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REPRESENTATION OF GROUPS AS GRAPHS

MS.V.S. Selvi[1,a] and J. Miriam Daffodil[2,b]

[1)]Vice Principal and Assistant Professor, Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuarm 605401, Tamil Nadu, India

[2)]Post Graduate Student Department of Mathematics Theivanai Ammal College for Women (Autonomous), Villupuram 605401, Tamil Nadu, India

Abstract

In this paper an attempt is made to explore and express groups as graphs. From the structure of the graphs we try to study the properties of groups. Groups are described in terms of graph and we express the concept of identity in group so we call the graph associated with a group as identity graphs. This paper gives a detailed study on the connection between the cyclic groups and graphs with cycles especially containing complete graphs. We derive a few results on the number of complete graphs in a cyclic group with respect to the generators that are generating the cyclic groups. We proved the results depending on the order of different cyclic groups and represent graphs. We also have proved that some theorems relating to identity sub graphs need not in general associated with a sub group. Identity graph forms a cyclic group of observed P which as a final result gives triangles. This paper delivers the groups represented as graphs.

Key Words. Cyclic Group, Identity graph , Zero divisor graphs, Generator of a cyclic Group, Generator Graph of a cyclic group,.

INTRODUCTION

Studying groups is not an easy task as it has abstract nature. This study of representing finite groups as graphs makes the research more fascinating. We try here to study the properties of cyclic groups using cyclic graphs. The graph is mentioned as generator graph, as it plays a significant role in obtaining the graph by the generators of the cyclic group. Primarily, we begin by a diagrammatic representation of cyclic graphs using the generators. Taking additive modular group that are cyclic in nature let's discuss on drawing generator graphs. After drawing generator graphs let's compare the pattern obtained by the even and odd values of n in additive modular group(Z_n, $+_n$). Finally we can derive a conclusion that through graphical representation studying Groups can be simple and understandable.

Definition 1 [3]

An algebraic structure (G,*) is said to form a group if G is a non-empty set and * is a binary operation defined on G with the following properties satisfied.

(i) $a*(b*c)=(a*b)*c\ \forall\ a, b, c \in G$

(ii) If $\forall\ a \in G\quad \exists\ e \in G$ *such that* $a * e = a = e * a$.

(iii) If $\forall\ a \in G\quad \exists\ a^{-1} \in G$ *h that* $a * a^{-1} = e = a^{-1} * a$.

Definition 2 [1]

A graph G is an order triple (V(G), E(G) , ψ_G) consisting of a non empty set V(G) of vertices , a set E(G) , disjoint from V(G) , of edges, and an incidence function ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G . If *e* is an edge and *u* and *v* are vertices such that ψ_G $(e) = uv$ then e is said to join *u* and *v* ; the vertices *u* and *v* are called the ends of *e* .

Example

G = (V(G), E(G) , ψ_G)

Where

V(G) = $\{v_1, v_2, v_3, v_4, v_5\}$

E(G) = $\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\}$

ψ_G is defined by

$\psi_G(e_1) = v_1,v_2$, $\psi_G(e_2) = v_2,v_3$, $\psi_G(e_3) = v_3,v_3$, $\psi_G(e_4) = v_3,v_4$, $\psi_G(e_5) = v_2,v_4$, $\psi_G(e_6) = v_4,v_5$,
$\psi_G(e_7) = v_2,v_5$, $\psi_G(e_8) = v_2,v_5$

Definition 3[1]

A trail is closed if its origin and terminus are same . A closed trail whose origin and internal vertices are distinct is a cycle.

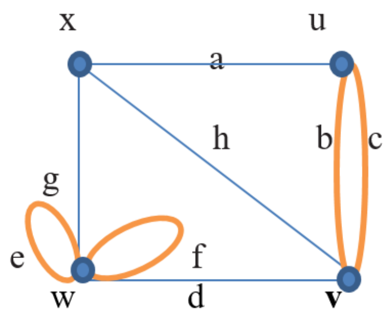


FIGURE 1 Closed trail : ucvhxgwfwdvbu

Cycle: xaubvhx

Definition 4 [3]

Group (G,*) is said to be a cyclic group if it possess a generator a∈G.A cyclic group is of the form $G = \{a, a^2, \ldots\ldots\ldots, a^n = e\}$ where e is the identity and a is the generator. It is possible for G to have many generators. Ifa ∈ G is a generator, then a^{-1} ∈ G is also a generator.

ANALYSIS OF GROUP AS GRAPH

Zero divisor

The zero divisor graph of R , denoted Γ (R) is the graph whose vertex set is the set of zero divisors of R and whose edge set is

E= { { a, b}⊆ Z(R) │ab} =0

Example

Γ (Z_{16})

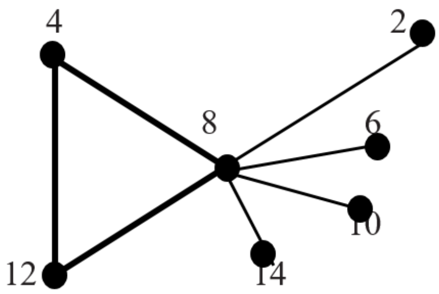
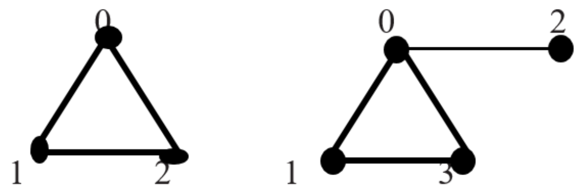


FIGURE 2

Identity Graph

The identity element will be adjacent with all the remaining vertices and the vertices other than identity element will be adjacent.

Example:



$Z_3 = \{0,1,2\}$ $Z_4 = \{0,1,2,3\}$

FIGURE 3

Theorem: [2]

Let G be a group. G_i denote the identity graph related to G. Every subgroup of G has an identity graph which is a special identity sub graph of G_i and every identity subgraph of G_i need not in general be associated with a subgroup of G.

Proof: Given G is a group. G the related identity graph of G. Suppose H is a subgroup of G then since H itself is a group and H a subset of G the identity graph associated with H will be aspecial identity subgraph of G_i.

Conversely if H_i is a identity sub graph of G i then we may not in general have a subgroup associated with it.Let $G = \{g \mid g^{11} = 1\}$ be the cyclic group of order 11. The identity graph associated with G be G_i which is as follows:

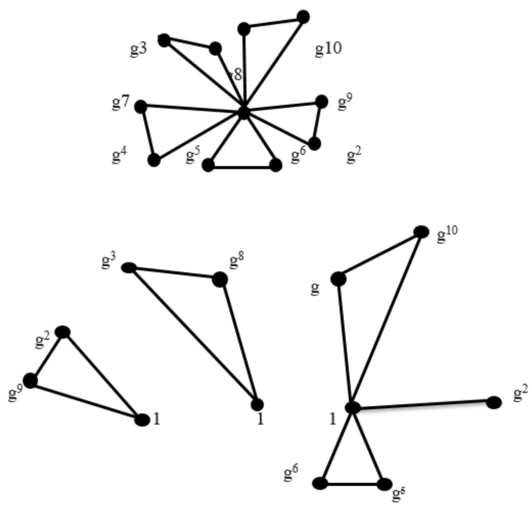


FIGURE 4

as some identity sub graphs of G_i. Clearly no subgroups can be associated with them as G has no proper subgroups, as o (G) =11, a prime. Hence the theorem.

ANALYSIS OF CYCLIC GROUPS AS GRAPHS

A minor cyclic group gives rise to a separate vertex. The cyclic group forms cyclic graphs whose adjacency is described with respect to their generators as follows. u is adjacent to v only if $u * a = v$ where ‘a’ isthe generator .

Example 1

This example shows us the fourth root of unity, which are cyclic groups under multiplication. For the fourthroots of unity G ={1, -1, i, -i} under multiplication, ‘i’ is the generator as in the below figure.

The four elements gives the following four vertices :$1 \times i = i$; $i \times i = -1$; $-1 \times i = -i$ *and* $-i \times i = 1$. The adjacent vertices in the graph are 1 i, -1, –i and 1 in the order as shown in figure and the graph we get is a cycle. Alternatively, if we choose the generator (-i) which is the inverse of (i) then again we can derive down to these vertices represented by1, -i, -1, i and back to 1 again. We get the graph with its elements and its inverse in opposite direction.

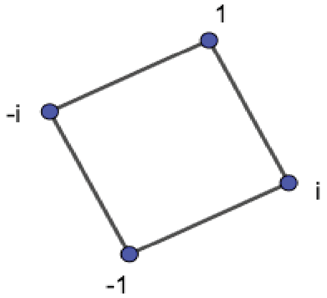


FIGURE 5 Cyclic Graph of square roots of unity under multiplication

Example 2

Modular groups like (Z_{12}, $+_{12}$), which has more than a pair of generators there are more than one cyclic graph .

Let Z_{12} = {0,1,2,3,4,5,6,7,8,9,10,11} under $+_{12}$ where 1 is a generator and thus its inverse 11 will also be a generator. The second pair of generators is 5 and 7. For 1 the vertices are in the order 0→ 1 → 2 → 3 → 4 → 5 → 6 → 7 → 8 → 9 → 10 → 11 →0 for the cyclic graph and in the reverse order for the generator 11.For the next pair 5 and 7 the cyclic graph will be with vertices in the order 0→ 5 → 10 → 3 → 8 → 1 → 6 → 11 → 4 →9 → 2 → 7 → 0 respectively. The different cyclic graphs obtained by the two pair of generators is shown in the below figure.

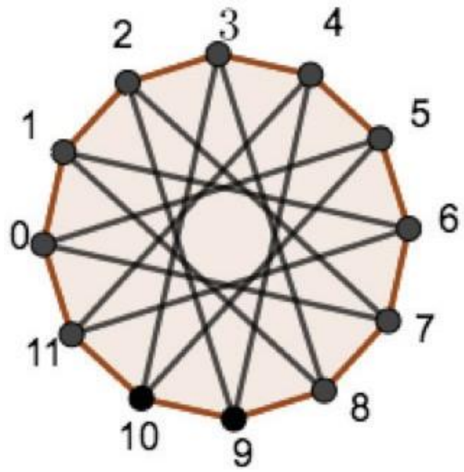


FIGURE 6 Cyclic Graph for (Z_{12}, $+_{12}$)

Generator Graphs

The generator graphs are known as the generators with the vertices connected to left vertices among themselves and to all the other non-generator elements on the right vertices in the cyclic group. The adjacency is mentioned as *a* is adjacent with y if *a***a*= *b* where 'a' is a generator. An edge connects a generator vertex to any other vertex, if and only if that particular element is generated by the generator. Take the below examples of the generator graphs formed by the additive modular groups (Z_n, $+_n$) when n is odd or even.

Examples for odd values of n

Consider modular additive cyclic groups (Z_7, $+_7$), (Z_9, $+_9$) and for odd values of n represented by fig 5, fig 6 and fig 7 respectively.

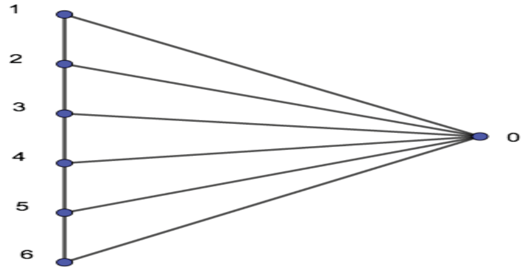


FIGURE 7 Generator Graph for (Z_7, $+_7$)

1 generates 2 and 0 and 2 generates 0.So the triangle formed between 1,2,0 can be considered as a K_3 graph. The number of K_3 graphs with respect to the cyclic group $(Z_7, +_7)$ is 15.

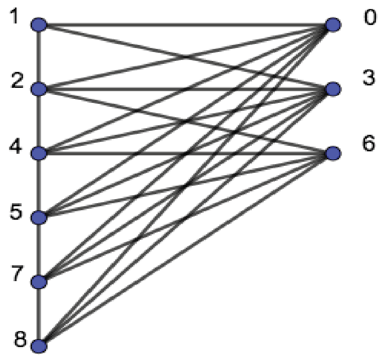


FIGURE 8 Generator Graph for $(Z_9, +_9)$

Examples for even values of n

For even values of n consider fig 8, fig 9 and fig 10.

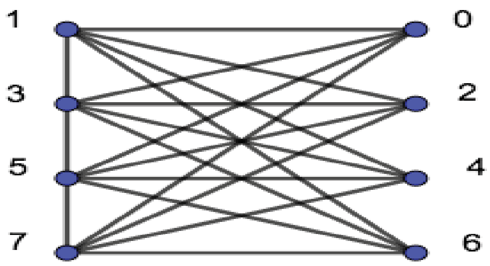


FIGURE 9 Generator Graph for $(Z_8, +_8)$

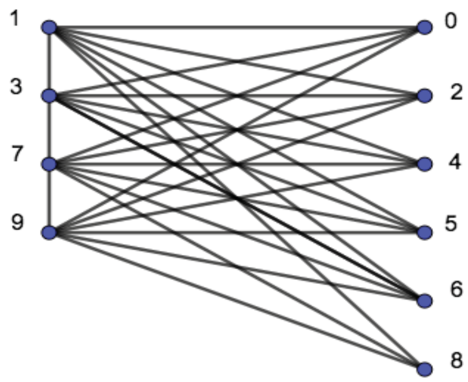


FIGURE 10 Generator Graphs for $(Z_{10}, +_{10})$

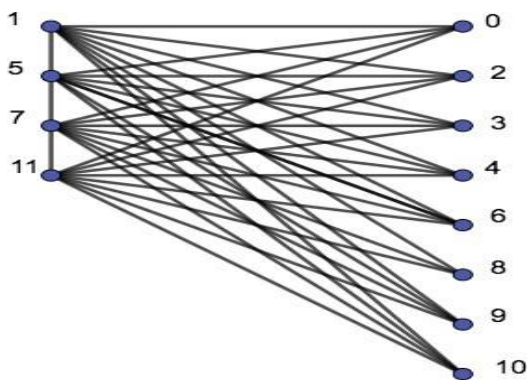


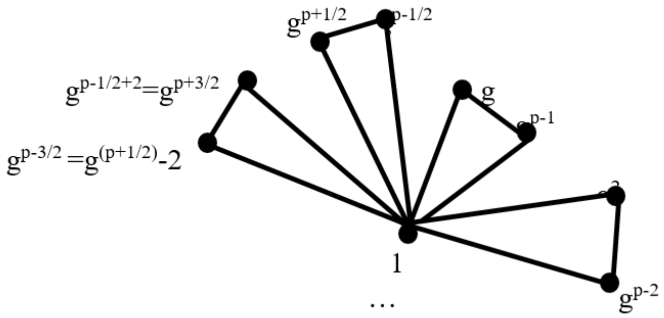
FIGURE 11 Generator Graph for $(Z_{12}, +_{12})$

Theorem [2]

If G = < g\ g^p =1> be a cyclic group of order p ,p a prime . Then the identity graph formed by G has only triangles in fact (p-1)/2 space triangles.

Proof

Given G= < g\ g^p =1> is a cyclic group of order p , p a prime. G has no proper subgroups. So no element in G is a self inversed element i.e, for no g_i in G is such that $(g_i)^2 = 1$. By Cauchy theorem G cannot have elements of order two. Thus for every g^i in G there exists a unique g^j in G such that $g^i g^j = 1$. So for every g^i the g^j is such that j =(p-i) so from this the elements $1, g^1, g^{p-I}$ form a triangle. Hence the identity graph will not have any line any graphs. Thus a typical identity graph of these G will be of the following form.



Generator graphs for Modular Groups under Multiplication

The number of generators for the cyclic group $(Z_n, \times_n)$ are different for different values of prime 'n'. Z_5 and Z_7 have two generators each while Z_{11} and Z_{13} have 4 generators each. Z_{17} have 8 generators which are 3,5,6,7,10,11,12 and 14. Moreover the generator occur in pairs like a number and its inverse. For example in the cyclic group $(Z_{17}, \times_{17})$ 3 and 6,5 and 7,10 and 12,14 and 11 are pairs of generators in it.

The result mentioned above in finding K_3 graphs is applicable in generator graphs drawn for modular groups under multiplication also.

Example

Consider the cyclic group $(Z_{13}, \times_{13})$as in fig(xiii).The number of K_3 graphs in $(Z_{13}, \times_{13})$ is 48. i.e $8\times \sum(4-1)$ where there are 8 non generators and 4 generators as in fig(xiii). the total number of K_3 graphs obtained in the generator graph is $[[12 - \emptyset(12)] \times \sum(\emptyset(12) - 1]$ where Ø(12) is the number of generators and [12 - Ø (12)] is the number of non generators .

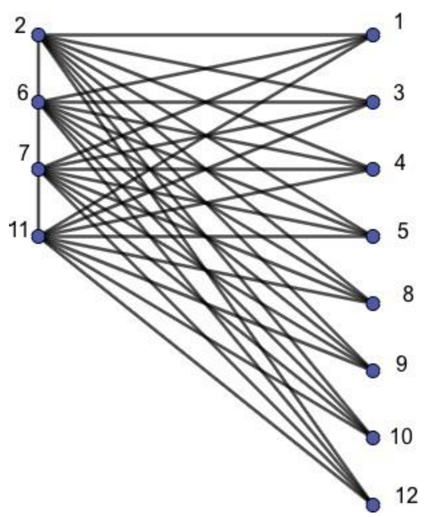


FIGURE 12 Generator Graph for $(Z_{13}, \times_{13})$

Theorem [4]

If (Z_n, $+_n$) be a cyclic group of order n, then the generator graph will have $[[n - \emptyset(n)] \times \Sigma(\emptyset(n) - 1)]$ where $\emptyset(n)$ is the number of generators in the cyclic group.

Proof:

The cyclic group (Z_n, $+_n$) have $\emptyset(n)$ generators and [n- $\emptyset(n)$] non-generators as the number of integers less to n and prime to n are the generators .Fixing the first generator and the first non –zero generator the number of K_3 graphs formed is ($\emptyset(n)$ – 1) . But there is [n - $\emptyset(n)$] non generators in the graph. Hence the total number of K_3 graphs obtained by fixing a vertex as the first generator is

[n - $\emptyset(n)$] ×($\emptyset(n)$ – 1). Similary with the second vertex the number of K_3 graphs formed is [n - $\emptyset(n)$] ×($\emptyset(n)$ – 2) as the combination of the second vertex with the first is considered. Continuing the process we obtain

[n - $\emptyset(n)$] ×($\emptyset(n)$ – 1) which is [n- $\emptyset(n)$ ×1]

Hence the total number of K_3 graphs obtained in the generator graph is

$= [n - \emptyset(n)] \text{ X } [(\emptyset(n)-1) + (\emptyset(n)-2) + \ldots\ldots + (\emptyset(n) - (n-1))]$

$= [n - \emptyset(n)] \times [(\emptyset(n) - 1) + (\emptyset(n) - 2) + \ldots\ldots + 1]$

$= [n - \emptyset(n)] \times \Sigma[(\emptyset(n) - 1)].$

Hence, the theorem.

CONCLUSION

This study can be used in analyzing the group and cyclic group represented as graphs. This enables us to learn quickly about the structure of the group. The examples we have seen and derived also shows that all the graphs are complete in nature.In future we may represent ring as graph. Finally it is observed through the above study that groups and cyclic groups can be represented as graphs and it also helps study groups easily through graphs.

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