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A Study on Graphs with Power Dominator Chromatic Number 3

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Abstract - Let G = (V, E) be a finite, undirected and connected graph without loops and multiple edges. Power dominator coloring of a graphs is highly useful in all the power related circuits. The objective of power dominator coloring is to reduce the number of colors required to color the graph with certain restrictions. This will enable us to understand the reliability of all nodes present in the graphs. We find the power dominator coloring for fan graph, double fan graph, planter graph, lilly graph, octopus graph, drum graph, venessa graph, flower pot graph and umbrella graph. All results are diagrammatically illustrated.

Keywords — Coloring, Power dominator coloring, Fan graph, Octopus graph

AMS Mathematics Subject Classification (2010): 05C15, 05C69.

INTRODUCTION

In graph theory, the prime research areas of coloring and domination has wide range of applications in the real life. Teresa W. et. al. published a book in 1998, on domination which lists 1222 papers in this area [1].

When Haynes et al [2] were trying to place Minimum number of PMU to observe current flow in the circuit, power domination was introduced.

Recent variation on coloring named dominator coloring is the main focus for many research, which was introduced by Gera [3]. In 2016, on linking the power domination and dominator coloring, K. Sathish Kumar et al, introduced power dominator coloring [4].

A. Uma Maheswari and Bala Samuvel J, found power dominator chromatic number for special graphs [5]. Also, power dominator chromatic number for various special graphs were found [6]–[8].

Study on properties of fan related graphs was published by Edward Samuel A. and Kalaivani S in [9]–[15]. Chidambaram found properties of fan graphs in [16].

Here, in this paper, we study the power dominator coloring for fan graph, double fan graph, planter graph, lilly graph, octopus graph, drum graph, venessa graph, flower pot graph and umbrella graph. All results are diagrammatically illustrated.

PRELIMINARIES

The definitions required for this paper are recalled below [17].

Definition 1: Dominator Coloring [3]

A dominator coloring [[18]–[20]] of a graph is a proper coloring such that each vertex dominates every vertex of color class. The chromatic number $\chi_d(G)$ of a graph is the minimum number of colors needed for a dominator coloring of G.

Definition 2: Power Dominator Coloring [4]

The power dominator coloring [[4],[5]] of G is a proper coloring of G, such that every single vertex of G power dominates all vertices of some color class. The minimum number of color classes in a power dominator coloring of the graph, is the power dominator chromatic number. It is denoted by $\chi_{pd}(G)$.

Definition 3: Double Fan Graph[21]

The double fan graph DF_n is defined as the graph join $\overline{K_2} + P_n$, where $\overline{K_2}$ is the empty graph on 2 nodes and P_n is the path on n nodes.

Definition 4: Planter Graph[12]

The planter graph R_n, $(n \geq 3)$ can be constructed by joining a fan graph Fn, $(n \geq 2)$ and cycle graph C_n, $(n \geq 3)$ with sharing a common vertex, where n is any positive integer. i.e., $R_n = F_n + C_n$.

Definition 5: Lilly graph [10]

The lilly graph I_n, n ≥ 2 is constructed by 2 stars $2K_{1,n}$, n ≥ 2 joining 2 path graphs $2P_n$, n ≥ 2 with sharing of a common vertex. i.e; $I_n = 2K_{1,n} + 2P_n$. □

Definition 6: Octopus graph [9]

An octopus graph O_n , $(n \geq 2)$ can be constructed by a fan graph F_n, $(n \geq 2)$ joining a star graph $K_{1,n}$ with sharing a common vertex, where n is any positive integer. i.e., $O_n = F_n + K_{1,n}$.

Definition 7: Drums graph[13]

The drums graph D_n, $n \geq 3$ can be constructed by two cycle graphs $2C_n$, n ≥ 3 joining two path graphs P_n, $n \geq 2$ with sharing a common vertex. i.e., D_n = $2C_n$ + $2P_n$.

Definition 8: Venessa graph[11]

The venessa graph V_n, $n \geq 3$ can be constructed by two fan graphs $2F_n$, n ≥ 2 of same order, sharing same common vertex v_0, with n number of Pendent vertices K_n,. i.e., V_n = $2F_n$ + K_n.

Definition 9: Udukkai graph[14]

An udukkai graph U_n, n ≥3 is a graph constructed by joining two fan graphs $F_n n \geq 2$ with two paths P_n, $n \geq 2$ by sharing a common vertex at the center.

Definition 10: Flower pot graph [22]

A flower pot graph FP_n is the graph attained by linking a star graph $K_{1,n}$ with the central vertex of a cycle graph C_n.

Definition 11: Umbrella graph[23]

An umbrella graph $U(m,n)$ is the graph attained by linking a path P_n with the central vertex of a fan f_m.

Here, in this paper, we study the power dominator coloring for the fan graph, double fan graph, planter graph, lilly graph, octopus graph, drum graph, venessa graph, flower pot graph and umbrella graph. All results are diagrammatically illustrated.

MAIN RESULTS

Theorem 1

For any $n \geq 3$, the power dominator chromatic number of fan graph F_n is 3**.**

Proof: Let F_n be the fan graph. Let the vertex v_1, be the apex vertex, $v_2, v_3, v_4, \dots, v_{n+1}$ be the vertices of fan graph F_n. Let $E(F_n)$ = $\{v_1 v_i / 2 \leq i \leq n+1\} \cup \{v_i v_{i+1} / 2 \leq i \leq n+1\}$.

Assign the color c_1 to the apex vertex v_1. The vertices $\{v_2, v_3, v_4, \dots, v_{n+1}\}$ of fan F_n assigned color c_2 and c_3 alternatively.

This coloring is proper. The vertices $\{v_2, v_3, v_4, \dots, v_{n+1}\}$ of fan graph F_n will power dominate the color class $c_1 = \{v_1\}$**.** Vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the fan graph F_n will power dominate atleast one color class. The power dominator coloring the fan graph F_n *is* 3. i.e., $\chi_{pd}(F_n) = 3$.

Example 1: In figure 1, the power dominator coloring of fan graph F_3 is shown.

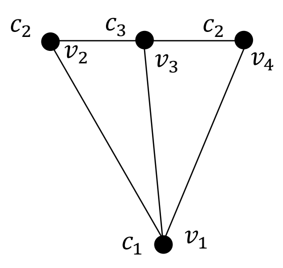

Fig.1 fan graph F_3

Theorem 2

For any $n \geq 3$, the power dominator coloring of double fan graph DF_n is 3.

Proof: Let DF_n be the double fan graph. Let the vertex v_1, v_1', be an apex vertex and $v_2, v_3, v_4, \ldots, v_{n+1}$ be the vertices of path attached to vertices v_1, v_1'. Let $E(DF_n) = \{v_1 v_i / 2 \leq i \leq n+1\} \cup \{v_1' v_i / 2 \leq i \leq n+1\} \cup \{v_i v_{i+1} / 2 \leq i \leq n+1\}$.

Assign the color c_1 to the apex vertices v_1, v_1'. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of double fan graph DF_n assigned color c_2 and c_3 alternatively.

This coloring is proper. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of double fan graph DF_n will power dominate the color class $c_1 = \{v_1, v_1'\}$. Vertices $\{v_1, v_1'\}$ will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the double fan graph DF_n will power dominate atleast one color class. The power dominator chromatic number for the double fan graph DF_n, is 3. i.e., $\chi_{pd}(DF_n) = 3$.

Example 2: In figure 2, the power dominator coloring of double fan graph DF_3 is shown.

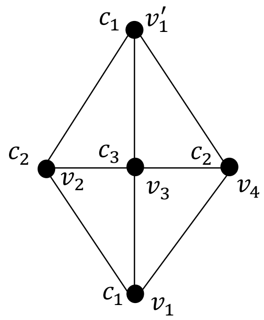

Fig.2 Double fan graph DF_3

Theorem 3: For any $n \geq 2$, the power dominator coloring of octopus graph O_n is 3.

Proof: Let O_n be an octopus graph. Let the vertex v_1,be the apex vertex, $v_2, v_3, v_4, \ldots, v_{n+1}$ be the vertices of fan graph F_n, *and* $\{v_{n+2}, v_{n+3}, v_{n+4}, \ldots, v_{2n+1}\}$ be the vertices of star graph $K_{1,n}$. Let $E(O_n) = \{v_1 v_i / 2 \leq i \leq 2n+1\} \cup \{v_i v_{i+1} / 2 \leq i \leq n\}$.

Assign the color c_1 to the apex vertices v_1. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of fan F_n assigned color c_2 and c_3 alternatively, and the vertices $\{v_{n+2}, v_{n+3}, v_{n+4}, \ldots, v_{2n+1}\}$ of star graph $K_{1,n}$ assigned color c_2 and c_3 alternatively.

□

This coloring is proper. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of fan graph F_n will power dominate the color class $c_1 = \{v_1\}$, every vertex of the star $K_{1,n}, \{v_i, n+2 \leq i \leq 2n+1$, will power dominate the color class $c_1 = \{v_1\}$. Vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator chromatic number for octopus graph O_n, is 3. i.e., $\chi_{pd}(O_n) = 3$.

Example 3: In figure 3, the power dominator coloring of octopus graph O_5 is shown.

□

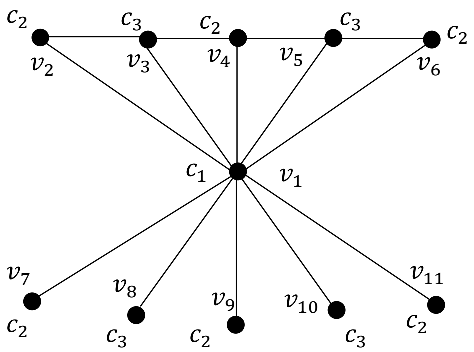

Fig.3 Octopus graph O_5

Theorem 4: For any $n \geq 2$, the power dominator chromatic number of lilly graph I_n is 3.

Proof: Let I_n be the lilly graph. Let the vertex v_1 be the apex vertex, $v_2, v_3, v_4, \ldots, v_{n+1}$ be the vertices of star graph $K_{1,n}$, and $\{v_{n+2}, v_{n+3}, v_{n+4}, \ldots, v_{2n+1}\}$ be the vertices of second star graph $K_{1,n}$. The vertices of first path P_n be $\{v_{2n+2}, v_{2n+2}, v_{2n+3}, \ldots, v_{3n}\}$, and the vertices of second path P_n be $\{v_{3n+1}, v_{3n+2}, v_{3n+3}, \ldots, v_{4n-1}\}$. The vertices $v_2, \ldots, v_n, v_{n+1}, \ldots,\ v_{2n+1}, v_{3n}, v_{4n-1}$ represents the pendant vertices. Let $E(I_n) = \{v_1\, v_i / 2 \le i \le n+1\} \cup \{v_1 v_{i+1} / n+2 \le i \le 2n+1\} \cup \{v_i v_{i+1} / 2n+2 \le i \le 3n-1\} \cup \{v_i v_{i+1} / 3n+1 \le i \le 4n-1\}\} \cup \{v_1 v_{2n+2}, v_1 v_{3n+1}\}$.

Assign the color c_1 to the vertices v_1. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of star $K_{1,n}$ assigned color c_2 and c_3 alternatively, and the vertices $\{v_{n+2}, v_{n+3}, v_{n+4}, \ldots, v_{2n+1}\}$ of star $K_{1,n}$ assigned color c_2 and c_3 alternatively. The vertices of first path P_n $\{v_{2n+2}, v_{2n+2}, v_{2n+3}, \ldots, v_{3n}\}$, and the vertices of second path P_n $\{v_{3n+1}, v_{3n+2}, v_{3n+3}, \ldots, v_{4n-1}\}$ are assigned color c_2 and c_3 alternatively. This coloring is proper. The vertices $v_2, v_3, v_4, \ldots, v_{n+1}, v_{n+2}, v_{n+3}, v_{n+4}, \ldots,$

$$v_{2n+1}, v_{2n+2}, v_{2n+2}, v_{2n+3}, \ldots, v_{3n}, v_{3n+1}, v_{3n+2}, v_{3n+3}, \ldots,$$

v_{4n-1} will power dominate the color class $c_1 = \{v_1\}$. Vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator chromatic number of lilly graph I_n, is 3. i.e., $\chi_{pd}(I_n) = 3$.

Example 4: In figure 4, the power dominator coloring of lilly graph I_5 is shown.

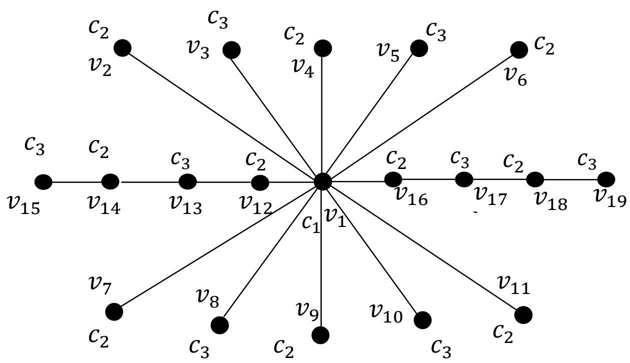

Fig.4 Lilly graph I_5

Theorem 5: For any $n \ge 2$, the power dominator chromatic number of planter graph R_n is 3.

Proof: Let R_n be the planter graph. Let the vertex v_1 be the apex vertex, $v_2, v_3, v_4, \ldots, v_{n+1}$ be the vertices of fan graph F_n, and $\{v_{n+2}, v_{n+3}, v_{n+4}, \ldots, v_{2n+1}\}$ be the vertices of cycle graph C_n. Let $E(R_n) = \{v_1\, v_i / 2 \le i \le n+1\} \cup \{v_i v_{i+1} / 2 \le i \le n+1\} \cup \{v_i v_{i+1} / n+2 \le i \le 2n-1\}$.

Assign the color c_1 to the apex vertex v_1. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of fan F_n assigned color c_2 and c_3 alternatively, and the vertices $\{v_{n+2}, v_{n+3}, v_{n+4}, \ldots, v_{2n}\}$ of cycle graph c_n assigned color c_2 and c_3 alternatively.

This coloring is proper. The vertices $\{v_2, v_3, v_4, \ldots, v_{n+1}\}$ of fan graph F_n will power dominate the color class $c_1 = \{v_1\}$, every vertex of the cycle $c_n, \{v_i, n+2 \le i \le 2n,\}$ will power dominate the color class $c_1 = \{v_1\}$. Vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator chromatic number of planter graph R_n, 3. i.e., $\chi_{pd}(R_n) = 3$.

Example 5: In figure 5, the power dominator coloring of planter graph R_3 is shown.

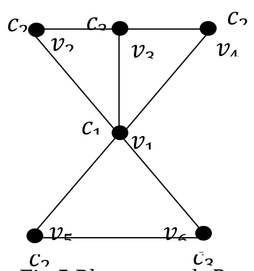

Fig.5 Planter graph R_3

Theorem 6

For any $n \geq 3$, the power dominator coloring of venessa graph V_n is 3.

Proof: Let V_n be the venessa graph. Let the vertex v_1,be the apex vertex, $v_2, v_3, v_4, \dots, v_{n+1}$ be the vertices of fan graph F_n, *and* $\{v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}\}$ be the vertices of second fan graph F_n.and $v_{2n+2}, u_{2n+3}, u_{2n+4}, \dots, u_{3n+1}$ be vertices of a star $K_{1,n}$ Let $E(V_n) = \{v_1\, v_i/2 \leq i \leq n+1\} \cup \{v_i v_{i+1}/2 \leq i \leq n+1\} \cup \{v_1\, v_j/n+2 \leq j \leq 2\,n+1\} \cup \{v_\text{i} v_\text{i+1}/\text{n} + 2 \leq \text{i} \leq 2\text{n} + 1\} \cup \{v_1\, u_j/2n+2 \leq k \leq 3\,n+1\}$.

Assign the color c_1 to the apex vertex v_1. The even indexed vertices $\{v_2, v_4, v_6, \dots,\}$ of venessa graph assigned color c_2 and the odd indexed vertices $\{v_3, v_5, v_7, \dots,\}$ of venessa graph assigned color c_3. This procedure ensures the coloring is proper. Every vertex $v_i.\, 2 \leq i \leq 3n+1$ of venessa graph will power dominate the color class $c_1 = \{v_1\}$. Vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator coloring of venessa graph V_n, $n \geq 3$ is 3. i.e., $\chi_{pd}(V_n) = 3$.

Example 6: In figure 5, the power dominator coloring of venessa graph V_3 is shown

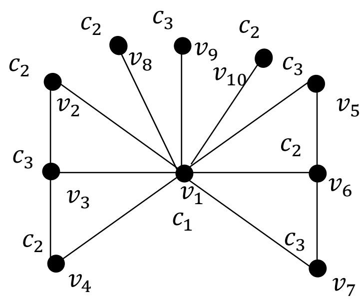

Fig.6 Venessa graph V_3

Theorem 7

For any $n \geq 3$, the power dominator chromatic number for flower pot graph FP_n is 3.

Proof: Let FP_n, $n \geq 3$ be a flower pot graph with $n \geq 3$ vertices. Let the vertex v_1,be the apex vertex, $v_2, v_3, v_4, \dots, v_{n+1}$ be the vertices of star graph $K_{1,n}$, and $\{v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}\}$ be the vertices of cycle graph C_n. Let $(FP_n) = \{v_1 v_i/2 \leq i \leq n+1\} \cup \{v_i v_{i+1}/n+2 \leq i \leq 2n+1\} \cup \{v_1 v_{n+2}, v_{2n+1} v_1\}$.

Assign the color c_1 to the apex vertices v_1. The pendent vertices $\{v_2, v_3, v_4, \dots, v_{n+1}\}$ of star graph $K_{1,n}$ assigned color c_2, and the vertices $\{v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}\}$ of cycle graph C_n assigned color c_2 and c_3 alternatively. This coloring is proper. The vertices $\{v_2, v_3, v_4, \dots, v_{n+1}\}$ of star graph associated in flower pot graph FP_n will power dominate the color class $c_1 = \{v_1\}$, every vertex of the cycle $C_n = \{v_i, n+2 \leq i \leq 2n+1$, will power dominate the color class $c_1 = \{v_1\}$ and c_3. Vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator coloring of flower pot graph FP_n, $n \geq 3$ is 3. i.e., $\chi_{pd}(FP_n) = 3$.

Example 7: In figure 7, the power dominator coloring of flower pot graph FP_5 is shown

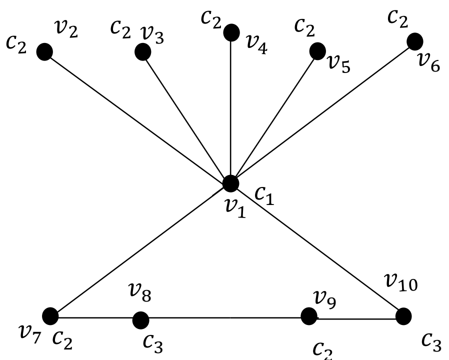

Fig.7 Flower pot graph FP_5

Theorem 8

For any $n \geq 2, m \geq 3$, the power dominator chromatic number of umbrella graph, $U(m,n)$ is 3.

Proof: Let $U(m,n)$, $m \geq 3, n \geq 2$ be a be umbrella graph with $m \geq 3, n \geq 2$ vertices. Let the vertex $v_1$be the apex vertex, $v_2, v_3, v_4, \dots, v_{m+1}$ be the vertices of fan F_m, and $\{v_{m+2}, v_{m+3}, v_{m+4}, \dots, v_{m+n}\}$ be the vertices of path P_n . Let $E(U(m,n)) = \{v_1 v_i / 2 \leq i \leq m+1\} \cup \{v_i v_{i+1} / 2 \leq i \leq m\} \cup \{v_1 v_{m+2}\} \cup \{v_j v_{j+1} / m+2 \leq j \leq m+n-1\}$.

Assign the color c_1 to the apex vertices v_1. The vertices $\{v_2, v_3, v_4, \dots, v_{m+1}\}$ of fan graph F_m assigned color c_2 and c_3 alternatively, and the vertices $\{v_{m+2}, v_{m+3}, v_{m+4}, \dots, v_{n+m}\}$ of path P_n assigned color c_2 and c_3 alternatively. This coloring is proper. The vertices $\{v_2, v_3, v_4, \dots, v_{m+1}\}$ of fan graph associated in flower pot graph $U(m,n)$ will power dominate the color class $c_1 = \{v_1\}$, every vertex of the path $P_n = \{v_i, m+2 \leq j \leq m+n$, will power dominate the color class $c_1 = \{v_1\}$. And vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator coloring for umbrella graph $U(m,n), m \geq 3, n \geq 2$, is 3. i.e., $\chi_{pd}(U(m,n)) = 3$.

Example 8: In figure 8, the power dominator coloring of umbrella graph $U(5,4)$ is shown.

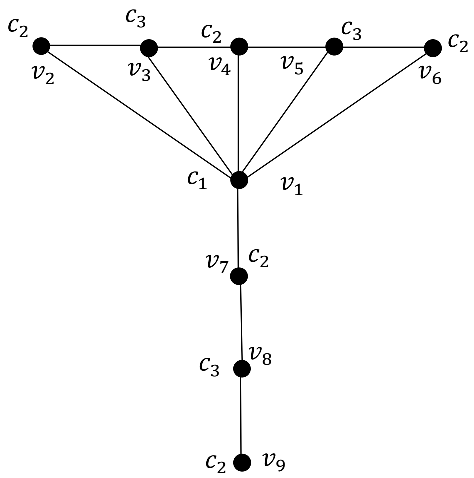

Fig.8 Umbrella graph U(5,4) .

Theorem 9: For any $n \geq 2$, the power dominator coloring of drums graph D_n is 3.

Proof: Let D_n be the drums graph. Let the vertex v_1, be the apex vertex, $v_2, v_3, v_4, \dots, v_n$ be the vertices of cycle graph C_n, and $\{v_{n+1}, v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n-1}\}$ be the vertices of second cycle graph C_n. The vertices of first path P_n be $\{v_{2n}, v_{2n+1}, v_{2n+2}, v_{2n+3}, \dots, v_{3n-2}\}$, and the vertices of second path P_n be $\{v_{3n-1}, v_{3n}, v_{3n+1}, v_{3n+2}, \dots, v_{4n-3}\}$. Let $E(D_n) = \{v_i v_{i+1} / 1 \leq i \leq n\} \cup \{v_j v_{j+1} / n+1 \leq j \leq 2n-2\} \cup \{v_1 v_{n+1}, v_1 v_{2n-1}, v_1 v_{2n}, v_1 v_{3n-1}\} \cup \{v_k v_{k+1} / 2n \leq k \leq 3n-3\} \cup \{v_l v_{l+1} / 3n-1 \leq l \leq 4n-4\}$.

Assign the color c_1 to the vertices v_1. The vertices $\{v_2, v_3, v_4, \dots, v_n\}$ of first cycle graph C_n assigned color c_2 and c_3 alternatively, and the vertices $\{v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n-1}\}$ of second cycle graph C_n assigned color c_2 and c_3 alternatively. The vertices of first path P_n be $\{v_{2n}, v_{2n+1}, v_{2n+2}, \dots, v_{3n-2}\}$, and the vertices of second path P_n be $\{v_{3n}, v_{3n+1}, v_{3n+2}, v_{3n+3}, \dots, v_{4n-3}\}$ are assigned color c_2 and c_3 alternatively. This coloring is proper. The vertices $v_2, v_3, v_4, \dots, v_{n+1} v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}$,

$v_{2n+2}, v_{2n+2}, v_{2n+3}, \dots, v_{3n}, v_{3n+1}, v_{3n+2}, v_{3n+3}, \dots, v_{4n-3}$ will power dominate the color class $c_1 = \{v_1\}$. vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator coloring of drums graph D_3 is 3. i.e., $\chi_{pd}(D_n) = 3$.

Example 9: In figure 3, the power dominator coloring of drums graph I_5 is shown.

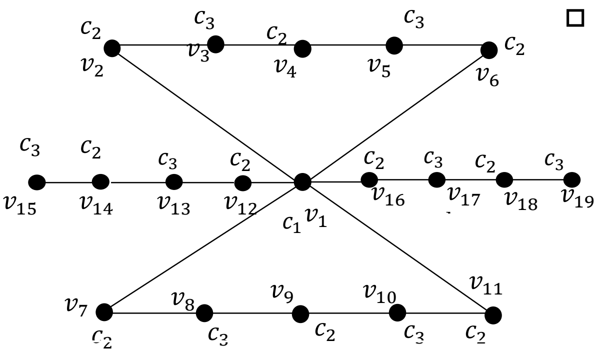

Fig.9 Drums graph D_5

Theorem 10: For any $n \geq 3$, the power dominator chromatic number of udukkai graph U_n is 3.

Proof: Let $U_n, n \geq 3$ be the udukkai graph. Let the vertex v_1, be the apex vertex. The vertices, $v_2, v_3, v_4, \dots, v_{n+1}$ be the vertices of first fan graph F_n, and $\{v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}\}$ be the vertices of second fan graph F_n. The vertices of first path P_n be $\{v_{2n+2}, v_{2n+2}, v_{2n+3}, \dots, v_{3n}\}$, and the vertices of second path P_n be $\{v_{3n+1}, v_{3n+2}, v_{3n+3}, \dots, v_{4n-1}\}$. Let $E(U_n) = \{v_1\, v_i / 2 \leq i \leq n+1\} \cup \{v_i v_{i+1} / 2 \leq i \leq n\} \cup \{v_1 v_{i+1} / n+2 \leq i \leq 2n+1\} \cup \{v_i v_{i+1} / n+2 \leq i \leq 2n\} \cup \{v_i v_{i+1} / 2n+2 \leq i \leq 3n-1\} \cup \{v_i v_{i+1} / 3n+1 \leq i \leq 4n-1\}\} \cup \{v_1 v_{2n+2}, v_1 v_{3n+1}\}$.

Assign the color c_1 to the vertices v_1. The vertices $\{v_2, v_3, v_4, \dots, v_{n+1}\}$ of first fan graph F_n assigned color c_2 and c_3 alternatively, and the vertices $\{v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}\}$ of second fan graph F_n assigned color c_2 and c_3 alternatively. The vertices of first path P_n be $\{v_{2n+2}, v_{2n+2}, v_{2n+3}, \dots, v_{3n}\}$, and the vertices of second path P_n be $\{v_{3n+1}, v_{3n+2}, v_{3n+3}, \dots, v_{4n-1}\}$ are assigned color c_2 and c_3 alternatively. This coloring is proper. The vertices $v_2, v_3, v_4, \dots, v_{n+1} v_{n+2}, v_{n+3}, v_{n+4}, \dots, v_{2n+1}$,

$v_{2n+2}, v_{2n+2}, v_{2n+3} \dots, v_{3n}, v_{3n+1}, v_{3n+2}, v_{3n+3}, \dots, v_{4n-1}$ will power dominate the color class $c_1 = \{v_1\}$, And vertex v_1 will power dominate itself and color class c_2 and c_3. Therefore, every vertex in the graph will power dominate atleast one color class. The power dominator coloring of udukkai graph $U_n, n \geq 3$ is 3. i.e., $\chi_{pd}(U_n) = 3$.

Example 10: In figure 3, the power dominator coloring of udukkai graph I_5 is shown.

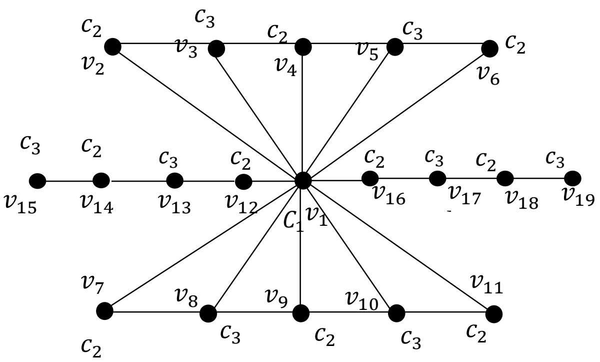

Fig.10 Udukkai graph U_5

CONCLUSION

On connecting graph coloring problem with power domination, power dominator coloring was introduced. The main objective of this paper is to study the power dominator coloring of the fan graph, double fan graph, planter graph, lilly graph, octopus graph, drums graph, venessa graph, flower pot graph and umbrella graph. There is scope for studying power dominator coloring for some platonic graphs and chemical structures.

References

[1] T. W. Haynes, S. M. Hedetniemi, S. T. Hedetniemi, and M. A. Henning, *Fundamentals of Domination in Graphs*. 1998.

[2] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power System Observability with Minimal Phasor Measurement Placement," *IEEE Transactions on Power Systems*, vol. 8, no. 2, pp. 707–715, 1993, doi: 10.1109/59.260810.

[3] S. H. Ralucca Gera, Craig Rasmussen, "Dominator Colorings and Safe Clique Partitions," *Congressus Numerantium*, 2006.

[4] K. S. Kumar, N. G. David, and K. G. Subramanian, "Graphs and Power Dominator Colorings," vol. 11, no. 2, pp. 67–71, 2016.

[5] A. Uma Maheswari and Bala Samuvel J., "Power dominator chromatic number for some special graphs," *International Journal of Innovative Technology and Exploring Engineering*, vol. 8, no. 12, pp. 3957–3960, 2019, doi: 10.35940/ijitee.L3466.1081219.

[6] A. Uma Maheswari and Bala Samuvel J, "Power Dominator Coloring for Various Graphs," *Journal of the Maharaja Sayajirao University of Baroda*, vol. 54, no. 2(I), pp. 119–123, 2020.

[7] A. Uma Maheswari, Bala Samuvel J., and S. Azhagarasi, "Power Dominator Coloring of Special Kind of Graphs," *Kala Sarovar*, vol. 23, no. 04(XI), pp. 58–62, 2020.

[8] A. Uma Maheswari, Bala Samuvel J., and S. Azhagarasi, "Power Dominator Chromatic Number for vertex duplication of some Graphs," *Design Engineering*, vol. 2021, no. 06, pp. 5755–5774, 2021.

[9] A. E. Samuel and S. Kalaivani, "Prime Labeling For Some Octopus Related Graphs," *IOSR Journal of Mathematics*, vol. 12, no. 6, pp. 57–64, doi: 10.9790/5728-1206035764.

[10] A. E. Samuel and S. Kalaivani, "Prime Labeling for Some Lilly related Graphs." [Online]. Available: http://acadpubl.eu/hub

[11] A. E. Samuel and S. Kalaivani, "PRIME LABELING FOR SOME VANESSA RELATED GRAPHS Mathematics," 2017.

[12] A. E. Samuel and S. Kalaivani, "Prime Labeling for Some Planter Related Graphs," 2016. [Online]. Available: http://www.irphouse.com

[13] A. E. Samuel and S. Kalaivani, "Prime Labeling to Drums Graphs," *Annals of Pure and Applied Mathematics*, vol. 16, no. 2, pp. 307–312, Feb. 2018, doi: 10.22457/apam.v16n2a7.

[14] A. E. Samuel and S. Kalaivani, "PRIME LABELING TO UDUKKAI GRAPHS," 2018. [Online]. Available: www.ijma.info

[15] A. E. Samuel and S. Kalaivani, "PRIME LABELING FOR SOME VANESSA RELATED GRAPHS Mathematics," 2017.

[16] Chidambaram, "Prime Labeling For Some Fan Related Graphs." [Online]. Available: www.ijert.org

[17] J. A. Bondy and U. S. R. Murty, "GRAPH THEORY WITH APPLICATIONS."

[18] K. Kavitha and N. G. David, "Dominator Coloring of Some Classes of Graphs," *International Journal of Mathematical Archive-3(11)*, vol. 3, no. 11, pp. 3954–3957, 2012.

[19] R. Gera, "On the Dominator Colorings in Bipartite Graphs," in *Fourth International Conference on Information Technology (ITNG'07)*, 2007, pp. 947–952.

[20] S. Arumugam and J. A. Y. Bagga, "On dominator colorings in graphs," vol. 122, no. 4, pp. 561–571, 2012.

[21] Chidambaram, "Prime Labeling For Some Fan Related Graphs." [Online]. Available: www.ijert.org

[22] J. Suresh Kumar and S. M. Nair, "Some Results on Prime Graphs," JETIR, 2020. Accessed: Apr. 11, 2022. [Online]. Available: www.jetir.org

[23] Lavanya.S and Ganesan V, "Vertex Prime Labeling of Umbrella U(m,n) Graphs," Nov. 2020, doi: 10.15680/IJIRSET.2020.0911026.