ISSN: 0974-5823 Vol. 7 No. 4 April, 2022

International Journal of Mechanical Engineering

Fuzzy Chaos Model Based Cryptosystem with Sine Chaotic Map

Bushra Hussien Aliwi and Ruma Kareem K. Ajeena

Mathematics Department, Education College for Pure Sciences, University of Babylon, Babylon, Iraq,

Abstract.

In based on a Knapsack problem approach, a fuzzy chaos based cryptography method for a sine chaotic map is proposed, where the system work through designing the Takagi-Sugeno (TS) fuzzy model on a chaotic map as a seed map in sine map. The basic idea of the method is to apply chaotification method to use a sine chaotic map approached on the out puts of TS fuzzy model to compute a real super increasing sequence applied in Knapsack equation. The proposed method only requires the application on Lozi chaotic map covered by sine map in generating the secure key at the real super increasing sequence and in encryption, decryptions models. Algorithmic performance of the method are also derived. Also some numerical simulations are presented to demonstrate the effectiveness of the method.

Keywords: Chaotic cryptosystem, Sine chaotic map, TS Fuzzy Model, Knapsack problem, .

1. Introduction :

Cryptography, defined as the science and study of secret writing, concerns the ways in which communications and data can be encoded to prevent disclosure of their contents through eavesdropping or message interception, using codes, ciphers, or other methods, so that only certain people can see the real messages [1].

A Cryptosystems are continuously improved. So there are many method combined with it such: Chaos maps as Chaos based Cryptography [2]–[4], and Fuzzy Logic that combined with Chaotic Systems [5] as Fuzzy Modeling of Chaotic Systems [6], for both discrete and continuous time models. fuzzy logic systems are lacked of a systematic modeling and control design methodology if it used individually, and in most controllers the stability of the closed-loop controlled system is not easy. So a fuzzy model for chaotic dynamics were approaches in many applications [6].

All were used to increase the security and speed in process data. The sine map employed to compose chaotic map to get Sine Chaotic Map SCM based on Cosine Chaotic Map CCM as in reference [7]. The SCM is in the interval [-1,1] for any value for the chaotic map that used as seed map and it with zero value as it started with initial zero value. That SCC determine the outputs within sine outputs interval**.**

Previously in our work the suggested method performed the CCM, now the SCC will performed. Through this performance the method aim to improve the performance of fuzzy chaos-based cryptosystem in [1], [8].

In this paper, a knapsack problem [9] with the TS fuzzy model [8] is applied to synthesis a cryptosystem by chaotic system, where a the discrete-time model is used for the performance. The proposed method is efficient in that it inherits the using of TS fuzzy model in knapsack problem. The TS fuzzy model determine the trajectory of premise variable in the system's region. The generation of the super-increasing based on the secret key values that corresponding to trajectory values. The sine map implemented on a chaotic map(seed map), that is sine for all trajectory, an then avoiding the negative values for sine map. Within the encryption stage, each message encrypted and then modified. A discrete time chaotic map that use for this method is Lozi map with appropriate parameters that ensure existing the chaotic behavior for trajectory values. The proposed method can provide an perfect combination between the TS chaotic fuzzy model with Knapsack problem. However, our proposed method could be implemented on any discrete-time chaotic map. In compared with the standard Knapsack, it more complex [10].

We organize this paper as follows. In Section2, we shall give a mathematical approaches for chaotification method sine chaotic map, and its terminology. Section3, presents steps of suggesting the TS fuzzy model for chaotic system and then provides the system algorithms. In Section4, we shall study and implement a simulation example which illustrate the effectively performance of the proposed method. Finally, in section5, we would give conclusion of the paper.

2.Sine Chaotic Map:

Sine chaotic Map (SCM) is a new chaotification method that enhances the chaotic complexity of existing chaotic maps. This method perform the sine function alongside a proposed chaotic map or the original maps or seed maps that cascade in used system, the way of performing is similar to the cosine function. So the result(s) pay a new chaotic maps have a wide chaotic range within the closed interval [-1,1] such; elevated sensitivity, complex characteristics, high nonlinearity, and an extended cycle length in compared with the seed map.

Theoretically SCM has properties based on the properties of the underlying seed maps. It take any chaotic maps(continuous or discrete) as seed map and improve its chaotic complexity, or chaotic parameter(s) range. So it will employed on Lure type discrete-time systems in [5] such: Lozi map, that will performed in this work .see the reference [8].

1.1 Sine Chaotic Maps Terminologies :

The sine function is a trigonometric function, which is a real valued function, periodic, and produce waves. Also this performance leverage the sine function (wave like) properties to enhance the chaotic behavior for different chaotic maps. In this paper the resulting SCM is bounded map in [-1,1], and it depict the chaotic behavior instead of periodic patterns.

A SCM can be directly formulated as cascade system with form: $x_{n+1} = C(F(x_n))$

Where $C(x) = Sin(x)$ is the sine function, and F is the chaotic map as seed map.

From properties of sine function the largest difference between the outputs is equal to 2 which is the bounds on the outputs interval, since $(1-(-1)) = 2$.

So from trying to increase chaotic complexity it must use an existing chaotic map instead of linear functions. That interpreted a new control parameter(s) to gain large differences between seed values for seed map. Since from the chaos map features is the sensitivity to initial conditions (Sidc) [11], [12]. The sensitivity refer to that small differences pay a large differences. So with new control parameter s the SCM could be formulated as;

$$x_{n+1} = C\left(2^{(s+F(x_n))}\right)$$

where s is chosen within interval $[10, 25]$, if $s < 10$ the result will be in a periodic state

$$x_{n+1} = C\left(2^{(9+F(x_n))}\right) = C\left(2^{(9)}.\,2^{(F(x_n))}\right)$$

if $s < 25$ it will bounds the computational complexity of the resulting system

$$x_{n+1} = C\left(2^{(30+F(x_n))}\right) = C\left(2^{(30)}.\,2^{(F(x_n))}\right)$$

Since whatever the angle is small or large a small difference in $F(x)$ led to wide diverge in output(s) from the accumulation error .

2. Suggest TS Fuzzy model for Chaotic system

There are two types for the process of TS fuzzy modeling of chaotic systems, the continuous-time and discrete-time TS fuzzy model [1]. We will work on the discrete-time type with two dimension. The idea is to use the discrete-time model to get a trajectory for point(s) in phase space of chaotic system for a prime variable. Details of the design and the algorithm for the fuzzy chaotic cryptosystem in explained in the steps below:

Step 1: Suppose a 2-dimensional discrete-time chaotic system $x(t) = [x_1(t),\ x_2(t)]$.

Step 2: Determine which the primitive variables in the system $x_1(t),\ x_2(t)$

*Step 3:*Design the TS fuzzy model through if-then rules on values of primitive variable at the chaotic system such; $x_1(t)$ on the interval $[-d, d]$.

*Step 4:*Calculate the table values for iterations of the TS system implemented on initial values $x_1(0), x_2(0)$ to find the phase trajectory.

Step 5: Perform the sine function alongside a chaotic map (seed map).

Step 6: Formulate the resulted map sine chaotic system on $x(t) = [x_1(t),\ x_2(t)]$.

Step 7: Evaluate the parameters in the system and the values of variables to get the system matrix A_i.

Step 8: Determine the output values that represent the trajectory for $x_1(t)$in the phase trajectory after t=T iteration .

Consider a class of discrete-time nonlinear control system without input term constructed as;

$$x(t+1) = f\big(x(t)\big)$$

where $x(t) \in R^2$, $[x_1(t)\ \ x_2(t)]^T$ is the state vector and $f\big(x(t)\big) \in R^2$is a nonlinear function with appropriate dimension defined on $x(t)$. Where $(t+1)$ is index of time steps in discrete case of the system. The TS fuzzy model here is composed for the rules as a set of rules (fuzzy implications) :

$$\boldsymbol{Plant\ Rule\ i}: IF\ x_1(t)\ is\ \Gamma_1^i\ and\ x_2(t)\ is\ \Gamma_2^i$$
$$THEN\ x(t+1) = A_i x(t) + b_i(t)$$

For $i = 1, 2, \ldots, q$. $x_1(t), x_2(t)$ are the premise variables which consist of the state values in states space for the system, q is the number of rules of this TS fuzzy model, Γ_j^i are fuzzy sets for $x_j(t)$ and $j = 1,2$. $A_i \in R^{2\times 2}$ is a discrete-time control system matrix and $b_i(t) \in R$ denotes the bias terms.

These rules characterize local relation(s) of the chosen system in the state space. Mainly, the essential feature for TS model is expressing the local dynamics of each fuzzy implication through a linear state-space system model. The fuzzy system is then modeled through fuzzy blending of the local linear system models by some appropriate membership functions.

3.1 Generate Secret keys from TS fuzzy chaotic-based model.

The input to the system is $x(t)$ initial values for vector states from the vector variable $x(t) = [x_1(t) \quad x_2(t)]^T$namely $x_1(t)$to be the output that generates a secure ephemeral key $k(t)$ In the other words, first user computes the key $k(t)$ by;

$$k(t) = [k(t-0) \quad k(t-1) \quad \cdots \quad k(t-j+1)$$

$$= [x_1(t-0) \quad x_1(t-1) \quad \cdots \quad x_1(t-j+1)$$

And compute $sin\big(x_1(t)\big) = [sin(x_1(t-0)) \quad sin(x_1(t-1)) \quad \cdots \quad sin(x_1(t-j+1))]$

Then the sequence will be as $\{|s(t-0)|, \quad |s(t-1)|, \quad \cdots, \quad |s(t-j+1)|\}$

Now, a super-increasing sequence S_i, for i =1, 2, ..., l, is generated easily through the following form: $S_1(t) = |sin(x_1(t-0))| + \tau = |s(t)| + \tau$, and

$S_j(t) = \sum_{i=1}^{j-1} S_i(t) + |sin(x_1(t-j+1))| + \tau$, for $j = 2, 3, \ldots, l$ and $\tau > 0$.

Absolute value ensure all the terms in the sequence are positive though the positive real super-increasing sequence $S = \{ S_1, S_2, \ldots, S_l \}$are computed secretly. Also, a first user computes $T = \sum_{j=1}^{l} S_j$ and will chooses $M > T$. Also selects W in $[2, M-1]$ and $gcd(W, M) = 1$. That is, W and M are relatively prime to ensure that W has multiplicative inverse $modM$. First user computes a public hard knapsack $O = \{ o_1, o_2, \ldots, o_l\}$, where $o_i = WS_i(mod\ M)$ for $i = 1,2, \ldots, l$. Finally, (S, W, M) are keep as a private key and O as public key.

3.2 Encryption Stages to Generate the Ciphertext :

Assume that the other user(s) would encrypt plaintext N and send ciphertext to first user, by choose a binary plaintext N breaks up into l sets each of t elements long, namely $N = \{n_1, \ldots, n_l\}$. For each set $n_i \in \{0,1\}$from message, compute;

$$e_i = \sum_{j=1}^{t} n_{ij} o_j$$

Where e_i is a ciphertext that corresponds to the plaintext p_i. So the ciphertext for a plaintext N is $E = \{e_1, \ldots, e_l\}$, Next, the second user will modify the encryption function into $\xi(t)$ as;

$$\xi(t) = \left(\frac{2E - T}{\gamma T}\right)$$

Where γ is a scalar is sufficient small chosen such that $\xi(t) \in (-0.01, 0.01)$

The user will publishes the modified form $\xi(t)$ and γ into public channel, that first user could decrypt the ciphertext .

The proposed cryptosystem is mainly suppose TS fuzzy model employ a chaotic map . Using one state from the vector variable $x(t) = [x_1(t) \ \ x_2(t)]^T$ to be the output that use in generation the secure key $k(t)$ in the system.

3.3 Decryption Stages for the Suggested system :

At this stage the first user will try to recover ciphertext $\xi(t)$, through implementing the steps;

The received values $\xi(t)$and γ ;

Step1:The user know the value T use it in Step2.

Step2:Demodify $\xi(t)$ into the form $E = (\gamma\xi(t) + 1)T/2$

Step3:Compute v the multiplicative inverse for $W\ mod\ M$, as $vW \equiv 1(modM)$

Through the connection between hard and easy knapsacks which could be defined as;

$$vo_i = S_i(modM), \text{ For } i = 1, \ldots, n$$

Step4:Calculate Z_i for each e_i,by applying the form ; $Z_i = ve_i(modN)$

By substituting the values of e_i, the form be;

$$Z_i = v \sum_{j=1}^{n} m_{ij}\, o_j(modN) = \sum_{j=1}^{n} m_{ij}\, vo_j(modN) = \sum_{j=1}^{n} m_{ij}\, S_j(modN)$$

Step5:Testing condition; if that $Z_i < N$ and $N > T$, then the formula for finding plaintext be;

$$Z_i = \sum_{j=1}^{n} m_{ij}\, S_j$$

Step6:Applying the polynomial time algorithm since S is an easy knapsack.

4. A Simulation Example on the Suggested Fuzzy Chaotic Cryptosystem

In this simulation we will consider the chaotic map of Rene Lozi (known as Lozi map) [13] in a chaotic cryptosystem. That is a discrete-time chaotic Lozi system in view of step by step of performing the suggested system. Its chaotic behavior exhibited in a single scroll, since it is well-known two-dimensional map on the interval [0,1], the Lozi map as implicational example for theoretical derivative is like the Hénon map but with absolute formulate the dynamical equations as the equation system bellow [14]:

$$x_1(t+1) = -1.8\,|x_1(t)| + x_2(t) + 3$$

$$x_2(t+1) = 0.25x_1(t)$$

The Lozi Map is with only one variable in the nonlinear term [14], which has the nonlinear term$|x_1(t)|$. Since$|x_1(t)|$ is not well defined at $x_1(t) = 0$, let $\varphi(x_1(t) = |x_1(t)|$ and choose

$x_1(t)$ to be the premise variable. The equivalent fuzzy model can be constructed with System matrices as:

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix}$$

the system matrices evaluated to be equaled, and the non-common bias terms as;

$$b_1 = \begin{bmatrix} 3-1.8d \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

The fuzzy sets for prime variable values are determined through the membership functions as;

$$F_1(x_1(t)) = (|x_1(t)|/d)$$

$$F_2(x_1(t)) = 1 - (|x_1(t)|/d)$$

With $d = 3.5$, so $x_1(t) \in [-3.5, 3.5]$

Then general TS-fuzzy model of discrete time chaotic system at Lozi map with dynamical equations can be written as follows [3;

$$\boldsymbol{Rule\ i}: IF\ x_1(t) is\ \Gamma_i$$

$$THEN\ x(t+1) = A_i x(t) + bi(t)\ , \text{For } i = 1,2,\dots,r$$

where the premise variable $x_1(t)$is a proper state variable, and Γ_1, Γ_2 are triangular fuzzy sets "about -3.5",and "about 3.5", respectively. The model infer the rules $r = 2$;

$$\boldsymbol{Rule\ 1}: IF\ x_1(t) is\ \Gamma_1$$

$$THEN\ x(t+1) = A_1 x(t) + b_1(t)$$

And

$$\boldsymbol{Rule\ 2}: IF\ x_1(t) is\ \Gamma_2$$

$$THEN\ x(t+1) = A_2 x(t) + b_2(t)$$

So

$$\boldsymbol{Rule\ 1}: IF\ x_1(t) is\ "about - 3.5"$$

$$THEN\ x(t+1) = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} x(t) + \begin{bmatrix} -3.3 \\ 0 \end{bmatrix}$$

$$\boldsymbol{Rule\ 2}: IF\ x_1(t) is\ "about\ 3.5"$$

$$THEN\ x(t+1) = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

the model is with tiny difference in values at bias terms. From the observation of bias terms, the systems Lozi map has non-common bias terms, while there are other systems have common bias terms in fuzzy models. Since $\varphi(x) = \sum_{i=1}^{r} \mu_i d_i$, Here $\varphi(x) = 1$

4.1 The Sine Lozi Chaotic SLC map :

The model performed on Lozi map as well-known system to be exact represented by TS fuzzy models on the vector variables $x(t) = [x_1(t)\ \ x_2(t)]^T$, where $x(t) \in R^2$, $x_1(t),\ x_2(t)$ is the state vector.

These features will help in encrypting the message in this work through composing the increasing sequence μ_n with the super increasing sequence S

$$\begin{cases} x_1(t+1) = 3 - a|x_1(t)| + x_2(t) \\ x_2(t+1) = \beta\, x_1(t) \end{cases}$$

Where α, β are the parameters of the system, such that $\alpha \in [1, 1.8]$, $\beta \in [0, 0.4]$, if we take some values for these parameters to view the chaotic behavior the dynamical system be as;

$$\begin{cases} x_1(t+1) = 3 - 1.8|x_1(t)| + x_2(t) \\ \quad x_2(t+1) = 0.25\, x_1(t) \end{cases}$$

The fuzzy sets that defined on values of premise variable $x_1(t)$ are as;

$F_1(x_1) = |x_1|/d$, $F_2(x_1) = 1 - |x_1|/d$

For $d \in R^+$, suppose $d = 3.5$, that is $x_1(t) \in [-d, d] = [-3.5, 3.5]$, and $\sum_{i=1}^{2} F_i = 1$

The system matrices that associated with the equations system are;

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix}$$

Since there is no nonlinear terms in the system of equations, and so the bias terms b on two rules will be as;

$$b_1 = \begin{bmatrix} 3 - 1.8d \\ 0 \end{bmatrix} \; b_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

To control on $x_1(t)$ values. Since the system may be seen as;

$$\begin{cases} x_1(t+1) = 3 - 1.8|x_1(t)| + 0\, x_1(t) + x_2(t) \\ x_2(t+1) = 0 \qquad\qquad\quad + 0.25\, x_1(t) + x_2(t) \end{cases}$$

$x(t),\ b_i \in R^2, A_i \in R^{2\times 2}, for\ i = 1, 2$. In designing the sine Lozi map(CLZ)map the system be;

$$f\big(x_n(t+1)\big) = C\big(2^{(s+F(x_n(t)))}\big)$$

So $$\begin{cases} x_1(t+1) = \sin\big(2^{(s+3-1.8|x_1(t)|+x_2(t))}\big) \\ \quad x_2(t+1) = \sin\big(2^{(s+0.25\, x_1(t))}\big) \end{cases}$$

Where $C(x) = sin(x)$, and $s \in [10, 25]$. So then $x_1(t+1),\ x_2(t+1) \in [-1, 1]$, for initial conditions let the values $x(0) = [x_1(0) \;\; x_2(0)]^T = [1 \quad 0.25]$,

The Experiment : The system performed with initial values for premise variables as; $x_1(0) = 1,\ x_2(0.25)$, and $s = 10$ after 40 iterations, to extract the secure key $k(t)$ from the state trajectory for the state $x_1(t)$. The sequence of sine chaotic values be ;

$$sin\big(x(t)\big) = \{-0.123689894, 0.351946819, -0.528245544, 0.93393977\}$$

With value for $\tau = 7$, then $S_1(t) = |-0.123689894| + 7$, and so the 4-terms super-increasing sequence;

$$S = \{7.123689894, 7.351946819, 7.528245544, 7.93393977\}$$

$$T = \sum_{j=1}^{5} S_j = 29.93782203$$

Take $M = 39 > T$. Also select $W = 28$ in $[2, 38]$ and $gcd(W, M) = 1$. Compute a public hard knapsack; O For each number with $(mod 39)$ be ;

$O = \{4.463317032, 10.85451093, 15.79087523, 27.15031356\}$. So (S, W, M) are a private key and B as public key. To encrypt the binary plaintext $N = \{0111,\ 1001, \ldots, n_l\}$ with four terms. For each set n_i from sender message, compute for instance the set $n_1 = \{0111\}$, and using public key;

$$E = 0 * 4.463317032 + 1 * 10.85451093 + 1 * 15.79087523 + 1 * 27.15031356$$

Then $E = 53.79569972$, Modify the ciphertext to;

$$\xi(t) = \left(\frac{2E - T}{\gamma T}\right)$$

$$\xi(t) = \left(\frac{2(53.79569972) - 29.93782203}{\gamma(29.93782203)}\right)$$

$$= \frac{77.65357741}{\gamma(29.93782203)}$$

$$= \frac{2.593828547}{\gamma}$$

Now, we need randomly take $\gamma \in R$ such that $\xi(t) \in (-0.01, 0.01)$, There are many values but here we will take only one, for instance; 288.2031719, That $\xi(t) = 0.009$.

To decrypt $\xi(t)$, by find the invertible of $W mod\ M$as; $v28 = 1(mod\ 39)$, Then $v = 7$.

The user perform the connection between easy and hard knapsacks through form ;

$vo_i = S_i(modN)$, For $i = 1, \dots ,5$. In the set

$$\{7 * 4.463317032, 7 * 10.85451093, 7 * 15.79087523, 7 * 27.15031356\}$$

$$\{31.24322, 75.98158, 110.5361, 190.0522\}$$

For mod 39 ;

$$\{31.24321922, 36.98157652, 32.53612662, 34.05219492\}$$

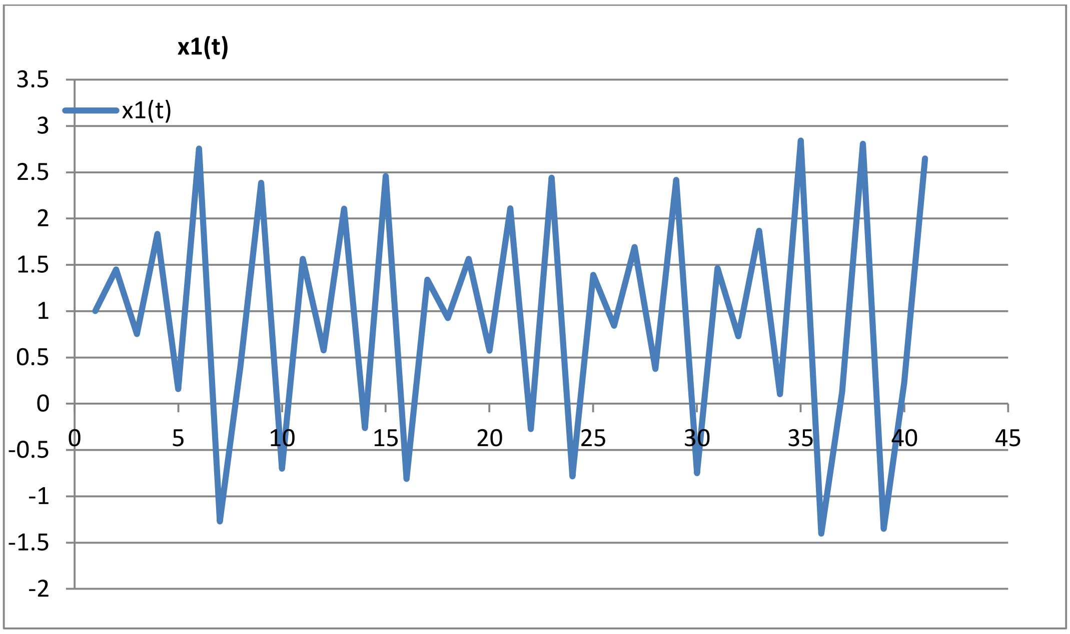
Here, Z_i for each e_i For $i = 1, \dots ,4$, and by substituting the values of e_i, testing condition; if that $\mathrm{Z_i} < 39$ and $\mathrm{T} < 39$. The message is $N = \{0111\}$.

Table(1):Sine function values on TS chaotic fuzzy model values for initial values

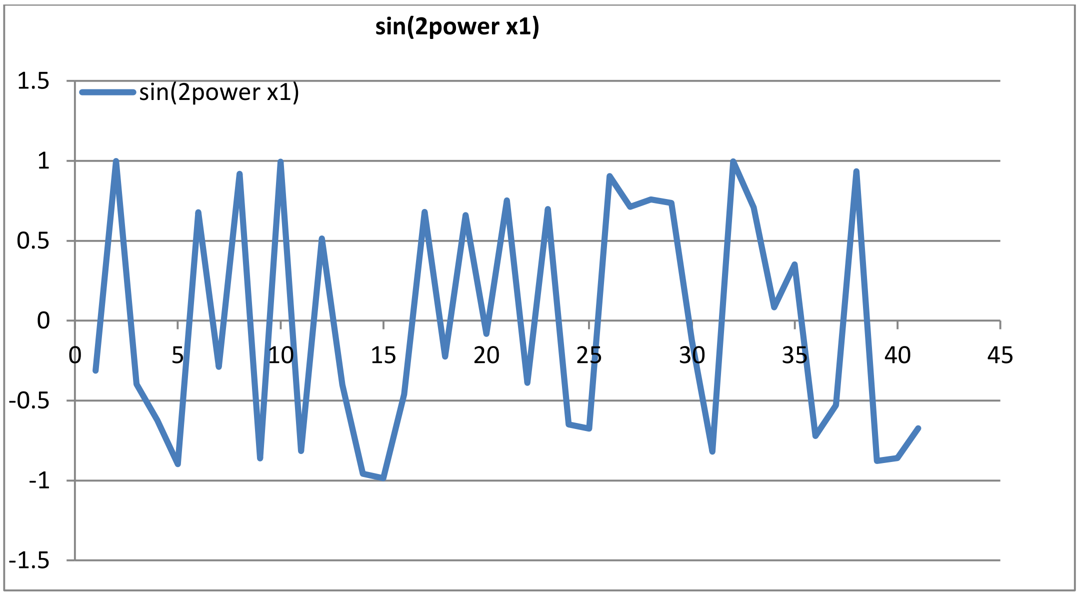
$x_1(0) = 1,\ x_2(0.25),\ and\ s = 10$

time t	x1(t)	x2(t)	sin(2power x1)	abs sin x	sin(2power x2)
0	1	0.25	-0.313057013	0.313057013	-0.92831835
1	1.45	0.3625	0.998066961	0.998066961	-0.179182443
2	0.7525	0.188125	-0.397093223	0.397093223	-0.888460467
3	1.833625	0.45840625	-0.622320818	0.622320818	-0.42037907
4	0.15788125	0.039470313	-0.898656896	0.898656896	0.031437117
5	2.755284063	0.688821016	0.678144995	0.678144995	-0.967890576
6	-1.270690297	-0.317672574	-0.289285932	0.289285932	-0.995633513
7	0.395084891	0.098771223	0.917849586	0.917849586	-0.145308166
8	2.387618418	0.596904605	-0.861776073	0.861776073	0.037952507
9	-0.700808548	-0.175202137	0.994655	0.994655	0.852846436
10	1.563342476	0.390835619	-0.815126054	0.815126054	-0.916887848
11	0.576819163	0.144204791	0.514472534	0.514472534	0.62169329
12	2.105930298	0.526482575	-0.400518843	0.400518843	-0.99998642
13	-0.264191962	-0.066047991	-0.957244315	0.957244315	-0.909413844
14	2.458406478	0.614601619	-0.986620123	0.986620123	-0.225684679
15	-0.810530041	-0.20263251	-0.461563713	0.461563713	-0.680105406
16	1.338413417	0.334603354	0.680110472	0.680110472	-0.100645571
17	0.925459204	0.231364801	-0.223844926	0.223844926	0.895841474
18	1.565538233	0.391384558	0.659943601	0.659943601	-0.994966707
19	0.573415738	0.143353935	-0.082994123	0.082994123	0.003691331
20	2.111205606	0.527801401	0.752602264	0.752602264	-0.214935454
21	-0.272368689	-0.068092172	-0.389376691	0.389376691	0.24075505
22	2.441644188	0.610411047	0.698242097	0.698242097	-0.923978473
23	-0.784548491	-0.196137123	-0.648345379	0.648345379	0.998712809
24	1.391675594	0.347918898	-0.676282801	0.676282801	0.472568703
25	0.842902829	0.210725707	0.903755763	0.903755763	-0.616447945
26	1.693500614	0.423375154	0.712247936	0.712247936	-0.362316663
27	0.375074048	0.093768512	0.759033837	0.759033837	-0.486825173
28	2.418635226	0.604658807	0.735526156	0.735526156	-0.898354744
29	-0.748884601	-0.18722115	-0.123689894	0.123689894	0.770372491
30	1.464786569	0.366196642	-0.820122736	0.820122736	0.404285776

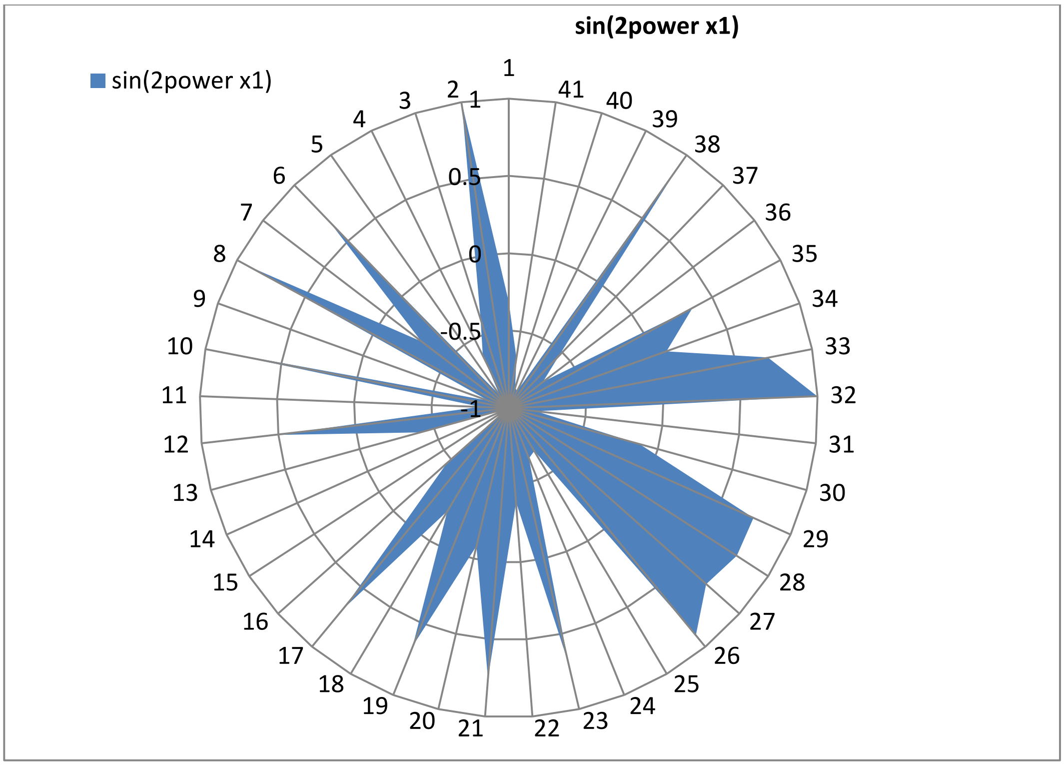
31	0.729580818	0.182395205	0.996973621	0.996973621	-0.378889699
32	1.869149732	0.467287433	0.709360782	0.709360782	0.920939027
33	0.102817916	0.025704479	0.084254622	0.084254622	-0.565190935
34	2.84063223	0.710158057	0.351946819	0.351946819	-0.701896588
35	-1.402979956	-0.350744989	-0.722139753	0.722139753	-0.948468961
36	0.123891089	0.030972772	-0.528245544	0.528245544	-0.071205426
37	2.807968812	0.701992203	0.93393977	0.93393977	0.679799012
38	-1.352351658	-0.338087915	-0.877713171	0.877713171	-0.440067509
39	0.227679101	0.056919775	-0.860323267	0.860323267	-0.206824369
40	2.647097394	0.661774348	-0.672508969	0.672508969	-0.87528235



Figure(1): Values for $x_1(t)$ for iteration 40, as in the table (1) .



Figure(2): Sine Chaotic values for $x_1(t)$ with iteration 40, as in the table (1) . Iteration



Figure(3): Sine Chaotic values for $x_1(t)$ with iteration 40 as area

5. Conclusions:

In this work, a designing of fuzzy model-based for chaotic map and cryptosystem has been proposed. The Knapsack Problem was performed in combination with the T–S fuzzy model for discrete-time chaotic systems, that were exactly derived with only one premise variable. Following the sine map implemented on the chaotic map at Lozi chaotic map, with non-common bias terms and the same premise variable and driving signal. In this fuzzy model-based, a sine chaotic map were applied with numerical data for chaotic behavior, parameters data and premise variable with initial values in interval $[-3.5, 3.5]$. The advantage of this performance design is that all well-known chaotic systems stated achieved the results in more secure, since the trajectory values for premise variable, converted into values for sine map in interval $[-1, 1]$. Numerical simulations with the values table and figures are shown to be consistent with theoretical design. The resulted trajectory values use to generate secret key by choosing super increasing sequence from the cosine values for trajectory from the chaotic map under TS model. After masking and modifying the ciphertext between the users, the ciphertext be ready to decrypted. A good result were gained and the security of the system shown in flexibility in changing the supposed values that will make the system behavior unpredictable. Also it could be extracted through; Employing a discrete chaotic map, and then compose the cosine map on it, and choosing the parameters values for chaotic map with chaotic behavior; Assume initial values randomly from closed interval in state space; Choosing the chaotic map is unpredictable; Finding a super-increasing sequence from the trajectory(orbit) of points, with increase iterations for absolute value for sine map for the seed chaotic map, the primitive variable trajectory has more than one super increasing sequence, we discussed the standard one with the longer sequence with four terms.'

6. References

[1] Z. Li, *Fuzzy chaotic systems*. Springer, 2006.

[2] L. Kocarev and S. Lian, "Chaos-based cryptography, studies in cumputational intelligence 354." Springer-Verlag Berlin Heidelberg, 2011.

[3] G. Zhang *et al.*, "Reinforced concrete deep beam shear strength capacity modelling using an integrative bio-inspired algorithm with an artificial intelligence model," *Eng Comput*, pp. 1–14, 2020.

[4] H. Tao *et al.*, "Artificial intelligence models for suspended river sediment prediction: state-of-the art, modeling framework appraisal, and proposed future research directions," *Eng Appl Comput Fluid Mech*, vol. 15, no. 1, pp. 1585–1612, 2021.

[5] K.-Y. Lian, C.-S. Chiu, T.-S. Chiang, and P. Liu, "LMI-based fuzzy chaotic synchronization and communications," *IEEE Trans Fuzzy Syst*, vol. 9, no. 4, pp. 539–553, 2001.

[6] K.-Y. Lian, T.-S. Chiang, C.-S. Chiu, and P. Liu, "Synthesis of fuzzy model-based designs to synchronization and secure communications for chaotic systems," *IEEE Trans Syst Man, Cybern Part B*, vol. 31, no. 1, pp. 66–83, 2001.

[7] M. Alawida, A. Samsudin, and J. Sen Teh, "Digital cosine chaotic map for cryptographic applications," *IEEE Access*, vol. 7, pp. 150609–150622, 2019.

[8] Z. Li, W. A. Halang, and G. Chen, *Integration of fuzzy logic and chaos theory*, vol. 187. Springer Science & Business Media, 2006.

[9] D. M. Burton, *Elementary number theory*. Tata McGraw-Hill Education, 2006.

[10] Z. Hua, Y. Zhou, and H. Huang, "Cosine-transform-based chaotic system for image encryption," *Inf Sci (Ny)*, vol. 480, pp. 403–419, 2019.

[11] M. Brin and G. Stuck, *Introduction to dynamical systems*. Cambridge university press, 2002.

[12] R. L. Devaney, *An introduction to chaotic dynamical systems*. CRC press, 2018.

[13] L. Merah, A. Ali-Pacha, N. Hadj-Said, B. Mecheri, and M. Dellassi, "FPGA hardware co-simulation of new chaos-based stream cipher based on Lozi map," *Int J Eng Technol*, vol. 9, no. 5, pp. 420–425, 2017.

[14] M. Seidi, M. Hajiaghamemar, and B. Segee, "Fuzzy control systems: Lmi-based design," *Fuzzy Control Adv theory Appl*, vol. 18, pp. 441–464, 2012.