w_iw_{i+1}) = p_{11} + q_{11} + 3k + 5l + 4m + 3n + 3p + 4i - 2, h_{11}(w'_iw''_i) = p_{11} + q_{11} + 3k + 5l + 4m + 3n + 3p + 4i$.

For each $1 \le i \le p-2, h_{11}(w_iw'_i) = p_{11} + q_{11} + 3k + 5l + 4m + 3n + 3p + 4i - 1, h_{11}(w_{i+1}w''_i) = p_{11} + q_{11} + 3k + 5l + 4m + 3n + 3p + 4i + 1$. $h_{11}(w_{p-1}w'_{p-1}) = p_{11} + q_{11} + 3k + 5l + 4m + 3n + 7p - 3, h_{11}(w_pw''_{p-1}) = p_{11} + q_{11} + 3k + 5l + 4m + 3n + 7p - 5$.

Hence $G^*_{11} = G_{11} \odot H_{11}$ is vertex edge neighborhood prime graph. ■

**Theorem 2.12.** If $G_{12}(p_{12}, q_{12})$ has vertex edge neighborhood prime graph, then $G_{12} \odot [BC_{z_1} \cup BC_{z_2} \cup \ldots \cup BC_{z_y}]$ that admits vertex edge neighborhood prime graph.

Proof. Let $G_{12}(p_{12}, q_{12})$ be vertex edge neighborhood prime graph with bijection $g_{12}: V(G_{12}) \cup E(G_{12}) \to \{1,2,\ldots,|V(G_{12}) \cup E(G_{12})|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_{12} = BC_{z_1} \cup BC_{z_2} \cup \ldots \cup BC_{z_y}$ with

$V(H_{12}) = \{c'_{st}, d'_{st}, e'_{st}: 1 \le s \le y, 1 \le t \le z_s\}$ and

$E(H_{12}) = \{c'_{st}d'_{st}, d'_{st}e'_{st}: 1 \le s \le y, 1 \le t \le z_s\} \cup \{c'_{s1}c'_{sz_s}, c'_{sz_s}d'_{s1}: 1 \le s \le y\} \cup \{c'_{st}c'_{s(t+1)}, c'_{st}d'_{s(t+1)}: 1 \le s \le y, 1 \le t \le z_s - 1\}$.

We overlay one of the vertex say $c'_{11}$ of $H_{12}$ on selected vertex of $z_1$ in $G_{12}$ with $g_{12}(z_1) = 1$.

Let us construct a new graph $G^*_{12} = G_{12} \odot H_{12}$ with $V(G^*_{12}) = V(G_{12}) \cup V(H_{12})$ and $E(G^*_{12}) = E(G_{12}) \cup E(H_{12})$.

$|V(G^*_{12})| = p_{12} + 3(z_1 + z_2 + \ldots + z_y) - 1$ and $|E(G^*_{12})| = q_{12} + 4(z_1 + z_2 + \ldots + z_y)$.

Define $h_{12}: V(G^*_{12}) \cup E(G^*_{12}) \to \{1,2,\ldots,p_{12} + q_{12} + 7(z_1 + z_2 + \ldots + z_y) - 1\}$ by

$g_{12}(z_{12}) = h_{12}(z_{12})$ for all $z_{12} \in V(G_{12})$ and $g_{12}(e_{12}) = h_{12}(e_{12})$ for all $e_{12} \in E(G_{12})$.

$h_{12}(c'_{11}) = h_{12}(z_1) = 1$.

For each $1 \le s \le y, h_{12}(c'_{sz_s}d'_{s1}) = p_{12} + q_{12} + 4(z_1 + z_2 + \ldots + z_y) + 3\sum_{b=1}^{s-1} z_b$, $h_{12}(c'_{s1}c'_{sz_s}) = p_{12} + q_{12} + 4(z_1 + z_2 + \ldots + z_y) + 3\sum_{b=1}^{s-1} z_b + 2z_s$.

For each $1 \le s \le y$ and $1 \le t \le z_s - 1, h_{12}(c'_{st}d'_{s(t+1)}) = p_{12} + q_{12} + 4(z_1 + z_2 + \ldots + z_y) + 3\sum_{b=1}^{s-1} z_b + 2t, h_{12}(c'_{st}c'_{s(t+1)}) = p_{12} + q_{12} + 4(z_1 + z_2 + \ldots + z_y) + 3\sum_{b=1}^{s-1} z_b + 2z_s + t$.

$h_{12}(c'_{st}d'_{st}) = p_{12} + q_{12} + 4(z_1 + z_2 + \ldots + z_y) + 3\sum_{b=1}^{s-1} z_b + 2t - 1$ for $1 \le s \le y$ and $1 \le t \le z_s$.

Consider the following two cases.

**Case 1.** $p_{12} + q_{12}$ is odd.

$h_{12}(c'_{1t}) = p_{12} + q_{12} + 2t - 2$ for $2 \le t \le z_1$.

$h_{12}(c'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2t - 2$ for $2 \le s \le y$ and $1 \le t \le z_s$.

For each $1 \le s \le y$ and $1 \le t \le z_s, h_{12}(d'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2z_s + 2t - 2, h_{12}(e'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2t - 1, h_{12}(d'_{st}e'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2z_s + 2t - 1$.

**Case 2.** $p_{12} + q_{12}$ is even.

$h_{12}(c'_{1t}) = p_{12} + q_{12} + 2t - 3$ for $2 \le t \le z_1$.

$h_{12}(c'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2t - 1$ for $2 \le s \le y$ and $1 \le t \le z_s$.

$h_{12}(e'_{1t}) = p_{12} + q_{12} + 2t$ for $1 \le t \le z_1 - 1$.

$h_{12}(e'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2t - 2$ for $2 \le s \le y$ and $1 \le t \le z_s$.

For each $1 \le s \le y$ and $1 \le t \le z_s, h_{12}(d'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2z_s + 2t - 1, h_{12}(d'_{st}e'_{st}) = p_{12} + q_{12} + 4\sum_{b=1}^{s-1} z_b + 2z_s + 2t - 2$.

Hence $G^*_{12} = G_{12} \odot H_{12}$ is vertex edge neighborhood prime graph. ■

## 3 Graph identification of duplicating of graphs

In this section, we discuss about duplication of graphs.

**Theorem 3.1.** If $G_1(p_1, q_1)$ has vertex edge neighborhood prime graph, then $G_1 \odot$ [duplicating all the vertices of $K_s$] that admits vertex edge neighborhood prime.

Proof. Let $G_1(p_1, q_1)$ be vertex edge neighborhood prime graph with bijection $g_1: V(G_1) \cup E(G_1) \to \{1,2,\ldots,|V(G_1) \cup E(G_1)|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_1$ the duplicating all the vertices of $K_s$ with

$V(H_1) = \{z_b, z'_b : 1 \le b \le s\}$ and

$E(H_1) = \{z_b z_{c+1} : 1 \le b \le s-1, b \le c \le s-1\} \cup \{z'_b z_{c+1} : 1 \le b \le s-1, b \le c \le s-1\} \cup \{z'_{b+1} z_c : 1 \le b \le s-1, 1 \le c \le b\}$.

We identify one of the vertex say $z_1$ of $H_1$ on selected vertex of $x_1$ in $G_1$ with $g_1(x_1) = 1$.

Let us construct a new graph $G_1^* = G_1 \odot H_1$ with $V(G_1^*) = V(G_1) \cup V(H_1)$ and $E(G_1^*) = E(G_1) \cup E(H_1)$.

$|V(G_1^*)| = p_1 + 2s - 1$ and $|E(G_1^*)| = q_1 + s(s-1) + \frac{s(s-1)}{2}$.

Define $h_1: V(G_1^*) \cup E(G_1^*) \to \left\{1,2,\ldots,p_1 + q_1 + 2s + s(s-1) + \frac{s(s-1)}{2} - 1\right\}$ by

$g_1(z_1) = h_1(z_1)$ for all $z_1 \in V(G_1)$ and $g_1(e_1) = h_1(e_1)$ for all $e_1 \in E(G_1)$.

$h_1(z_1) = h_1(x_1) = 1$.

$h_1(z_{b+1}) = p_1 + q_1 + b$ for $1 \le b \le s-1$.

$h_1(z'_b) = p_1 + q_1 + \frac{s(s-1)}{2} + s - 1 + b$ for $1 \le b \le s$.

For each $1 \le b \le s-1$ and $b \le c \le s-1, h_1(z_b z_{c+1}) = p_1 + q_1 + bs - \frac{b(b+1)}{2} + c, h_1(z'_b z_{c+1}) = p_1 + q_1 + \frac{s(s-1)}{2} + (b+1)s - \frac{b(b+1)}{2} + c$.

$h_1(z'_{b+1} z_c) = p_1 + q_1 + s(s-1) + 2s - [\frac{b(b-1)}{2} - 1] + c$ for $1 \le b \le s-1$ and $1 \le c \le b$.

Hence $G_1^* = G_1 \odot H_1$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.2**. If $G_2(p_2, q_2)$ has vertex edge neighborhood prime graph, then $G_2 \odot$ [duplicating all the vertices of Petersen graph $P(n,2)$] that admits vertex edge neighborhood prime for all $n \ge 5$.

Proof. Let $G_2(p_2, q_2)$ be vertex edge neighborhood prime graph with bijection $g_2: V(G_2) \cup E(G_2) \to \{1,2,\ldots,|V(G_2) \cup E(G_2)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_2$ the duplicating all the vertices of Petersen graph $P(n,2)$ when $n \ge 5$ with

$V(H_2) = \{u_s, v_s, u'_s, v'_s : 1 \le s \le n\}$ and

$E(H_2) = \{u_s v_s, u_s v'_s, u'_s v_s : 1 \le s \le n\} \cup \{v_s v_{s+1}, v'_s v_{s+1}, v_s v'_{s+1} : 1 \le s \le n-1\} \cup \{u_s u_{s+2}, u'_s u_{s+2}, u_s u'_{s+2} : 1 \le s \le n-2\} \cup \{v_1 v_n\} \cup \{u_1 u_{n-1}\} \cup \{u_2 u_n\} \cup \{v_1 v'_n\} \cup \{v'_1 v_n\} \cup \{u_1 u'_{n-1}\} \cup \{u_2 u'_n\} \cup \{u'_1 u_{n-1}\} \cup \{u'_2 u_n\}$.

We superimposing one of the vertex say $v_1$ of $H_2$ on selected vertex of $s_1$ in $G_2$ with $g_2(s_1) = 1$.

Let us construct a new graph $G_2^* = G_2 \odot H_2$ with $V(G_2^*) = V(G_2) \cup V(H_2)$ and $E(G_2^*) = E(G_2) \cup E(H_2)$.

$|V(G_2^*)| = p_2 + 4n - 1$ and $|E(G_2^*)| = q_2 + 9n$.

Define $h_2: V(G_2^*) \cup E(G_2^*) \to \{1,2,\ldots,p_2 + q_2 + 13n - 1\}$ by

$g_2(z_2) = h_2(z_2)$ for all $z_2 \in V(G_2)$ and $g_2(e_2) = h_2(e_2)$ for all $e_2 \in E(G_2)$.

$h_2(v_1) = h_2(s_1) = 1, h_2(v_1 v_n) = p_2 + q_2 + 5n - 1, h_2(u'_1 u_{n-1}) = p_2 + q_2 + 7n + 2, h_2(u'_2 u_n) = p_2 + q_2 + 7n + 5, h_2(v_1 v'_n) = p_2 + q_2 + 13n - 2, h_2(v'_1 v_n) = p_2 + q_2 + 10n + 2$.

For each $1 \le s \le n, h_2(u_s v_s) = p_2 + q_2 + 3n + s - 1, h_2(v'_s) = p_2 + q_2 + 6n + s - 1, h_2(u'_s) = p_2 + q_2 + 5n + s - 1, h_2(u'_s v_s) = p_2 + q_2 + 7n + 3s - 3, h_2(u_s v'_s) = p_2 + q_2 + 12n + 3s - 3$.

For each $1 \le s \le n-1, h_2(v_s v_{s+1}) = p_2 + q_2 + 4n + s - 1, h_2(v'_s v_{s+1}) = p_2 + q_2 + 10n + 3s - 2, h_2(v_s v'_{s+1}) = p_2 + q_2 + 10n + 3s + 2$.

For each $1 \le s \le n-2, h_2(u'_s u_{s+2}) = p_2 + q_2 + 7n + 3s - 2, h_2(u_s u'_{s+2}) = p_2 + q_2 + 7n + 5 + 3s$.

Let us consider the following four cases.

**Case 1.** $n$ is odd

$h_2(u_s u_{s+2}) = p_2 + q_2 + 2n + s - 1$ for $1 \le s \le n-2$.

$h_2(u_1 u_{n-1}) = p_2 + q_2 + 3n - 2, h_2(u_2 u_n) = p_2 + q_2 + 3n - 1$.

**Case 2.** $n$ is even

$h_2(u_s u_{s+2}) = p_2 + q_2 + 2n + s - 2$ for $2 \leq s \leq n - 2$.

$h_2(u_1 u_{n-1}) = p_2 + q_2 + 3n - 3, h_2(u_2 u_n) = p_2 + q_2 + 3n - 2, h_2(u_1 u_3) = p_2 + q_2 + 3n - 1$.

**Case 3.** $p_2 + q_2$ is odd

$h_2(u_s) = p_2 + q_2 + 2s - 1$ for $1 \leq s \leq n$.

$h_2(v_{s+1}) = p_2 + q_2 + 2s$ for $1 \leq s \leq n - 1$.

**Case 4.** $p_2 + q_2$ is even

For each $1 \leq s \leq n - 1, h_2(u_s) = p_2 + q_2 + 2s, h_2(v_{s+1}) = p_2 + q_2 + 2s - 1. h_2(u_n) = p_2 + q_2 + 2n - 1$.

Hence $G_2^* = G_2 \odot H_2$ is vertex edge neighborhood prime graph. ■

**Theorem 3.3**. If $G_3(p_3, q_3)$ has vertex edge neighborhood prime graph, then $G_3 \odot$ [ duplicating all the vertices of lotus inside circle $LC_r$] that admits vertex edge neighborhood prime.

Proof. Let $G_3(p_3, q_3)$ be vertex edge neighborhood prime graph with bijection

$g_3: V(G_3) \cup E(G_3) \to \{1,2,\ldots,|V(G_3) \cup E(G_3)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_3$ the duplicating all the vertices of lotus inside circle $LC_r$ with

$V(H_3) = \{b_0\} \cup \{b'_0\} \cup \{b_x, b'_x, c_x, c'_x: 1 \leq x \leq r\}$ and

$E(H_3) = \{b_0 b_x, b'_0 b_x, b_x c_x, b_0 b'_x, b'_x c_x, b_x c'_x: 1 \leq x \leq r\} \cup \{b_1 c_r\} \cup \{c_1 c_r\} \cup \{b'_1 c_r\} \cup \{b_1 c'_r\} \cup \{c_1 c'_r\} \cup \{c'_1 c_r\} \cup \{b_{x+1} c_x, c_x c_{x+1}, b'_{x+1} c_x, b_{x+1} c'_x, c'_x c_{x+1}, c_x c'_{x+1}: 1 \leq x \leq r - 1\}$.

We superimposing one of the vertex say $b_0$ of $H_3$ on selected vertex of $y_1$ in $G_3$ with $g_3(y_1) = 1$.

Let us construct a new graph $G_3^* = G_3 \odot H_3$ with $V(G_3^*) = V(G_3) \cup V(H_3)$ and $E(G_3^*) = E(G_3) \cup E(H_3)$

$|V(G_3^*)| = p_3 + 4r + 1$ and $|E(G_3^*)| = q_3 + 12r$.

Define $h_3: V(G_3^*) \cup E(G_3^*) \to \{1,2,\ldots,p_3 + q_3 + 16r + 1\}$ by

$g_3(z_3) = h_3(z_3)$ for all $z_3 \in V(G_3)$ and $g_3(e_3) = h_3(e_3)$ for all $e_3 \in E(G_3)$.

$h_3(b_0) = h_3(y_1) = 1, h_3(b'_0) = p_3 + q_3 + 8r + 1, h_3(c_1 c_r) = p_3 + q_3 + 3r, h_3(b_1 c_r) = p_3 + q_3 + 5r, h_3(b'_1 c_r) = p_3 + q_3 + 9r + 2, h_3(c'_1 c_r) = p_3 + q_3 + 11r + 2, h_3(b_1 c'_r) = p_3 + q_3 + 15r, h_3(c_1 c'_r) = p_3 + q_3 + 15r + 1$.

For each $1 \leq x \leq r, h_3(b_x) = p_3 + q_3 + x, h_3(c_x) = p_3 + q_3 + r + x, h_3(b_x c_x) = p_3 + q_3 + 3r + 2x - 1, h_3(b_0 b_x) = p_3 + q_3 + 5r + x, h_3(b'_x) = p_3 + q_3 + 6r + x, h_3(c'_x) = p_3 + q_3 + 7r + x, h_3(b_0 b'_x) = p_3 + q_3 + 8r + 1 + x, h_3(b'_x c_x) = p_3 + q_3 + 9r + 1 + 2x, h_3(b_x c'_x) = p_3 + q_3 + 11r - 1 + 4x, h_3(b'_0 b_x) = p_3 + q_3 + 15r + 1 + x$.

For each $1 \leq x \leq r - 1, h_3(c_x c_{x+1}) = p_3 + q_3 + 2r + x, h_3(b_{x+1} c_x) = p_3 + q_3 + 3r + 2x, h_3(b'_{x+1} c_x) = p_3 + q_3 + 9r + 2 + 2x, h_3(b_{x+1} c'_x) = p_3 + q_3 + 11r + 4x, h_3(c'_x c_{x+1}) = p_3 + q_3 + 11r + 1 + 4x, h_3(c_x c'_{x+1}) = p_3 + q_3 + 11r + 4x + 2$.

Hence $G_3^* = G_3 \odot H_3$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.4.** If $G_4(p_4, q_4)$ has vertex edge neighborhood prime graph, then $G_4 \odot$ [duplicating all the vertices of double triangular snake $DT_c$] that admits vertex edge neighborhood prime for all $c > 1$.

Proof. Let $G_4(p_4, q_4)$ be vertex edge neighborhood prime graph with bijection $g_4: V(G_4) \cup E(G_4) \to \{1,2,\ldots,|V(G_4) \cup E(G_4)|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_4$ the duplicating all the vertices of double triangular snake $DT_c$ when $c > 1$ with

$V(H_4) = \{x_a, x'_a: 1 \leq a \leq c\} \cup \{y_a, z_a, y'_a, z'_a: 1 \leq a \leq c - 1\}$ and

$E(H_4) = \{x_a x_{a+1}, x_a y_a, x_{a+1} y_a, x_a z_a, x_{a+1} z_a, x_a y'_a, x_{a+1} y'_a, x_a z'_a, x_{a+1} z'_a: 1 \leq a \leq c - 1\} \cup \{x'_a x_{a+1}, x'_a z_a, x_a x'_{a+1}, x'_{a+1} z_a: 1 \leq a \leq c - 1\} \cup \{x'_1 y_1\} \cup \{x'_c y_{c-1}\}$.

We overlay one of the vertex say $x_1$ of $H_4$ on selected vertex of $t_1$ in $G_4$ with $g_4(t_1) = 1$.

Let us construct a new graph $G_4^* = G_4 \odot H_4$ with $V(G_4^*) = V(G_4) \cup V(H_4)$ and $E(G_4^*) = E(G_4) \cup E(H_4)$.

$|V(G_4^*)| = p_4 + 6c - 5$ and $|E(G_4^*)| = q_4 + 13c - 11$.

Define $h_4: V(G_4^*) \cup E(G_4^*) \to \{1,2,\ldots,p_4 + q_4 + 19c - 16\}$ by

$g_4(z_4) = h_4(z_4)$ for all $z_4 \in V(G_4)$ and $g_4(e_4) = h_4(e_4)$ for all $e_4 \in E(G_4)$. $h_4(x_1) = h_4(t_1) = 1, h_4(x'_1 y_1) = p_4 + q_4 + 15c - 13, h_4(x'_c y_{c-1}) = p_4 + q_4 + 15c - 12$.

$h_4(x'_a) = p_4 + q_4 + 8c - 8 + a$ for $1 \leq a \leq c$.

For each $1 \leq a \leq c - 1, h_4(x_a x_{a+1}) = p_4 + q_4 + 3c + 5a - 3, h_4(x_a y_a) = p_4 + q_4 + 3c + 5a - 7, h_4(x_{a+1} y_a) = p_4 + q_4 + 3c + 5a - 6, h_4(x_a z_a) = p_4 + q_4 + 3c + 5a - 4, h_4(x_{a+1} z_a) = p_4 + q_4 + 3c + 5a - 5, h_4(y'_a) = p_4 + q_4 + 9c - 8 +$

$a, h_4(z'_a) = p_4 + q_4 + 10c + a - 9, h_4(x_a y'_a) = p_4 + q_4 + 11c + 2a - 11, h_4(x_{a+1} y'_a) = p_4 + q_4 + 11c + 2a - 10, h_4(x_a z'_a) = p_4 + q_4 + 13c + 2a - 13, h_4(x_{a+1} z'_a) = p_4 + q_4 + 13c + 2a - 12, h_4(x'_a x_{a+1}) = p_4 + q_4 + 15c + 2a - 13, h_4(x'_a z_a) = p_4 + q_4 + 15c + 2a - 12, h_4(x'_{a+1} z_a) = p_4 + q_4 + 17c + 2a - 15, h_4(x_a x'_{a+1}) = p_4 + q_4 + 17c + 2a - 14.$

Consider the following two cases.

**Case 1.** $p_4 + q_4$ is odd

For each $1 \leq a \leq c - 1, h_4(y_a) = p_4 + q_4 + 2a - 1, h_4(x_{a+1}) = p_4 + q_4 + 2a, h_4(z_a) = p_4 + q_4 + 2c - 2 + a.$

**Case 2.** $p_4 + q_4$ is even

$h_4(z_1) = p_4 + q_4 + 1.$

For each $1 \leq a \leq c - 1, h_4(y_a) = p_4 + q_4 + 2a, h_4(x_{a+1}) = p_4 + q_4 + 2a + 1.$

$h_4(z_{a+1}) = p_4 + q_4 + 2c - 1 + a$ for $1 \leq a \leq c - 2.$

Hence $G_4^* = G_4 \odot H_4$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.5.** If $G_5$ has vertex edge neighborhood prime graph, then $G_5 \odot$ [ duplicating all the vertices of helm graph $H_t$] that admits vertex edge neighborhood prime.

Proof. Let $G_5(p_5, q_5)$ be vertex edge neighborhood prime graph with bijection $g_5: V(G_5) \cup E(G_5) \to \{1,2,\ldots,|V(G_5) \cup E(G_5)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_5$ the duplicating all the vertices of helm graph $H_t$ with

$V(H_5) = \{c_0\} \cup \{c'_0\} \cup \{c_r, c'_r, d_r, d'_r: 1 \leq r \leq t\}$ and

$E(H_5) = \{c_0 c_r, c'_0 c_r, c_0 c'_r, c_r d'_r, c'_r d_r, c_r d_r: 1 \leq r \leq t\} \cup \{c_1 c_t\} \cup \{c'_1 c_t\} \cup \{c_1 c'_t\} \cup \{c_r c_{r+1}, c'_r c_{r+1}, c_r c'_{r+1}: 1 \leq r \leq t - 1\}.$

We identify one of the vertex say $c_0$ of $H_5$ on selected vertex of $u_1$ in $G_5$ with $g_5(u_1) = 1.$

Let us construct a new graph $G_5^* = G_5 \odot H_5$ with $V(G_5^*) = V(G_5) \cup V(H_5)$ and $E(G_5^*) = E(G_5) \cup E(H_5).$

$|V(G_5^*)| = p_5 + 4t + 1$ and $|E(G_5^*)| = q_5 + 9t.$

Define $h_5: V(G_5^*) \cup E(G_5^*) \to \{1,2,\ldots,p_5 + q_5 + 13t + 1\}$ by

$g_5(z_5) = h_5(z_5)$ for all $z_5 \in V(G_5)$ and $g_5(d_5) = h_5(d_5)$ for all $d_5 \in E(G_5).$

$h_5(c_0) = h_5(u_1) = 1, h_5(c_1 c_t) = p_5 + q_5 + 6t, h_5(c'_1 c_t) = p_5 + q_5 + 9t + 1, h_5(c_1 c'_t) = p_5 + q_5 + 11t, h_5(c'_0) = p_5 + q_5 + 12t + 1.$

For each $1 \leq r \leq t, h_5(c'_r) = p_5 + q_5 + 3r - 2, h_5(c_r) = p_5 + q_5 + 3r - 1, h_5(d'_r) = p_5 + q_5 + 3t + r, h_5(d_r) = p_5 + q_5 + 4t + r, h_5(c_r d'_r) = p_5 + q_5 + 3r, h_5(c_0 c_r) = p_5 + q_5 + 6t + r, h_5(c'_r d_r) = p_5 + q_5 + 7t + 2r - 1, h_5(c_r d_r) = p_5 + q_5 + 7t + 2r, h_5(c_0 c'_r) = p_5 + q_5 + 11t + r, h_5(c'_0 c_r) = p_5 + q_5 + 12t + r + 1.$

For each $1 \leq r \leq t - 1, h_5(c_r c_{r+1}) = p_5 + q_5 + 5t + r, h_5(c'_r c_{r+1}) = p_5 + q_5 + 9t + 2r, h_5(c_r c'_{r+1}) = p_5 + q_5 + 9t + 2r + 1.$

Hence $G_5 \odot H_5$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.6.** If $G_6$ has vertex edge neighborhood prime graph, then $G_6 \odot$ [ duplicating all the vertices of closed helm graph $CH_s$] that admits vertex edge neighborhood prime.

Proof. Let $G_6(p_6, q_6)$ be vertex edge neighborhood prime graph with bijection $g_6: V(G_6) \cup E(G_6) \to \{1,2,\ldots,|V(G_6) \cup E(G_6)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_6$ the duplicating all the vertices of closed helm graph $CH_s$ with

$V(H_6) = \{a_0\} \cup \{a'_0\} \cup \{a_u, a'_u, b_u, b'_u: 1 \leq u \leq s\}$ and

$E(H_6) = \{a_0 a_u, a'_0 a_u, a_u b_u, a_u b'_u, a'_u b_u, a_0 a'_u: 1 \leq u \leq s\} \cup \{a_1 a_s\} \cup \{b_1 b_s\} \cup \{a_u a_{u+1}, b_u b_{u+1}, b'_u b_{u+1}, b_u b'_{u+1}, a'_u a_{u+1}, a_u a'_{u+1}: 1 \leq u \leq s - 1\} \cup \{a'_1 a_s\} \cup \{b'_1 b_s\} \cup \{a_1 a'_s\} \cup \{b_1 b'_s\}$. We superimposing one of the vertex say $b_1$ of $H_6$ on selected vertex of $k_1$ in $G_6$ with $g_6(k_1) = 1.$

Let us construct a new graph $G_6^* = G_6 \odot H_6$ with $V(G_6^*) = V(G_6) \cup V(H_6)$ and $E(G_6^*) = E(G_6) \cup E(H_6)$

$|V(G_6^*)| = p_6 + 4s + 1$ and $|E(G_6^*)| = q_6 + 12s.$

Define $h_6: V(G_6^*) \cup E(G_6^*) \to \{1,2,\ldots,p_6 + q_6 + 16s + 1\}$ by

$g_6(z_6) = h_6(z_6)$ for all $z_6 \in V(G_6)$ and $g_6(d_6) = h_6(d_6)$ for all $d_6 \in E(G_6).$

$h_6(b_1) = h_6(k_1) = 1, h_6(a_s) = p_6 + q_6 + 2s - 1, h_6(a_0) = p_6 + q_6 + 2s, h_6(b_1 b_s) = p_6 + q_6 + 3s, h_6(a_1 a_s) = p_6 + q_6 + 5s, h_6(a'_1 a_s) = p_6 + q_6 + 11s + 1, h_6(a_1 a'_s) = p_6 + q_6 + 13s, h_6(b'_1 b_s) = p_6 + q_6 + 13s + 1, h_6(b_1 b'_s) = p_6 + q_6 + 15s, h_6(a'_0) = p_6 + q_6 + 15s + 1.$

For each $1 \leq u \leq s, h_6(a_u b_u) = p_6 + q_6 + 3s + u, h_6(a_0 a_u) = p_6 + q_6 + 5s + u, h_6(a'_u) = p_6 + q_6 + 6s + u, h_6(b'_u) = p_6 +$

$q_6+7s+u, h_6(a_u b'_u)=p_6+q_6+8s+u, h_6(a_0 a'_u)=p_6+q_6+9s+u, h_6(a'_u b_u)=p_6+q_6+10s+u, h_6(a'_0 a_u)=p_6+q_6+15s+1+u$.

For each $1\le u\le s-1, h_6(a_u)=p_6+q_6+2u, h_6(b_{u+1})=p_6+q_6+2u-1, h_6(b_u b_{u+1})=p_6+q_6+2s+u, h_6(a_u a_{u+1})=p_6+q_6+4s+u, h_6(a'_u a_{u+1})=p_6+q_6+11s+2u, h_6(a_u a'_{u+1})=p_6+q_6+11s+1+2u, h_6(b'_u b_{u+1})=p_6+q_6+13s+2u, h_6(b_u b'_{u+1})=p_6+q_6+13s+1+2u$.

Hence $G_6\odot H_6$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.7.** If $G_7$ has vertex edge neighborhood prime graph, then $G_7\odot$ [ duplicating all the vertices of planter graph $R_c$] that admits vertex edge neighborhood prime for all $c\ge 3$.

Proof. Let $G_7(p_7,q_7)$ be vertex edge neighborhood prime graph with bijection $g_7: V(G_7)\cup E(G_7)\to\{1,2,\dots,|V(G_7)\cup E(G_7)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_7$ the duplicating all the vertices of planter graph $R_c$ where $c\ge 3$ with

$V(H_7)=\{u_0\}\cup\{u'_0\}\cup\{u_z,u'_z: 1\le z\le c-1\}\cup\{v_z,v'_z: 1\le z\le c\}$ and

$E(H_7)=\{u_0v_z,u_0v'_z,u'_0v_z: 1\le z\le c\}\cup\{u'_z u_{z+1},u_z u'_{z+1},u_z u_{z+1}: 1\le z\le c-2\}\cup\{u_0u_1\}\cup\{u_0u_{c-1}\}\cup\{u'_0u_1\}\cup\{u'_0u_{c-1}\}\cup\{u_0u'_1\}\cup\{u_0u'_{c-1}\}\cup\{v'_z v_{z+1},v_z v'_{z+1},v_z v_{z+1}: 1\le z\le c-1\}$.

We overlay one of the vertex say $u_0$ of $H_7$ on selected vertex of $s_1$ in $G_7$ with $g_7(s_1)=1$.

Let us construct a new graph $G_7^*=G_7\odot H_7$ with $V(G_7^*)=V(G_7)\cup V(H_7)$ and $E(G_7^*)=E(G_7)\cup E(H_7)$

$|V(G_7^*)|=p_7+4c-1$ and $|E(G_7^*)|=q_7+9c-3$.

Define $h_7: V(G_7^*)\cup E(G_7^*)\to\{1,2,\dots,p_7+q_7+13c-4\}$ by

$g_7(z_7)=h_7(z_7)$ for all $z_7\in V(G_7)$ and $g_7(d_7)=h_7(d_7)$ for all $d_7\in E(G_7)$.

$h_7(u_0)=h_7(s_1)=1, h_7(u_0u_1)=p_7+q_7+4c-1, h_7(u_0u_{c-1})=p_7+q_7+5c-2, h_7(u'_0u_1)=p_7+q_7+8c-1, h_7(u'_0u_{c-1})=p_7+q_7+8c, h_7(u_0u'_1)=p_7+q_7+8c+1, h_7(u_0u'_{c-1})=p_7+q_7+10c-2, h_7(u_0v'_c)=p_7+q_7+13c-4, h_7(u'_0)=p_7+q_7+7c-2$.

For each $1\le z\le c, h_7(u_0v_z)=p_7+q_7+2c+2z-2, h_7(u'_0v_z)=p_7+q_7+7c-2+z, h_7(v_z)=p_7+q_7+z, h_7(v'_z)=p_7+q_7+5c-2+z$.

For each $1\le z\le c-1, h_7(v_zv_{z+1})=p_7+q_7+2c+2z-1, h_7(u_0v'_z)=p_7+q_7+10c+2z-3, h_7(u'_z)=p_7+q_7+6c-2+z, h_7(v'_zv_{z+1})=p_7+q_7+10c+2z-2, h_7(v_zv'_{z+1})=p_7+q_7+12c+z-4$.

For each $1\le z\le c-2, h_7(u_zu_{z+1})=p_7+q_7+4c+z-1, h_7(u'_zu_{z+1})=p_7+q_7+8c+2z, h_7(u_zu'_{z+1})=p_7+q_7+8c+2z+1$. $h_7(u_{2z-1})=p_7+q_7+c+z$ for $1\le z\le\left\lfloor\frac{c}{2}\right\rfloor$.

$h_7(u_{2z})=p_7+q_7+c+\left\lfloor\frac{c}{2}\right\rfloor+z$ for $1\le z\le\left\lceil\frac{c}{2}\right\rceil-1$.

Hence $G_7\odot H_7$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.8.** If $G_8(p_8,q_8)$ has vertex edge neighborhood prime graph, then $G_8\odot$ [duplicating all the vertices of udukkai graph $A_t$ except the end vertices of path] that admits vertex edge neighborhood prime for all $t>2$.

Proof.Let $G_8(p_8,q_8)$ be vertex edge neighborhood prime graph with bijection $g_8: V(G_8)\cup E(G_8)\to\{1,2,\dots,|V(G_8)\cup E(G_8)|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_8$ the duplicating all the vertices of udukkai graph $A_t$ except the end vertices of path when $t>2$ with

$V(H_8)=\{u_0\}\cup\{u'_0\}\cup\{u_r,v_r: 1\le r\le t-1\}\cup\{u'_r,v'_r: 1\le r\le t-2\}\cup\{w_r,w'_r: 1\le r\le 2t\}$ and

$E(H_8)=\{u_0w_r,u_0w'_r,u'_0w_r: 1\le r\le 2t\}\cup\{u_0v_1\}\cup\{u_0u_1\}\cup\{u'_0u_1\}\cup\{u'_0v_1\}\cup\{u_ru_{r+1},v_rv_{r+1},u'_ru_{r+1},v'_rv_{r+1}: 1\le r\le t-2\}\cup\{u_ru'_{r+1},v_rv'_{r+1}: 1\le r\le t-3\}\cup\{w_rw_{r+1},w_{t+r}w_{t+r+1},w'_rw_{r+1},w_rw'_{r+1},w'_{t+r}w_{t+r+1},w_{t+r}w'_{t+r+1}: 1\le r\le t-1\}\cup\{u_0v'_1\}\cup\{u_0u'_1\}$.

We identify one of the vertex say $u_0$ of $H_8$ on selected vertex of $s_1$ in $G_8$ with $g_8(s_1)=1$.

Let us construct a new graph $G_8^*=G_8\odot H_8$ with $V(G_8^*)=V(G_8)\cup V(H_8)$ and $E(G_8^*)=E(G_8)\cup E(H_8)$.

$|V(G_8^*)|=p_8+8t-5$ and $|E(G_8^*)|=q_8+18t-14$.

Define $h_8: V(G_8^*)\cup E(G_8^*)\to\{1,2,\dots,p_8+q_8+26t-19\}$ by

$g_8(z_8)=h_8(z_8)$ for all $z_8\in V(G_8)$ and $g_8(e_8)=h_8(e_8)$ for all $e_8\in E(G_8)$.

$h_8(u_0)=h_8(s_1)=1, h_8(u'_0)=p_8+q_8+4t+1, h_8(u'_0u_1)=p_8+q_8+8t+2, h_8(u'_0v_1)=p_8+q_8+8t+3, h_8(u_0u_1)=p_8+q_8+23t-14, h_8(u_0v'_1)=p_8+q_8+23t-13, h_8(u_0u'_1)=p_8+q_8+20t-8, h_8(u_0v_1)=p_8+q_8+26t-19, h_8(u_0w'_t)=p_8+q_8+13t-1, h_8(u_0w'_{2t})=p_8+q_8+16t-3$.

For each $1\le r\le 2t, h_8(w_r)=p_8+q_8+r, h_8(w'_r)=p_8+q_8+2t+r, h_8(u_0w_r)=p_8+q_8+4t+2r, h_8(u'_0w_r)=p_8+$

$q_8 + 4t + 1 + 2r$.

For each $1 \le r \le t-2, h_8(u'_r u_{r+1}) = p_8 + q_8 + 20t + 2r - 9, h_8(u_r u_{r+1}) = p_8 + q_8 + 22t - 13 + (t-1-r), h_8(v'_r v_{r+1}) = p_8 + q_8 + 23t + 2r - 14, h_8(v_r v_{r+1}) = p_8 + q_8 + 25t - 18 + (t-1-r)$.

For each $1 \le r \le t-3, h_8(u'_r) = p_8 + q_8 + 18t - 3 + r, h_8(v'_r) = p_8 + q_8 + 19t - 6 + r, h_8(u_r u'_{r+1}) = p_8 + q_8 + 20t + 2r - 8, h_8(v_r v'_{r+1}) = p_8 + q_8 + 23t + 2r - 13$.

For each $1 \le r \le t-1, h_8(w_r w_{r+1}) = p_8 + q_8 + 8t + 3 + r, h_8(w_{t+r} w_{t+r+1}) = p_8 + q_8 + 9t + 2 + r, h_8(u_0 w'_r) = p_8 + q_8 + 10t + 2r, h_8(w'_r w_{r+1}) = p_8 + q_8 + 10t + 1 + 2r, h_8(w_r w'_{r+1}) = p_8 + q_8 + 12t + r - 1, h_8(u_0 w'_{t+r}) = p_8 + q_8 + 13t - 2 + 2r, h_8(w'_{t+r} w_{t+r+1}) = p_8 + q_8 + 13t + 2r - 1, h_8(w_{t+r} w'_{t+r+1}) = p_8 + q_8 + 15t + r - 3$.

Let us consider the following two cases.

**Case 1.** $t$ is odd

$h_8(u'_{t-2}) = p_8 + q_8 + 16t + \left\lceil \frac{t}{2} \right\rceil - 2, h_8(v'_{t-2}) = p_8 + q_8 + 17t + \left\lceil \frac{t}{2} \right\rceil - 2$.

For each $1 \le r \le \left\lfloor \frac{t}{2} \right\rfloor, h_8(u_{2r-1}) = p_8 + q_8 + 16t - 3 + r, h_8(u_{2r}) = p_8 + q_8 + 16t - 2 + \left\lfloor \frac{t}{2} \right\rfloor + r, h_8(v_{2r-1}) = p_8 + q_8 + 17t - 3 + r, h_8(v_{2r}) = p_8 + q_8 + 17t - 2 + \left\lfloor \frac{t}{2} \right\rfloor + r$.

**Case 2.** $t$ is even

$h_8(u'_{t-2}) = p_8 + q_8 + 17t - 3, h_8(v'_{t-2}) = p_8 + q_8 + 18t - 3$.

For each $1 \le r \le \frac{t}{2}, h_8(u_{2r-1}) = p_8 + q_8 + 16t - 3 + r, h_8(v_{2r-1}) = p_8 + q_8 + 17t - 3 + r$.

For each $1 \le r \le \frac{t}{2} - 1, h_8(u_{2r}) = p_8 + q_8 + 16t - 3 + \frac{t}{2} + r, h_8(v_{2r}) = p_8 + q_8 + 17t - 3 + \frac{t}{2} + r$.

Hence $G_8^* = G_8 \odot H_8$ is vertex edge neighborhood prime graph. ■

**Theorem 3.9.** If $G_9(p_9, q_9)$ has vertex edge neighborhood prime graph, then $G_9 \odot$ [duplicating all the vertices of butterfly graph $BF_{m,n}$] that admits vertex edge neighborhood prime.

Proof. Let $G_9(p_9, q_9)$ be vertex edge neighborhood prime graph with bijection $g_9: V(G_9) \cup E(G_9) \to \{1,2,\ldots,|V(G_9) \cup E(G_9)|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_9$ the duplicating all the vertices of butterfly graph $BF_{m,n}$ with

$V(H_9) = \{w_i, w'_i : 1 \le i \le m\} \cup \{u_j, v_j, u'_j, v'_j : 1 \le j \le n-1\} \cup \{u_0\} \cup \{u'_0\}$ and

$E(H_9) = \{u_j u_{j+1}, v_j v_{j+1}, u'_j u_{j+1}, u_j u'_{j+1}, v'_j v_{j+1}, v_j v'_{j+1}, : 1 \le j \le n-2\} \cup \{u_0 u'_1\} \cup \{u_0 v'_1\} \cup \{u_0 u'_{n-1}\} \cup \{u_0 v'_{n-1}\} \cup \{u'_0 u_1\} \cup \{u'_0 u_{n-1}\} \cup \{u'_0 v_1\} \cup \{u'_0 v_{n-1}\} \cup \{u_0 u_1\} \cup \{u_0 u_{n-1}\}\{u_0 v_1\} \cup \{u_0 v_{n-1}\} \cup \{u'_0 w_i, u_0 w_i, u_0 w'_i : 1 \le i \le m\}$.

We overlay one of the vertex say $u_0$ of $H_9$ on selected vertex of $z_1$ in $G_9$ with $g_9(z_1) = 1$.

Let us construct a new graph $G_9^* = G_9 \odot H_9$ with $V(G_9^*) = V(G_9) \cup V(H_9)$ and $E(G_9^*) = E(G_9) \cup E(H_9)$.

$|V(G_9^*)| = p_9 + 2m + 4n - 3$ and $|E(G_9^*)| = q_9 + 3m + 6n$.

Define $h_9: V(G_9^*) \cup E(G_9^*) \to \{1,2,\ldots,p_9 + q_9 + 5m + 10n - 3\}$ by

$g_9(z_9) = h_9(z_9)$ for all $z_9 \in V(G_9)$ and $g_9(e_9) = h_9(e_9)$ for all $e_9 \in E(G_9)$.

$h_9(u_0) = h_9(z_1) = 1, h_9(u'_0) = p_9 + q_9 + 2m + 4n - 3, h_9(u'_0 u_1) = p_9 + q_9 + 5m + 4n - 2, h_9(u'_0 u_{n-1}) = p_9 + q_9 + 5m + 4n - 1, h_9(u'_0 v_1) = p_9 + q_9 + 5m + 4n, h_9(u'_0 v_{n-1}) = p_9 + q_9 + 5m + 4n + 1, h_9(u_0 u_1) = p_9 + q_9 + 5m + 4n + 2, h_9(u_0 u_{n-1}) = p_9 + q_9 + 5m + 5n + 1, h_9(u_0 v_1) = p_9 + q_9 + 5m + 5n + 2, h_9(u_0 v_{n-1}) = p_9 + q_9 + 5m + 6n + 1, h_9(u_0 u'_1) = p_9 + q_9 + 5m + 6n + 2, h_9(u_0 u'_{n-1}) = p_9 + q_9 + 5m + 8n - 1, h_9(u_0 v'_1) = p_9 + q_9 + 5m + 8n, h_9(u_0 v'_{n-1}) = p_9 + q_9 + 5m + 10n - 3$.

For each $1 \le i \le m, h_9(w_i) = p_9 + q_9 + i, h_9(w'_i) = p_9 + q_9 + m + i, h_9(u_0 w'_i) = p_9 + q_9 + 2m + 4n - 3 + i, h_9(u_0 w_i) = p_9 + q_9 + 3m + 4n - 4 + 2i, h_9(u'_0 w_i) = p_9 + q_9 + 3m + 4n - 3 + 2i$.

For each $1 \le j \le n-2, h_9(u_j u_{j+1}) = p_9 + q_9 + 5m + 4n + 2 + j, h_9(v_j v_{j+1}) = p_9 + q_9 + 5m + 5n + 2 + j, h_9(u'_j u_{j+1}) = p_9 + q_9 + 5m + 6n + 1 + 2j, h_9(u_j u'_{j+1}) = p_9 + q_9 + 5m + 6n + 2 + 2j, h_9(v'_j v_{j+1}) = p_9 + q_9 + 5m + 8n + 2j - 1, h_9(v_j v'_{j+1}) = p_9 + q_9 + 5m + 8n + 2j$.

For each $1 \le j \le n-1, h_9(u'_j) = p_9 + q_9 + 2m + 2n - 2 + j, h_9(v'_j) = p_9 + q_9 + 2m + 3n - 3 + j$.

For each $1 \le j \le \left\lceil \frac{n}{2} \right\rceil, h_9(u_{2j-1}) = p_9 + q_9 + 2m + j, h_9(v_{2j-1}) = p_9 + q_9 + 2m + n - 1 + j$.

For each $1 \le j \le \left\lceil \frac{n}{2} \right\rceil - 1, h_9(u_{2j}) = p_9 + q_9 + 2m + \left\lceil \frac{n}{2} \right\rceil + j, h_9(v_{2j}) = p_9 + q_9 + 2m + n + \left\lceil \frac{n}{2} \right\rceil - 1 + j$.

Hence $G_9^* = G_9 \odot H_9$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.10.** If $G_{10}$ has vertex edge neighborhood prime graph, then $G_{10} \odot$ [duplicating all the vertices of shell graph $S_t$] that admits vertex edge neighborhood prime for all $t \geq 5$.

Proof. Let $G_{10}(p_{10}, q_{10})$ be vertex edge neighborhood prime graph with labeling

$g_{10}: V(G_{10}) \cup E(G_{10}) \rightarrow \{1,2,\ldots,|V(G_{10}) \cup E(G_{10})|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_{10}$ the duplicating all the vertices of shell graph $S_t$ where $t \geq 5$ with

$V(H_{10}) = \{u_x, u'_x : 1 \leq x \leq t\}$ and

$E(H_{10}) = \{u_1 u_{x+2}, u_1 u'_{x+2} : 1 \leq x \leq t-3\} \cup \{u_1 u_t\} \cup \{u_1 u'_t\} \cup \{u_x u_{x+1}, u'_1 u'_{x+1}, u_x u'_{x+1} : 1 \leq x \leq t-1\} \cup \{u'_{x+1} u_{x+2} : 1 \leq x \leq t-2\}$

We identify one of the vertex say $u_1$ of $H_{10}$ on selected vertex of $z_1$ in $G_{10}$ with $g_{10}(z_1) = 1$.

Let us construct a new graph $G_{10}^* = G_{10} \odot H_{10}$ with $V(G_{10}^*) = V(G_{10}) \cup V(H_{10})$ and $E(G_{10}^*) = E(G_{10}) \cup E(H_{10})$.

$|V(G_{10}^*)| = p_{10} + 2t - 1$ and $|E(G_{10}^*)| = q_{10} + 6t - 9$.

Define $h_{10}: V(G_{10}^*) \cup E(G_{10}^*) \rightarrow \{1,2,\ldots,p_{10} + q_{10} + 8t - 10\}$ by

$g_{10}(z_{10}) = h_{10}(z_{10})$ for all $z_{10} \in V(G_{10})$ and $g_{10}(e_{10}) = h_{10}(e_{10})$ for all $e_{10} \in E(G_{10})$.

$h_{10}(u_1) = h_{10}(z_1) = 1, h_{10}(u_1 u_t) = p_{10} + q_{10} + 2t - 1, h_{10}(u_{t-1} u'_t) = p_{10} + q_{10} + 8t - 10$.

For each $1 \leq x \leq t-1, h_{10}(u_{x+1}) = p_{10} + q_{10} + x, h_{10}(u_x u_{x+1}) = p_{10} + q_{10} + t - 1 + x, h_{10}(u'_1 u_{x+1}) = p_{10} + q_{10} + 4t - 4 + x$.

For each $1 \leq x \leq t-2, h_{10}(u_x u'_{x+1}) = p_{10} + q_{10} + 5t - 6 + 2x, h_{10}(u'_{x+1} u_{x+2}) = p_{10} + q_{10} + 5t - 5 + 2x, h_{10}(u_1 u'_{x+2}) = p_{10} + q_{10} + 7t - 9 + x$. $h_{10}(u'_x) = p_{10} + q_{10} + 3t - 4 + x$ for $1 \leq x \leq t$. $h_{10}(u_1 u_{x+2}) = p_{10} + q_{10} + 2t - 1 + x$ for $1 \leq x \leq t-3$.

Hence $G_{10}^* = G_{10} \odot H_{10}$ is vertex edge neighborhood prime graph. ∎

**Theorem 3.11.** If $G_{11}$ has vertex edge neighborhood prime graph, then $G_{11} \odot$ [ duplicating all the vertices of octopus graph $O_n$] that admits vertex edge neighborhood prime for all $n \geq 2$.

Proof. Let $G_{11}(p_{11}, q_{11})$ is vertex edge neighborhood prime graph with bijection $g_{11}: V(G_{11}) \cup E(G_{11}) \rightarrow \{1,2,\ldots,|V(G_{11}) \cup E(G_{11})|\}$ satisfying the condition of vertex edge neighborhood prime graph.

Consider $H_{11}$ the duplicating all the vertices of octopus graph $O_n$ where $n \geq 2$ with

$V(H_{11}) = \{u_0\} \cup \{u'_0\} \cup \{u_k, u'_k, v_k, v'_k : 1 \leq k \leq n\}$ and

$E(H_{11}) = \{u_0 u_k, u_0 u'_k, u'_0 u_k, u'_0 v_k, u_0 v_k, u_0 v'_k : 1 \leq k \leq n\} \cup \{u'_k u_{k+1}, u_k u'_{k+1}, u_k u_{k+1} : 1 \leq k \leq n-1\}$

We superimposing one of the vertex say $u_0$ of $H_{11}$ on selected vertex of $r_1$ in $G_{11}$ with $g_{11}(r_1) = 1$.

Define a new graph $G_{11}^* = G_{11} \odot H_{11}$ with $V(G_{11}^*) = V(G_{11}) \cup V(H_{11})$ and $E(G_{11}^*) = E(G_{11}) \cup E(H_{11})$.

$|V(G_{11}^*)| = p_{11} + 4n + 1$ and $|E(G_{11}^*)| = q_{11} + 9n - 3$.

Define $h_{11}: V(G_{11}^*) \cup E(G_{11}^*) \rightarrow \{1,2,\ldots,p_{11} + q_{11} + 13n - 2\}$ by

$g_{11}(u_{11}) = h_{11}(u_{11})$ for all $u_{11} \in V(G_{11})$ and $g_{11}(e_{11}) = h_{11}(e_{11})$ for all $e_{11} \in E(G_{11})$. $h_{11}(u_0) = h_{11}(r_1) = 1, h_{11}(u'_0) = p_{11} + q_{11} + 4n + 1, h_{11}(u_0 u'_n) = p_{11} + q_{11} + 13n - 2$.

For each $1 \leq k \leq n, h_{11}(u_k) = p_{11} + q_{11} + k, h_{11}(v_k) = p_{11} + q_{11} + n + k, h_{11}(u'_k) = p_{11} + q_{11} + 2n + k, h_{11}(v'_k) = p_{11} + q_{11} + 3n + k, h_{11}(u_0 u_k) = p_{11} + q_{11} + 4n + 2k, h_{11}(u'_0 u_k) = p_{11} + q_{11} + 4n + 2k + 1, h_{11}(u'_0 v_k) = p_{11} + q_{11} + 6n + 2k, h_{11}(u_0 v_k) = p_{11} + q_{11} + 6n + 2k + 1, h_{11}(u_0 v'_k) = p_{11} + q_{11} + 8n + 1 + k$.

For each $1 \leq k \leq n-1, h_{11}(u_k u_{k+1}) = p_{11} + q_{11} + 9n + 1 + k, h_{11}(u_0 u'_k) = p_{11} + q_{11} + 10n + 2k - 1, h_{11}(u'_k u_{k+1}) = p_{11} + q_{11} + 10n + 2k, h_{11}(u_k u'_{k+1}) = p_{11} + q_{11} + 12n - 2 + k$.

Hence $G_{11}^* = G_{11} \odot H_{11}$ admits vertex edge neighborhood prime graph. ∎

**Theorem 3.12.** If $G_{12}(p_{12}, q_{12})$ has vertex edge neighborhood prime graph, then $G_{12} \odot H_{12}$ [duplicating all the vertices of Mycielskian graph $\mu(C_x)$, of cycle $C_x$] that admits vertex edge neighborhood prime graph for all $x$ is odd.

Proof. Let $G_{12}(p_{12}, q_{12})$ be vertex edge neighborhood prime graph with bijection $g_{12}: V(G_{12}) \cup E(G_{12}) \rightarrow \{1,2,\ldots,|V(G_{12}) \cup E(G_{12})|\}$ satisfying the property of vertex edge neighborhood prime graph.

Consider $H_{12}$ the duplicating all the vertices of Mycielskian graph $\mu(C_x)$, of cycle $C_x$ when $x$ is odd with

$V(H_{12}) = \{r_0\} \cup \{r'_0\} \cup \{r_y, s_y, r'_y, s'_y : 1 \leq y \leq x\}$ and

$E(H_{12}) = \{s_y r_{y+1}, s_{y+1} r_y, r_{y+1} s'_y, r_y s'_{y+1}, r'_y s_{y+1}, s_y r'_{y+1} : 1 \leq y \leq x-1\} \cup \{s_1 r_x\} \cup \{r_1 s_x\} \cup \{s'_1 r_x\} \cup \{r_1 s'_x\} \cup \{r'_1 s_x\} \cup \{s_1 r'_x\} \cup \{r_0 r_y, r'_0 r_y, r_0 r'_y : 1 \leq y \leq x\}$.

We overlay one of the vertex say $r_1$ of $H_{12}$ on selected vertex of $z_1$ in $G_{12}$ with $g_{12}(z_1) = 1$.

Let us construct a new graph $G^*_{12} = G_{12} \odot H_{12}$ with $V(G^*_{12}) = V(G_{12}) \cup V(H_{12})$ and $E(G^*_{12}) = E(G_{12}) \cup E(H_{12})$.

$|V(G^*_{12})| = p_{12} + 4x + 1$ and $|E(G^*_{12})| = q_{12} + 9x$.

Define $h_{12}: V(G^*_{12}) \cup E(G^*_{12}) \to \{1,2,\dots,p_{12} + q_{12} + 13x + 1\}$ by

$g_{12}(z_{12}) = h_{12}(z_{12})$ for all $z_{12} \in V(G_{12})$ and $g_{12}(e_{12}) = h_{12}(e_{12})$ for all $e_{12} \in E(G_{12})$.

$h_{12}(r_1) = h_{12}(z_1) = 1, h_{12}(r_0) = p_{12} + q_{12} + x, h_{12}(r'_0) = p_{12} + q_{12} + 5x + 1, h_{12}(s_1 r_x) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil - 1, h_{12}(s_1 r_2) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil, h_{12}(r_1 s_x) = p_{12} + q_{12} + 4x, h_{12}(s'_1 r_x) = p_{12} + q_{12} + 7x + 2, h_{12}(r_1 s'_x) = p_{12} + q_{12} + 9x + 1, h_{12}(r'_1 s_x) = p_{12} + q_{12} + 10x + 2, h_{12}(s_1 r'_x) = p_{12} + q_{12} + 12x + 1$.

For each $1 \le y \le \left\lceil \frac{x}{2} \right\rceil, h_{12}(r_{2y-1}) = p_{12} + q_{12} + y - 1, h_{12}(s_{2y-1}) = p_{12} + q_{12} + x + y$.

For each $1 \le y \le \left\lfloor \frac{x}{2} \right\rfloor$, $h_{12}(r_{2y}) = p_{12} + q_{12} + \left\lceil \frac{x}{2} \right\rceil + y - 1, h_{12}(s_{2y}) = p_{12} + q_{12} + \left\lceil \frac{x}{2} \right\rceil + x + y, h_{12}(r_{2y-1} s_{2y}) = p_{12} + q_{12} + 2x + 2y - 1, h_{12}(r_{2y} s_{2y+1}) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil + 2y - 1, h_{12}(r_{2y+1} s_{2y}) = p_{12} + q_{12} + 2x + 2y$.

For each $1 \le y \le x$, $h_{12}(r_0 r_y) = p_{12} + q_{12} + 4x + y, h_{12}(r'_y) = p_{12} + q_{12} + 5x + 1 + y, h_{12}(s'_y) = p_{12} + q_{12} + 6x + 1 + y, h_{12}(r_0 r'_y) = p_{12} + q_{12} + 9x + 1 + y, h_{12}(r'_0 r_y) = p_{12} + q_{12} + 12x + 1 + y$.

For each $1 \le y \le x - 1, h_{12}(s'_y r_{y+1}) = p_{12} + q_{12} + 7x + 2y + 1, h_{12}(r_y s'_{y+1}) = p_{12} + q_{12} + 7x + 2 + 2y, h_{12}(r'_y s_{y+1}) = p_{12} + q_{12} + 10x + 1 + 2y, h_{12}(s_y r'_{y+1}) = p_{12} + q_{12} + 10x + 2 + 2y$. $h_{12}(r_{2y} s_{2y-1}) = p_{12} + q_{12} + 2x + 2\left\lceil \frac{x}{2} \right\rceil + 2y - 2$ for $2 \le y \le \left\lfloor \frac{x}{2} \right\rfloor$.

Hence $G^*_{12} = G_{12} \odot H_{12}$ admits vertex edge neighborhood prime graph. ■

**Theorem 3.13.** The graph acquired by duplicating all the vertices of $K_t$ that admits vertex edge neighborhood prime.

Proof. Consider $G_{13}$ the duplicating all the vertices of $K_t$ with

$V(G_{13}) = \{d_x, d'_x : 1 \le x \le t\}$ and

$E(G_{13}) = \{d_x d_{y+1} : 1 \le x \le t - 1, x \le y \le t - 1\} \cup \{d'_x d_{y+1} : 1 \le x \le t - 1, x \le y \le t - 1\} \cup \{d'_{x+1} d_y : 1 \le x \le t - 1, 1 \le y \le x\}$.

$|V(G_{13})| = 2t$ and $|E(G_{13})| = t(t-1) + \frac{t(t-1)}{2}$.

Define $f_{13}: V(G_{13}) \cup E(G_{13}) \to \left\{1,2,\dots,2t + t(t-1) + \frac{t(t-1)}{2}\right\}$ by

For each $1 \le x \le t, f_{13}(d_x) = x, f_{13}(d'_x) = \frac{t(t-1)}{2} + t + x$.

For each $1 \le x \le t - 1$ and $x \le y \le t - 1$, $f_{13}(d_x d_{y+1}) = xt - \frac{x(x+1)}{2} + 1 + y$, $f_{13}(d'_x d_{y+1}) = \frac{t(t-1)}{2} + (x+1)t - \frac{x(x+1)}{2} + 1 + y$. $f_{13}(d'_{x+1} d_y) = t(t-1) + 2t + [\frac{x(x-1)}{2} - 1] + 1 + y$ for $1 \le x \le t - 1$ and $1 \le y \le x$.

Hence $G_{13}$ is vertex edge neighborhood prime graph. ■

**Theorem 3.14.** The graph acquired by duplicating all the vertices of double triangular snake $DT_z$ that admits vertex edge neighborhood prime for all $z > 1$.

Proof. Consider $G_{14}$ the duplicating all the vertices of double triangular snake $DT_z$, where $z > 1$ with

$V(G_{14}) = \{r_c, r'_c : 1 \le c \le z\} \cup \{s_c, s'_c, t_c, t'_c : 1 \le c \le z - 1\}$ and

$E(G_{14}) = \{r_c r_{c+1}, r_c s_c, s_c r_{c+1}, r_c t_c, r_{c+1} t_c, r_c s'_c, r_{c+1} s'_c, r_c t'_c, r_{c+1} t'_c : 1 \le c \le z - 1\} \cup \{r'_c r_{c+1}, r'_c t_c, r_c r'_{c+1}, r'_{c+1} t_c : 1 \le c \le z - 1\} \cup \{r'_1 s_1\} \cup \{r'_z s_{z-1}\}$.

$|V(G_{14})| = 6z - 4$ and $|E(G_{14})| = 13z - 11$.

Define $f_{14}: V(G_{14}) \cup E(G_{14}) \to \{1,2,\dots,19z - 15\}$ by $f_{14}(r'_1 s_1) = 15z - 12$, $f_{14}(r'_z s_{z-1}) = 15z - 11$.

For each $1 \le c \le z, f_{14}(r_c) = 2c - 1, f_{14}(r'_c) = 8z - 7 + c$.

For each $1 \le c \le z - 1, f_{14}(s_c) = 2c, f_{14}(t_c) = 2z - 1 + c, f_{14}(r_c r_{c+1}) = 3z + 5c - 2, f_{14}(r_c s_c) = 3z - 5c - 6, f_{14}(r_{c+1} s_c) = 3z + 5c - 5, f_{14}(r_c t_c) = 3z + 5c - 3, f_{14}(r_{c+1} t_c) = 3z + 5c - 4, f_{14}(s'_c) = 9z - 7 + c, f_{14}(t'_c) = 10z - 8 + c, f_{14}(r_c s'_c) = 11z - 10 + 2c, f_{14}(r_{c+1} s'_c) = 11z - 9 + 2c, f_{14}(r_c t'_c) = 13z - 12 + 2c, f_{14}(r_{c+1} t'_c) = 13z - 11 + 2c, f_{14}(r'_c r_{c+1}) = 15z - 12 + 2c, f_{14}(r'_c t_c) = 15z - 11 + 2c, f_{14}(r'_{c+1} t_c) = 17z - 14 + 2c, f_{14}(r_c r'_{c+1}) = 17z - 13 + 2c$.

Hence $G_{14}$ is vertex edge neighborhood prime graph. ■

**Theorem 3.15.** The graph acquired by duplicating all the vertices of rectangular book $B_{4,t}$ admits vertex edge neighborhood prime graph for all $t$.

Proof. Consider $G_{15}$ the duplicating all the vertices of rectangular book $B_{4,t}$ with

$V(G_{15}) = \{c_x, c'_x, d_x, d'_x : 1 \le x \le t+1\}$ and

$E(G_{15}) = \{c_1c_{x+1}, c'_1c_{x+1}, d_1d_{x+1}, d'_1d_{x+1}, d_1d'_{x+1}, c_1c'_{x+1} : 1 \le x \le t\} \cup \{c_xd_x, c'_xd_x, c_xd'_x : 1 \le x \le t+1\}$. and $|E(G_{15})| = 9t+3$.

Define a bijective function $f_{15}: V(G_{15}) \cup E(G_{15}) \to \{1,2,\dots,13t+7\}$ as follows: $f_{15}(c_1c_2) = 2t+3, f_{15}(c_2d_2) = 2t+4, f_{15}(d_1d_2) = 2t+5, f_{15}(c_1d_1) = 2t+6, f_{15}(c'_1d_1) = 11t+6, f_{15}(c_1d'_1) = 12t+7$.

For each $1 \le x \le t+1, f_{15}(c_x) = 2x-1, f_{15}(d_x) = 2x, f_{15}(c'_x) = 5t+3+x, f_{15}(d'_x) = 6t+4+x$.

For each $1 \le x \le t-1$, $f_{15}(c_1c_{x+2}) = 2t+3x+4$, $f_{15}(c_{x+2}d_{x+2}) = 2t+3x+5$, $f_{15}(d_1d_{x+2}) = 2t+3x+6$.

For each $1 \le x \le t, f_{15}(c_1c'_{x+1}) = 7t+2x+4$, $f_{15}(c'_{x+1}d_{x+1}) = 7t+2x+5$, $f_{15}(d_1d'_{x+1}) = 9t+2x+4$, $f_{15}(c_{x+1}d'_{x+1}) = 9t+2x+5$, $f_{15}(c'_1c_{x+1}) = 11t+x+6, f_{15}(d'_1d_{x+1}) = 12t+x+7$.

Hence $G_{15}$ is vertex edge neighborhood prime graph. ■

**Theorem 3.16.** The graph acquired by duplicating all the vertices of Mycielskian graph $\mu(C_k)$, of cycle $C_k$ that admits vertex edge neighborhood prime.

Proof. Consider $G_{16}$ be duplicating all the vertices of Mycielskian graph $\mu(C_k)$, of cycle $C_k$ with

$V(H_{16}) = \{r_0\} \cup \{r'_0\} \cup \{r_z, s_z, r'_z, s'_z : 1 \le z \le k\}$ and

$E(H_{16}) = \{s_zr_{z+1}, s_{z+1}r_z, r_{z+1}s'_z, r_zs'_{z+1}, r'_zs_{z+1}, s_zr'_{z+1} : 1 \le z \le k-1\} \cup \{s_1r_k\} \cup \{r_1s_k\} \cup \{s'_1r_k\} \cup \{r_1s'_k\} \cup \{r'_1s_k\} \cup \{s_1r'_k\} \cup \{r_0r_z, r'_0r_z, r_0r'_z : 1 \le z \le k\}$.

$|V(G_{16})| = 4k+2$ and $|E(G_{16})| = 9k$.

Define $f_{16}: V(G_{16}) \cup E(G_{16}) \to \{1,2,\dots,13k+2\}$ by

$f_{16}(r_0) = k+1$, $f_{16}(r'_0) = 5k+2$, $f_{16}(s_1r_k) = 2k+2\left\lceil\frac{k}{2}\right\rceil$, $f_{16}(s_1r_2) = 2k+2\left\lceil\frac{k}{2}\right\rceil+1$, $f_{16}(r_1s_k) = 4k+1$, $f_{16}(s'_1r_k) = 7k+3$, $f_{16}(r_1s'_k) = 9k+2$, $f_{16}(r'_1s_k) = 10k+3$, $f_{16}(s_1r'_k) = 12k+2$.

For each $1 \le z \le \left\lceil\frac{k}{2}\right\rceil, f_{16}(r_{2z-1}) = z, f_{16}(s_{2z-1}) = k+z+1$.

For each $1 \le z \le \left\lfloor\frac{k}{2}\right\rfloor, f_{16}(r_{2z}) = \left\lceil\frac{k}{2}\right\rceil + z, f_{16}(s_{2z}) = \left\lceil\frac{k}{2}\right\rceil + k + z + 1, f_{16}(r_{2z-1}s_{2z}) = 2k+2z, f_{16}(r_{2z}s_{2z+1}) = 2k+2\left\lceil\frac{k}{2}\right\rceil + 2z, f_{16}(r_{2z+1}s_{2z}) = 2k+2z+1$.

For each $1 \le z \le k, f_{16}(r_0r_z) = 4k+z+1, f_{16}(r'_z) = 5k+2+z, f_{16}(s'_z) = 6k+2+z, f_{16}(r_0r'_z) = 9k+2+z, f_{16}(r'_0r_z) = 12k+2+z$.

For each $1 \le z \le k-1$, $f_{16}(s'_zr_{z+1}) = 7k+2z+2$, $f_{16}(r_zs'_{z+1}) = 7k+3+2z, f_{16}(r'_zs_{z+1}) = 10k+2+2z, f_{16}(s_zr'_{z+1}) = 10k+3+2z$. $f_{16}(r_{2z}s_{2z-1}) = 2k+2\left\lceil\frac{k}{2}\right\rceil+2z-1$ for $2 \le z \le \left\lfloor\frac{k}{2}\right\rfloor$.

Hence $G_{16}$ is vertex edge neighborhood prime graph. ■

## References


[1] Balakrishnan. R and Ranganathan. K, *A Text Book of Graph Theory*, Second Edition, Springer, New York, 2012.

[2] Gallian. J. A, *A Dynamic Survey of Graph Labeling*, Electronic Journal of Combinatorics, DS6, 2015.

[3] Parul B Pandya and Shrimali. N. P, *Vertex Edge Neighborhood Prime Labeling of Some Graphs*, International Journal of Scientific Research and Review, Volume 7, Issue 10, 735 - 743, 2018.

[4] Simaringa. M and Vijayalakshmi. K, *Vertex Edge Neighborhood Prime Labeling of Some Graphs*, Malaya Journal of Mathematik, Volume 7, No. 4, Page 775 - 785, 2019.

[5] Simaringa. M and Vijayalakshmi. K, *Vertex Edge Neighborhood Prime Labeling For Union And One Point Union of Graphs*, International Journal of Advanced Science and Technology, Vol.29, No. 4, Page 309 - 320, 2020.

[6] Simaringa. M and Vijayalakshmi. K, *Graph identification of different operations of vertex edge neighborhood prime* , Malaya Journal of Mathematik, Volume 9, No. 1, Page 1057 -1070, 2021.