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FEKETE-SZEGO PROBLEM AND COEFFICIENT ESTIMATES RELATED WITH SINE FUNCTION

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ABSTRACT:

In this paper,we define a analytic functions associated with sine functions of unit disk in the region on the complex plane. Our aim to find the coefficient of the class S*sine using a star like function in Fekete-Szego Problem.

Key words: Analytic functions, trigonometric function,Subordination.

1.INTRODUCTION AND DEFINITION:

1. 1.INTRODUCTION

Let A be the class of functions f(z) of the form, $\mathbf{f(z)} = \sum_{\mathbf{n=1}}^{\infty} \boldsymbol{a_n z^n}, (\boldsymbol{z \in D})$ which are analytic

of the function in the region $D = \{Z: Z \in C: |Z| < 1\}$ and let A be the class of all analytic functions f(z) in the open unit disk D, which are normalized by f(0)=0,f′(0)=1. The functions fϵA has the Taylor's series expansion of the form,

$$\mathbf{f(z)} = \boldsymbol{z} + \sum_{\mathbf{n=2}}^{\infty} \boldsymbol{a_n z^n} \qquad \textbf{(1.1)}$$

and the subclass of A is comprise of univalent functions and such functions are called as normalized univalent functions in D. These all functions of classes is denoted by S and the class S of all regular univalent functions. Cho et al was introduced by the class S*sin of analytic function .They also analysis of ϕ(z) = 1+sin z is determine by the radii problems for this class of functions. An Analytic function f is subordinate to an analytic function g. we may written as $f \prec g$. If there exists an analytic function ω with ω(0)=0 and |ω(z)| <1 for $Z \in D$ such that f(z)=g(ω(z)), where ω is a Schwarz function. They also expand some new inequalities associate with coefficient bounds of some subclasses of univalent functions. Fekete-Szego inequality is one of the inequality for the coefficients of univalent analytic functions.

1.2. DEFINITION:

DEFINITION 1.2.1: Let f be given by (1.1) then $f \epsilon\, \mathbb{S}^*{}_{sin}$ if and only if

$$\mathrm{Re}\left\{(1-\lambda)f'(z) + \lambda\left(1 + \frac{z f''(z)}{f'(z)}\right)\right\} > 0 \text{ ,(Z}\epsilon\text{D)} \qquad (1.2)$$

DEFINITION 1.2.2: The class $\mathbb{S}^*{}_{\boldsymbol{sin}}$ of analytic function defined by,

$$\mathbb{S}^*{}_{\boldsymbol{sin}} = \left\{f \epsilon \mathbb{S} : \frac{z f'(z)}{f(z)} \prec 1 + \sin(z)\right\} \qquad (1.3)$$

2. COEFFICIENT ESTIMATION:

LEMMA 2.1 . If P ϵℙ and has the form $P(z) = 1 + c_1 z + c_2 z^2 + \dots\dots$(ZϵD), then for anycomplex number μ we have,

$$|c_2 - \mu c_1^2| \le \max\{2, 2|\mu - 1|\}$$

Then $|P_k| \le 1$, k ϵ N where P is the family of all functions analytic in D for which P(0)=1 and Re(P(z))>0, (Z ϵ D).

THEOREM 2.2. If f(z) given by (1.1) belongs to the class $f \epsilon \mathbb{S}^*{}_{sin}$ then,

$$|a_2| \le \left(\frac{1}{4(1-\lambda)+4\lambda}\right) c_1$$

$$|a_3| \le \left(\frac{1}{6(1-\lambda)+12\lambda}\right) \left[\ c_2 - \frac{c_1{}^2}{2}\left(1 + \frac{\lambda}{(1-\lambda)^2} - 1\right)\right.$$

PROOF:Let $f \epsilon \mathbb{S}^*{}_{sin}$. Then we can write using (1.2) and (1.3), in terms of Schwarz function as,

$$(1-\lambda)f'(z) + \lambda\left(1 + \frac{z f''(z)}{f'(z)}\right) = 1 + \sin(\omega(z))$$

$$f(z) = z + a_2 z^2 + a_3 z^3$$

$$f'(z) = 1 + 2a_2 z + 3a_3 z^2$$

$$f''(z) = 2a_2 + 6a_3 z + 12a_4 z^2$$

$$f'(z) = 1 + 2a_2 z + 3a_3 z^2$$

$$(1-\lambda) f'(z) = (1-\lambda)[1 + 2a_2 z + 3a_3 z^2]$$

$$(1-\lambda) f'(z) = \{(1-\lambda) + (1-\lambda)2a_2 z + (1-\lambda)3a_3 z^2\}$$

$$\left(1 + \frac{z f''(z)}{f'(z)}\right) = 1 + \frac{z(2a_2 + 6a_3 z + 12a_4 z^2)}{(1 + 2a_2 z + 3a_3 z^2 + 4a_4 z^3)} = 1 + \frac{(2a_2 z + 6a_3 z^2 + 12a_4 z^3)}{(1 + 2a_2 z + 3a_3 z^2 + 4a_4 z^3)}$$

$$= \frac{1+2a_2 z+3a_3 z^2+4a_4 z^3 + (2a_2 z+6a_3 z^2+12a_4 z^3)}{(1+2a_2 z+3a_3 z^2+4a_4 z^3)}$$

$$= \frac{(1+4a_2 z+9a_3 z^2+16a_4 z^3)}{(1+2a_2 z+3a_3 z^2+4a_4 z^3)}$$

By simple calculations, we get

$$= [1+2a_2 z + z^2 (6a_3 - 4a_2^2)]$$

$$\lambda\left(1 + \frac{z f''(z)}{f'(z)}\right) = \{\lambda + 2a_2 \lambda z + \lambda(6a_3 - 4a_2^2) z^2\}$$

From (1.1), we can write

$$(1-\lambda) f'(z) + \lambda\left(1 + \frac{z f''(z)}{f'(z)}\right) = \{ (1-\lambda) + (1-\lambda)2a_2 z + (1-\lambda) 3a_3 z^2 + \lambda + 2a_2 \lambda z + \lambda(6a_3 - 4a_2^2) z^2 \} \quad (2.1)$$

Using the above lemma and by simple calculations, we get

$$1 + \sin(w(z)) = 1 + \frac{1}{2} c_1 z + \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right) z^2 + \ldots\ldots \quad (2.2)$$

By comparing (2.1) and (2.2), on equating the coefficients of z,

$$(1-\lambda)2a_2 + 2a_2 \lambda = \frac{c_1}{2}$$

We get

$$a_2 = \left(\frac{1}{4(1-\lambda)+4\lambda}\right) c_1 \quad (2.3)$$

Equating the coefficients of z^2 ,

$$(1-\lambda)3a_3 + \lambda(6a_3 - 4a_2^2) = \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right)$$

Substituting the value a_2 in this equation,

$$(1-\lambda)3a_3 + \lambda\left[6a_3 - 4\left(\frac{c_1}{4(1-\lambda)+4\lambda}\right)^2 \right] = \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right)$$

$$(1-\lambda)3a_3 + \lambda\left[6a_3 - \left(\frac{4c_1^2}{16(1-\lambda)^2+16\lambda}\right) \right] = \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right)$$

By simple Calculations,

We get,

$$a_3 = \left(\frac{1}{6(1-\lambda)+12\lambda}\right) \left[c_2 - \frac{c_1^2}{2}\left(1 + \frac{\lambda}{(1-\lambda)^2} - 1\right) \right. \quad (2.4)$$

3.THE FEKETE-SZEGO PROBLEM

THEOREM 3.1. If $f\epsilon\, \mathbb{S}^*_{sin}$ then,

$$|a_3-\mu a_2^2|\le \max\{\ 2,2|\frac{1}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)-\frac{\mu}{16(1-\lambda)^2+16\lambda}-1|$$

PROOF:If $f\epsilon\, \mathbb{S}^*_{sin}$ then $a_3-\mu a_2^2$

Substituting the value a_2 and a_3 , we get

$$a_3-\mu a_2^2=\left(\frac{1}{6(1-\lambda)+12\lambda}\right)\ [\ c_2-\frac{c_1^2}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)\ ]\ -\mu\left(\frac{c_1}{4(1-\lambda)+4\lambda}\right)^2$$

$$=\left(\frac{1}{6(1-\lambda)+12\lambda}\right)\ [\ c_2-\frac{c_1^2}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)\ ]-\mu\left(\frac{c_1^2}{16(1-\lambda)^2+16\lambda}\right)$$

By simple calculations, we get

$$a_3-\mu a_2^2=\left(\frac{1}{6(1-\lambda)+12\lambda}\right)c_2-c_1^2[\ \frac{1}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)-\frac{\mu}{16(1-\lambda)^2+16\lambda}$$

Hence we have,

$$a_3-\mu a_2^2=\left(\frac{1}{6(1-\lambda)+12\lambda}\right)\ [\ c_2-c_1^2\nu \qquad (3.1)$$

Where $\nu=\frac{1}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)-\frac{\mu}{16(1-\lambda)^2+16\lambda}$

By taking modulus on both sides of (3.1),we get the required result.

Hence,

$$|a_3-\mu a_2^2|=|\left(\frac{1}{6(1-\lambda)+12\lambda}\right)\ [\ c_2-c_1^2[\ \frac{1}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)-\frac{\mu}{16(1-\lambda)^2+16\lambda}|$$

Corollary 1.

When $\lambda=1$, then

$$|a_3-\mu a_2^2|\le\frac{1}{12}\ [\ c_2-c_1^2\nu$$

Where,

$$\nu=\frac{1}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)-\frac{\mu}{16(1-\lambda)^2+16\lambda}$$

Corollary 2.

When $\lambda=0$, then

$$|a_3-\mu a_2^2|\le\frac{1}{6}[\ c_2-c_1^2\nu$$

Where,

$$\nu=\frac{1}{2}\left(1+\frac{\lambda}{(1-\lambda)^2}-1\right)-\frac{\mu}{16(1-\lambda)^2+16\lambda}$$

Conclusion:

In this paper, the focus is to venture of investigate a some new subclasses of analytic functions for to describe on the open unit disk D. Additionally determines using the sine functions to the coefficients bounds and classical fekete szego problem.

References:

[1] Ali, R, M,.Lee, S, K,Ravichandran, V., Subramaniam , S 2009, The Fekete-Szego Coefficient functional for transforms of analytic functions, Bull.Iranian Math.Soc., Vol. 35 no.2,pp. 119-142.

[2] Cho N.E., kumar V., kumar S.S., Ravichandran V.,Radius problems for starlike functions associated with the sine function, Bull.Iran.Math.Soc., 2019,45,213-232.

[3] Duren, P. L. 1983, Univalent functions, Grundlehren der mathematischen wissenschaften, Bd. Springer-Velag, New York, Berlin, Heidelberg, Tokyo, pp. 259

[4] Dziok, J & Raina, R.K 2004, Families of analytic functions associated with the Wright generalized hypergeometric function, Demonstratio Math., Vol 37, no. 3, pp. 533-542.

[5] Frasin,B.A 2010, Coefficient in equalities for certain classes of Sakaguchi type functions,Int.J.Nonlinear sei.,Vol 10,no 2 ,ppt.206-211.

[6] Fadipe-Joseph, O. A, Oladipo, A.T & Ezeafulukwe U. A 2013, odified sigmoid function in univalent function theory, J. of Math. Sci. and Engg. Appls, Vol 7, no. V. pp.313-317.

[7] Goodman , A . W. 1983, Univalent Functions ,Volume I and II Mariner publishing company,Tampa , Florida

[8] Keogh, F, R. Merkes,E,P 1969 ,A Coefficient inqualities for certain of analytic functions, Proc. Amer.Math Soc.,Vol.20,pp.8-12.

[9] Ma,W & Minda, D 1994, Aunified treatment of sum special classes of univalent functions, Procedings of the conference on complex analysis.pp.157-169.

[10] Robertson,Ms.S 1936,on the theory of univalent functions , Ann. Of Math,Vol 37 ,no.2 pp.374-408.