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SOLVING SECOND ORDER DIFFERENTIAL EQUATION AND COMPARISON BETWEEN LAPLACE, ELZAKI AND MAHGOUB TRANSFORMS

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ABSTRACT:

In this paper, we solved second order differential equation using Laplace and inverse Laplace transforms. Then the end result is compared with Laplace, Elzaki and Mahgoub. Hence the relationship between these transforms are closely connected.

1. INTRODUCTION:

Laplace transforms is one of the most important tools used for solving differential equations to an algebraic problem. It is a particular integral formula used to solve linear differential equations with certain conditions. Recently, in 2016, Mahgoub introduced a useful technique for solving ordinary and partial differential equations in the time domain. Hassan Eltayeb, introduced some relationship between Sumudu and Laplace transforms, further for the comparison purpose and applied both transforms to solve differential equations to see the differences and similarities [4]. Elzaki transform method is a powerful device for constructing analytic approximate solution of scientific problems. It was initially introduced by Elzaki in 2011 as a modification of the classical sumudu transform. The new method Elzaki transform was presented by P.R, Bhadane, that to solve system of homogenous and non-homogenous linear differential equations of first order satisfying certain conditions.

In 2016, Abdelbagy A. Alshikh Mohand M. Abdelrahim Mahgoub discussed some relationship between Laplace transform and the new two transform called Elzaki transform and Aboodh transform and solved first and second order ordinary differential equations using both transforms, and show that Elzaki transfom and Aboodh transform are closely connected with the Laplace transform [2]. In 2017, Raisinghania, M.D., Introduced integral equations and boundary value problems.

In this paper, solution of differential equation of second order is solved by both Laplace transforms, Elzaki transform and Mahgoub transform are discussed and we showed that these three transforms are closely connected and are efficient to find a solution of differential equation of second order satisfying initial conditions.

2. PRELIMINARIES

2.1 The Laplace Transform

If f(t) is a function defined for all positive values of t, then the Laplace Transform is defined as;

$$L[f(t)]= F(S) = \int_0^\infty e^{-st}\, f(t)dt$$

Provided that the integral exists. Here the parameter s is a real or complex number. The corresponding inverse Laplace transform is that the integral exists. Here the parameter s is a real or complex number. The corresponding inverse Laplace transform is L^{-1}[F(s)] = f(t). Here f(t) and F(s) are called as pair of Laplace transforms.

2.1.1 Laplace transform of some functions:

(1) $L\left(t^n\right) = \frac{n!}{s^{n+1}}$ = F(s)

Inversion formula: $L^{-1}\left(\frac{1}{s^{n+1}}\right)=\frac{t^n}{n!}$=f(t)

(2) $L(e^{at}) = \frac{1}{s-a}$=F(s)

Inversion formula: $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$=f(t)

(3) L(sin(at))= $\frac{a}{s^2+a^2}$=F(s)

Inversion formula: $L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin(at)}{a}$ =f(t)

(4) L(cos(at))= $\frac{s}{s^2+a^2}$=F(s)

Inversion formula: $L^{-1}\left(\frac{s}{s^2+a^2}\right)$=cos(at)=f(t)

2.1.2 Laplace Transform of derivatives:

(1) L [f '(t)] = s F(s) – f (0)

(2) L [f "(t)] = s^2 F(s) - s f(0) – f '(0)

2.2 The Elzaki Transform

Over the set of functions, {f(t) / ∃M, τ_1, τ_2>0,|f(t)| < M $e^{\frac{|t|}{\tau_j}}$, if t ϵ $(-1)^j \times [0, \infty)$},

The Elzaki transform is defined by

E[f(t)] = T(u) = u $\int_0^{\infty} e^{\frac{t}{u}} f(t)$ dt , t ≥ 0, $k_1 \leq u \leq k_2$, 0 ≤ t ≤ ∞.

2.2.1 Elzaki Transform of some functions:

(1) E(t^n) = n! u^{n+2} = T(u)

Inversion formula: $E^{-1}(u^{n+2}) = \frac{t^n}{n!}$ = f(t)

(2) $E(e^{at}) = \frac{u^2}{1-au}$ = T(u)

Inversion formula: $E^{-1}\left(\frac{u^2}{1-au}\right) = e^{at}$=f(t)

(3) E(sin(at))= $\frac{au^3}{1+a^2u^2}$= T(u)

Inversion formula: $E^{-1}\left(\frac{u^3}{1+a^2u^2}\right) = \frac{\sin(at)}{a}$ =f(t)

(4) E(cos(at))= $\frac{u^2}{1+a^2u^2}$ = T(u)

Inversion formula: $E^{-1}\left(\frac{u^2}{1+a^2u^2}\right)$=cos(at)=f(t)

2.2.2 Elzaki Transform of derivatives:

(1) E[f '(t)] = $\frac{T(u)}{u}$ – u f(0)

(2) E[f "(t)] = $\frac{T(u)}{u^2}$ – f(0) – u f '(0)

2.3 Mahgoub Transform:

A new transform is called the Mahgoub transform defined for function of exponential order we consider functions in the set A is defined by,

A={f(t) / ∃M, τ_1, τ_2>0,|f(t)| < M $e^{\frac{|t|}{\tau_j}}$, if t ϵ $(-1)^j \times [0, \infty)$}, For given function in the set A, the constant M must be finite number $k_1 k_2$ may be finite or infinite. The Mahgoub transform denoted by the operator M(.) defined by the integral equations M[f(t)] = H,

$$(V) = v\int_0^{\infty} e^{-vt} f(t)\,dt,\ t \geq 0,\ k_1 \leq v \leq k_2.$$

2.3.1 Mahgoub Transform of some functions:

(1) $M\left(\frac{t^n}{n!}\right) = \frac{1}{v^n}$= H(v)

Inversion formula: $M^{-1}\left(\frac{n!}{v^n}\right)$=$t^n$=f(t)

(2) M(e^{at})= $\frac{v}{v-a}$ = H(v)

Inversion formula: $M^{-1}\left(\frac{v}{v-a}\right)$=$e^{at}$=f(t)

(3) M(sin(at))= $\frac{av}{v^2+a^2}$= H(v)

Inversion formula: $M^{-1}\left(\frac{v}{v^2+a^2}\right)$= $\frac{\sin(at)}{a}$ =f(t)

(4) M(cos(at))= $\frac{v^2}{v^2+a^2}$ = H(v)

Inversion formula: $M^{-1}\left(\frac{v^2}{v^2+a^2}\right)$= cos(at) = f(t)

2.3.2 Mahgoub Transform of derivatives:

(1) M [f '(t)] = v H(v) – v f (0)

(2) L [f "(t)] = v^2 H(v) - v f '(0) – v^2 f (0)

3.APPLICATION

In this section, the effectiveness of Laplace, Elzaki and Mahgoub transform technique are demonstrated by finding exact solution of homogenous and non- homogenous differential equations of second order with constant coefficients and satisfying some initial conditions.

EXAMPLE: 1

Find the homogeneous second order differential equation, x"-6x'+9x=0 with initial conditions x (0) = 2, x' (0) = -4.

Solution:

(1) LAPLACE TRANSFORM:

Given, x"-6x'+9x=0

Applying Laplace transform,

L[x"(t)]-6L[x'(t)] +9L[x(t)] =0

Since L[x(t)] = F(s)

Using Laplace transform of derivatives,

s^2 F(s)- s x (0) - x' (0)- 6[s F(s) – x (0)] +9 F(s)=0

s^2 F(s)- s (2) – (-4)- 6[s F(s) – 2] +9 F(s)=0

s^2 F(s)- 2s +4 – 6 s F(s) +1 2 +9 F(s)=0

$F(s)$ [s^2-6s+9]-2s+16=0

F(s)= $\frac{2s-16}{[s^2-6s+9]}$ =$\frac{2(s-3)-10}{(s-3)^2}$=$\frac{2}{(s-3)}$ - $\frac{10}{(s-3)^2}$

Applying Inverse Laplace transform of both sides,

L^{-1}[F(s)] = 2 $L^{-1}\left[\frac{1}{(s-3)}\right]$ – 10 $L^{-1}\left[\frac{1}{(s-3)^2}\right]$

$f(t) = 2e^{3t} - 10te^{3t} = (2-10t)e^{3t}$

(2) ELZAKI TRANSFORM:

Given, x''-6x'+9x=0

Applying Elzaki transform,

E[x''(t)]-6E[x'(t)] +9E[x(t)] =0

Since E[x(t)] = T(u)

Using Elzaki transform of derivatives,

$\frac{T(u)}{u^2} - x(o) - u\,x'(0) - 6\left[\frac{T(u)}{u} - u\,x(0)\right] + 9\,T(u) = 0$

$\frac{T(u)}{u^2} - 2 - u\,(-4) - 6\,\frac{T(u)}{u} + 6u\,(2) + 9\,T(u) = 0$

$\frac{T(u)}{u^2} - 2 + 4u - 6\,\frac{T(u)}{u} + 12u + 9\,T(u) = 0$

$T(u)\left[\frac{1}{u^2} - \frac{6}{u} + 9\right] = 2-16u$

$T(u) = \frac{[2-16u]u^2}{[1-6u-9u^2]} = \frac{[2u^2-16u^3]}{(1-3u)^2} = \frac{2u^2(1-3u)-10u^3}{(1-3u)^2} = \frac{2u^2}{(1-3u)} - \frac{10u^3}{(1-3u)^2}$

Applying Inverse Elzaki transform of both sides,

$E^{-1}[T(u)] = 2\,E^{-1}\left[\frac{u^2}{(1-3u)}\right] - 10\,E^{-1}\left[\frac{u^3}{(1-3u)^2}\right]$

$f(t) = 2e^{3t} - 10te^{3t} = (2-10t)e^{3t}$

(3) MAHGOUB TRANSFORM:

Given, x''-6x'+9x=0

Applying Mahgoub transform,

M[x''(t)]-6M[x'(t)] +9M[x(t)] =0

Since $M[x(t)] = H(v)$

Using Mahgoub transform of derivatives,

$v^2 H(v) - v\,x'(0) - v^2 x(0) - 6[v\,H(v) - v\,x(0)] + 9\,H(v) = 0$

$v^2 H(v) - v\,(-4) - v^2(2) - 6v\,H(v) + 6v(2) + 9\,H(v) = 0$

$v^2 H(v) + 4v - 2v^2 - 6v\,H(v) + 12v + 9\,H(v) = 0$

$H(v)[v^2 - 6v + 9] = 2v^2 - 16v$

$H(v) = \frac{[2v^2-16v]}{[v^2-6v+9]} = \frac{[(2v-16)v]}{(v-3)^2} = \frac{[2v(v-3)-10v]}{(v-3)^2} = \frac{2v}{(v-3)} - \frac{10v}{(v-3)^2}$

Applying Inverse Mahgoub transform of both sides,

$M^{-1}[H(v)] = 2\,M^{-1}\left[\frac{v}{(v-3)}\right] - 10\,M^{-1}\left[\frac{v}{(v-3)^2}\right]$

$f(t) = 2e^{3t} - 10te^{3t} = (2-10t)e^{3t}$

EXAMPLE: 2

Find the non-homogeneous second order differential equation, x''+4x=2cos2t with initial conditions x (0) =0, x' (0) =4.

Solution:

(1) LAPLACE TRANSFORM:

Given, x''+4x=2cos2t

Applying Laplace transform,

L[x''(t)] + 4L[x(t)] = 2L[cos2t]

Since, L[x(t)] = F(s)

Using Laplace transform of derivatives,

s^2 F(s)- s x (0) - x' (0)+4 F(s)=$2\left(\frac{s}{s^2+4}\right)$

s^2 F(s)- s (0) - 4+4 F(s)=$2\left(\frac{s}{s^2+4}\right)$

F(s)[s^2+4]= $2\left(\frac{s}{s^2+4}\right)$+4

F(s)[s^2+4]=$\frac{2s+4(s^2+4)}{s^2+4}$

F(s) = $\frac{2s+4(s^2+4)}{(s^2+4)^2} = \frac{2s}{(s^2+4)^2} + \frac{4}{s^2+4}$

Applying Inverse Laplace transform of both sides,

L^{-1}[F(s)] = $L^{-1}\left[\frac{2s}{(s^2+4)^2}\right]+2L^{-1}\left[\frac{2}{s^2+4}\right]$

f(t)=$\frac{1}{2}$ t sin2t + 2 sint2t

(2) ELZAKI TRANSFORM:

Given, x''+4x=2cos2t

Applying Elzaki transform,

E[x''(t)] + 4E[x(t)] = 2E[cos2t]

Since, E[x(t)] = T(u)

Using Elzaki transform of derivatives,

$\frac{T(u)}{u^2}$ – x(o) – u x'(0) + 4 T(u) = $2\left(\frac{u^2}{1+4u^2}\right)$

$\frac{T(u)}{u^2}$ – 0 – u (4) + 4 T(u) = $2\left(\frac{u^2}{1+4u^2}\right)$

$\frac{T(u)}{u^2}$ – 4 u + 4 T(u) = $2\left(\frac{u^2}{1+4u^2}\right)$

T(u)$\left[\frac{1}{u^2}+4\right] = 2\left(\frac{u^2}{1+4u^2}\right)$+4u =$\left[\frac{2u^2+4u(1+4u^2)}{1+4u^2}\right]$

T(u) = $\frac{(2u^2+4u(1+4u^2))u^2}{(1+4u^2)^2} = \frac{2u^4}{(1+4u^2)^2}+\frac{4u^3}{(1+4u^2)}$

Applying Inverse Elzaki transform of both sides,

E^{-1}[T(u)] = $E^{-1}\left[\frac{2u^4}{(1+4u^2)^2}\right]+2E^{-1}\left[\frac{2u^3}{(1+4u^2)}\right]$

f(t) = $\frac{1}{2}$ t sin2t + 2 sint2t

(3) MAHGOUB TRANSFORM:

Given, x''+4x=2cos2t

Applying Mahgoub transform,

M[x''(t)] + 4M[x(t)] = 2M[cos2t]

Since, M[x(t)] = H(v)

Using Mahgoub transform of derivatives,

v^2 H(v)- v x$'$(0) - v^2x (0)+4 H(v) = $2\left(\frac{v^2}{v^2+4}\right)$

v^2 H(v)- v (4) - v^2(0)+4 H(v) = $2\left(\frac{v^2}{v^2+4}\right)$

H(v)(v^2 +4) = $2\left(\frac{v^2}{v^2+4}\right)$+4$v$ = $\frac{2v^2+4v(v^2+4)}{v^2+4}$

H(v) = $\frac{2v^2+4v(v^2+4)}{(v^2+4)^2} = \frac{2v^2}{(v^2+4)^2} + \frac{4v}{(v^2+4)}$

Applying Inverse Mahgoub transform of both sides,

M^{-1}[H(v)] = $M^{-1}\left[\frac{2v^2}{(v^2+4)^2}\right]+2M^{-1}\left[\frac{2v}{(v^2+4)}\right]$

f(t) = $\frac{1}{2}$ t sin2t + 2 sint2t

CONCLUSION:

The transforms like Elzaki and Mahgoub transforms are efficient tool for solving second order differential equations. Hence, we have solved Laplace, Elzaki and Mahgoub transforms and compared with each other. The comparison gives same and better results and thus they are closely connected with Laplace transforms.

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