SOLVING NONLINEAR VOLTERRA-FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS USING MODIFIED LAPLACE DECOMPOSITION METHOD

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Abstract:

In this paper, a brand new change of the Laplace Discrete Adomian decomposition method (LDADM) is called Modified Laplace Discrete Adomian decomposition method (MLDADM) is carried out to non-homogeneous non-linear Volterra-Fredholm integro-differential equations. This approach is primarily based totally upon the Laplace Discrete Adomian decomposition method. The overall performance of the proposed approach is established via absolute blunders measures among the exact solutions. An illustrative example is given to explain the applicability of the proposed method.

Keywords: Volterra-Fredholm integro-differential equations, Adomian decomposition method, Laplace Discrete decomposition method, Laplace transform.

1.Introduction:

An integro-differential equation is an equation that involves both integrals and derivatives of unknown function. The solution of integro-differential equations have a major role in the fields of science and engineering. In the case of non-linear equations in science and technology includes chemistry, biology, physics, vibration, acoustic signals, signal processing, fluid dynamics and viscoelasticity [6,7].

Laplace Adomian decomposition method (LADM) is one of the effective method to solve the non-linear Volterra-Fredholm integro-differential equations. LADM possess a combined form of Laplace Transformation and Adomian Decomposition method. On further modifications of LADM for finding the solutions by firstly discretizing the ADM followed by the quadrature rules [4]. Then the discretizing method is Laplace Discrete Adomian Decomposition method (LDADM).

In this paper, we aim at extending the Laplace Discrete Adomian Decomposition method (LDADM) into Modified Laplace Discrete Adomian Decomposition method (MLDADM). There are numerous techniques for solving integro-differential equations. Examples are ADM [3,4,5], MADM [2,3], Galerkin method, LDADM[1,8], and MLADM [6,7] etc.

ADM is an analytical method that gives the solution in the form of Adomian polynomials and it applied to both linear and non-linear equations.

In this, we use LDADM and MLDADM for solving the non-linear Volterra-Fredholm integro-differential equations of the second kind of the form,

$$y''(x) = f(x) + \int_a^x \kappa 1(x,t)\left(\chi 1\big(y(t)\big) + \phi 1\big(y(t)\big)\right) dt + \int_a^b \kappa 2(x,t)\left(\chi 2\big(y(t)\big) + \phi 2\big(y(t)\big)\right) dt \quad (1)$$

with the initial conditions $y(0) = \alpha$, $y'(0) = \beta$, $a \le x \le b$. (2)

Section 2.1 describes the LDADM. Section 2.2 describes the proposed MLDADM. Section 3 presents the numerical results with two examples and finally Section 4 concludes

2. Description of the method

In this section, we will consider a non-linear Volterra-Fredholm integrodifferential equations. In this equation, we use effective method such as LDADM and MLDADM. We will describe this approach in this sections.

2.1. Laplace Discrete Adomian Decomposition Method (LDADM)

To solve the nonlinear Volterra-Fredholm integro-differential eqns.(1)-(2) we use the Laplace transform method, we recall the Laplace transform of the second derivative of y(x),that is

$L\{y''(x)\}=s^2L\{y(x)\}-sy(0)-y'(0)$ (3)

Appplying the Laplace transform to both sides of eq.(1) we get

$$L\{y''(x)\}=L\{f(x)+\int_a^x \kappa1(x,t)\left(\chi1(y(t))+\phi1(y(t))\right)dt+\int_a^b \kappa2(x,t)\left(\chi2(y(t))+\phi2(y(t))\right)dt\} \quad (4)$$

From eq(3)

$$s^2\,L\{y(x)\}-sy(0)-y'(0)=L\{f(x)\}+L\{\int_a^x \kappa1(x,t)\left(\chi1(y(t))+\phi1(y(t))\right)dt+\int_a^b \kappa2(x,t)\left(\chi2(y(t))+\phi2(y(t))\right)dt\} (5)$$

By using eq(2)

$$s^2L\{y(x)\}-s\alpha-\beta = L\{f(x)\}+L\{\int_a^x \kappa1(x,t)\left(\chi1(y(t))+\phi1(y(t))\right)dt + \int_a^b \kappa2(x,t)\left(\chi2(y(t))+\phi2(y(t))\right)dt\}$$

$$L\{y(x)\}=\frac{\alpha}{s}+\frac{\beta}{s^2}+\frac{1}{s^2}L\{f(x)\}+\frac{1}{s^2}L\{\int_a^x \kappa1(x,t)\left(\chi1(y(t))+\phi1(y(t))\right)dt+\int_a^b \kappa2(x,t)\left(\chi2(y(t))+\phi2(y(t))\right)dt\} (6)$$

The decomposition method defines the solution y(x) known by the series of the form

$$y(x)=\sum_{n=0}^{\infty} y_n(x) \quad (7)$$

and the non-linear operator ϕ1(y(t)), ϕ2(y(t))are decomposed into the infinite series as

$\phi1(y(t))=\sum_{n=0}^{\infty} A_n$,and $\phi2(y(t))=\sum_{n=0}^{\infty} B_n$ (8)

where An and Bn are the Adomian polynomials that is given by

$A_n=\frac{1}{n!}\frac{d^n}{d\gamma^n}[\phi1(\sum_{i=0}^{n}\gamma^i y_i)]_\gamma=0$ and $B_n=\frac{1}{n!}\frac{d^n}{d\gamma^n}[\phi2(\sum_{i=0}^{n}\gamma^i y_i)]_\gamma=0$ (9)

Substituting the equations (7), (8)and (9) into equation (6) and yields the following iterative algorithm:

$L\{y_0(x)\}=\frac{\alpha}{s}+\frac{\beta}{s^2}+\frac{1}{s^2}L\{f(x)\}$ (10)

and$L\{y_{k+1}(x)\}=\frac{1}{s^2}L\{\int_a^x \kappa1(x,t)(\chi1(y_k(t))+A_k)dt+\int_a^b \kappa2(x,t)(\chi2(y_k(t))+B_k)dt\}$ (11)

Applying the inverse Laplace transform equations (10) and (11) y_0(x) is given and therefore A_0 can be defined. The usage of A_0allows to assess y_1(x). Alsoy_0(x) and$y_1(x)$ leads toA_1so us to permit us to decidey_1(x) and so on. Then the recursive relation is given by

$$y_0(x)=L^{-1}\left\{\frac{\alpha}{s}+\frac{\beta}{s^2}+\frac{1}{s^2}L\{f(x)\}\right\}=h(x),$$

$$y_{k+1}(x)=L^{-1}\{\frac{1}{s^2}L\{\int_a^x \kappa1(x,t)(\chi1(y_k(t))+A_k)dt+\int_a^b \kappa2(x,t)(\chi2(y_k(t))+B_k)dt\}\}$$

2.2. Modified Laplace Discrete decomposition method.

This method is based on the assumption of the function h(x) can be divided into two parts namely h1(x) and h2(x). Under this assumption we set

$$h(x) = h1(x) + h2(x), \quad (12)$$

As a result of Modified Laplace discrete decomposition method introduces the recursive relation we obtain

$$y_0(x)=L^{-1}\left\{\frac{\alpha}{s}+\frac{\beta}{s^2}+\frac{1}{s^2}L\{f(x)\}\right\}=h_1(x)+h_2(x), \quad (13)$$

Therefore,

$$y_0(x)=h_1(x)$$

$$y_{k+1}(x) = h_2(x) + L^{-1}\{\frac{1}{s^2}L\{\int_a^x \kappa 1(x,t)\big(\chi 1\big(y_k(t)\big) + A_k\big)dt + \int_a^b \kappa 2(x,t)\big(\chi 2\big(y_k(t)\big) + B_k\big)dt\}\}.$$

3. Application and Numerical results:

In this section, we implemented the numerical solution of Laplace Discrete Adomian decomposition method (LDADM) and Modified Laplace Discrete Adomian decomposition method (MLDADM) for solving nonlinear Volterra-Fredholm integro-differential equations.

Example 3.1

Consider the nonlinear integro-differential equations:

$$y''(x)=\frac{1}{2}e^x + \frac{1}{2}\int_0^x e^{(x-2t)}\, y^2(t)dt, \qquad (14)$$

with initial conditions y(0)=y'(0)=1. (15)

Solution: Taking the laplace transform of both sides of eq.(14)

$$L\{y''(x)\} = L\{\frac{1}{2}e^x\} + L\{\frac{1}{2}\int_0^x e^{(x-2t)}\, y^2(t)dt\}, \qquad (16)$$

From eq(3)

$$s^2 L\{y(x)\} - sy(0) - y'(0)=\frac{1}{2}L\{e^x\} + \frac{1}{2}L\{\int_0^x e^{(x-2t)}\, y^2(t)dt\},$$

By eq(15)

$$L\{y(x)\} = \frac{1}{s} + \frac{1}{s^2}+\frac{1}{2s^2(s-1)} + \frac{1}{2s^2}L\{\int_0^x e^{(x-2t)}\, y^2(t)dt\},$$

The recursive relation is given by

LDADM:

$$y_0(x)=L^{-1}\{\frac{1}{s} + \frac{1}{s^2}+\frac{1}{2s^2(s-1)}\}$$

$$y_0(x)=\frac{1+x+e^x}{2}$$

$$y_1(x) = \frac{3e^{-x}}{2}+\frac{dirac(x)}{2} + \frac{29e^x}{32} + \frac{xe^{-x}}{16} - \frac{x^2e^{-x}}{32} + \frac{xe^x}{8}, \text{ for k=0.}$$

$$y_{k+1}(x) = L^{-1}\{\frac{1}{2s^2}L\{\int_0^x e^{(x-2t)}\, A_k dt\}\}, k>1.$$

MLDADM:

$$y_0(x)=L^{-1}\left\{\frac{1}{s} + \frac{1}{s^2}\right\}$$

$$y_0(x)=1+x$$

$$y_1(x) = \frac{3e^{-x}}{8}- \frac{x}{2} + \frac{x}{2} + \frac{9e^x}{8} + \frac{xe^{-x}}{4} - \frac{x^2e^{-x}}{4}\frac{1}{2}, \text{ for k=0.}$$

$$y_{k+1}(x) = L^{-1}\{\frac{1}{2s^2(s-1)}\frac{1}{2s^2}L\{\int_0^x e^{(x-2t)}\, A_k dt\}\}, \ k>1.$$

Example 3.2

Consider the nonlinear integro-differential equations:

$$y''(x) = \frac{x^2}{2} - \int_0^x (x-t)y^2(t)dt, (17)$$

with the initial conditions y(0) = 1, y′ (0) = 0. (18)

Solution:Taking the laplace transform of both sides of eq(17)

$$L\{y''(x)\} = L\left\{\frac{x^2}{2}\right\} + L\{\int_0^x (x-t)y^2(t)dt\},$$

From eq(3)

$$s^2 L\{y(x)\} - sy(0) - y'(0) = \frac{1}{2}L\{x^2\} + L\{\int_0^x (x-t)y^2(t)dt\},$$

By eq(18)

L{y(x)}=$\frac{1}{s}+\frac{1}{s^5}-\frac{1}{s^2}L\{\int_0^x (x-t)y^2(t)dt\}$,

The recursive relation is given by

LDADM:

$$y_0(x) = L^{-1}\{\frac{1}{s}+\frac{1}{s^5}\}$$

$$y_0(x) = 1+\frac{x^4}{24}$$

$$y_1(x) = -\frac{x^{12}}{6842880}-\frac{x^8}{20160}-\frac{x^4}{24}, for\ k = 0.$$

$y_{k+1}(x) = L^{-1}\{\frac{-1}{s^2}L\{\int_0^x (x-t)A_k dt\}\}$,k>1.

MLDADM:

$$y_0(x) = L^{-1}\left\{\frac{1}{s}\right\}\ then\ \ y_0(x) = 1.$$

$$y_1(x) = 0, for\ k = 0.$$

$y_{k+1}(x) = L^{-1}\{\frac{1}{s^5}-\frac{1}{s^2}L\{\int_0^x (x-t)A_k dt\}\}$, k>1.

4. Conclusion:

In this paper, the modified Laplace Discrete decomposition method (MLDADM) and Laplace discrete Adomian decomposition method(LDADM) is applied successfully to obtain the solution of non-linear Volterra-Fredholm integro-differential equations. The answer acquired via way of means of those approach are derived via way of means of endless approximate series. The illustrative examples are provided to make clear overall performance and accuracy of the methods. Then the proposed method is simple to execute.

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