ISSN: 0974-5823 Vol. 7 No. 4 April, 2022

International Journal of Mechanical Engineering

SOLVING MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEMS BY STATISTICAL AVERAGING AND NEW STATISTICAL AVERAGING METHOD

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Abstract

In this paper, we proposed a statistical averaging method (Quadratic mean) and a new statistical averaging method (New Quadratic mean) for multi-objective linear programming problem (MOLPP). We use Chandra Sen's technique for converting MOLPP into a single objective function. We have solved some numerical examples to provide the utility of these methods.

Keywords

Multi-objective linear programming problem, Chandra Sen, Quadratic averaging, New Quadratic averaging.

1.Introduction

A Multi-objective linear programming problem is a linear programming problem with more than one objective function considered simultaneously minimized or maximized subject to the common set of constraints. It is the special type of Goal programming problem where we determine target values for each goal. The three components of any optimization problem are the decision variables (activities), the objective (goal) and the constraints (restrictions).

2.Definition

Any set X = { $x_1, x_2, \ldots, x_{n+m}$ } of variables is called a feasible solution to LP problem, if it satisfies the set of constraints and non-negativity restrictions also.

3.A formal specification of Multi Objective optimization problem

Let us Consider a Multi Objective optimization problem with n decision variables and m objective functions

$$\text{Min or Max } y = f(x) = [y_1 \in f_1(x), y_2 \in f_2(x), \ldots, y_k \in f_k(x)] \qquad (1)$$

$$x = [x_1, x_2, \ldots, x_n] \in X$$

$$y = [y_1, y_2, \ldots, y_n] \in Y$$

where

x decision vector

y objective vector

X decision space

Y objective space

Subject to the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \, (\leq, =, \geq) b_i \qquad i = 1,2,\ldots,m \qquad (2)$$

$$x_j \geq 0 \qquad j = 1,2,\ldots,n$$

c_j coefficient represents per unit contribution of decision variable x_j to the value of objective function

a_{ij} technological coefficients represents the amount of resources (say i) consumed per unit of variable (activity), it can be positive/negative/zero

b_i represents the total availability of the i'th resourse, $b_i \geq 0$ for all i

4.MOLPP solving methodologies

4.1. Chandra Sen's technique

The Multi objective function can be solved and be changed into a single objective function by the following formula

$$\text{Max Z} = \sum_{i=1}^{m} \frac{z_i}{|\lambda_i|} - \sum_{i=m+1}^{n} \frac{z_i}{|\lambda_i|} \qquad (3)$$

This can be solved by using simplex method with the same constraints (2).

4.2. Algorithm for Chandra Sen's technique

The optimal solution for MOLPP can be obtain by the following steps:

Step 1: Find the values of each of the individual objective functions that has to be maximized or minimized.

Step 2: Solve the first objective function.

Step 3: Check the feasibility of the obtained solution, if it is feasible proceed further otherwise use the method of dual simplex and remove the infeasibility.

Step 4: Label the optimum value of the first objective function as λ_1.

Step 5: Repeat from step 2, i = 1,2,…,m.

Step 6: Determine Chandra Sen's technique by 3.

Step 7: Optimize the combined objective function (3), under the same constraints (2), by repeating steps 2-4.

5.Quadratic Averaging technique

For making multi-objective functions into a single objective function

$$\text{Max Z} = \sum_{i=1}^{m} \frac{z_i}{\text{Q.M}\,(AA_i)} - \sum_{i=m+1}^{n} \frac{z_i}{\text{Q.M}\,(AL_i)}$$

where

$$\text{A}A_i = |\lambda_i| \quad i = 1,2,\ldots,m$$

$$\text{A}L_i = |\lambda_i| \quad i = 1+m,\ldots,n$$

6.New Quadratic Averaging technique

Let

$$m_1 = \min\{\text{A}A_i\} \text{ where } \text{A}A_i = |\lambda_i|,\ \lambda_i \text{ is the minimum value of } z_i,\ i = 1,2,\ldots,m$$

$$m_2 = \min\{\text{A}L_i\} \text{ where } \text{A}L_i = |\lambda_i|,\ \lambda_i \text{ is the minimum value of } z_i,\ i = m+1,2,\ldots,n$$

$$\text{Q.Av} = \sqrt{\frac{1}{n} \sum_i x_i^2}$$

So

$$\text{Max Z} = \left[\sum_{i=1}^{m} z_i - \sum_{i=m+1}^{n} z_i\right] / \text{Q.Av}$$

7.Used Notations

$$\lambda AA_i = \text{value of objective functions which is to be maximized}$$

$$\lambda AL_i = \text{value of objective functions which is to be minimized}$$

Therefore

$$\text{A}A_i = |\lambda_i| \quad i = 1,2,\ldots,m$$

$$\text{A}L_i = |\lambda_i| \quad i = 1+m,\ldots,n$$

$$\text{SM} = \sum_{i=1}^{m} z_i \qquad \text{SN} = \sum_{i=m+1}^{n} z_i$$

$$m_1 = \min\{\text{A}A_i\} \qquad m_2 = \min\{\text{A}L_i\}$$

$$\text{Max Z} = (\text{SM} - \text{SN}) / \text{Q.Av}$$

8. Numerical Examples

Example (1)

$$\text{Max } z_1 = x_1 + 2x_2$$

$$\text{Max } z_2 = x_1$$

$$\text{Min } z_3 = -2x_1 - 3x_2$$

$$\text{Min } z_4 = -x_2$$

Subject to

$$6x_1 + 8x_2 \leq 48$$

$$x_1 + x_2 \geq 3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Solution

Finding the value of each individual objective function by using simplex method, we obtain

	λ_i	X_i	$AA_i=\lvert\lambda_i\rvert$	$AL_i=\lvert\lambda_i\rvert$
1	10	(4,3)	10	
2	4	(4,3)	4	
3	-17	(4,3)		17
4	-3	(4,3)		3

By Chandra Sen's technique,

$$\text{Max Z} = \sum_{i=1}^{m} \frac{z_i}{|\lambda_i|} - \sum_{i=m+1}^{n} \frac{z_i}{|\lambda_i|}$$

$$\text{Max Z} = \frac{x_1+2x_2}{10} + \frac{x_1}{4} - \left[\frac{-2x_1-3x_2}{-17} + \frac{-x_2}{-3}\right]$$

Thus, we get a single objective function as

$$\text{Max Z} = 0.2323x_1 - 0.3098x_2$$

Subject to the same given constraints

By simplex method, the optimal solution is

$$\text{Max Z} = 0.9292 \qquad x_1 = 4, x_2 = 0$$

Quadratic averaging,

$$\text{Q.M}\,(AA_i) = \sqrt{\frac{(10)^2+(4)^2}{2}} = 7.6157$$

$$\text{Q.M}\,(AL_i) = \sqrt{\frac{(17)^2+(3)^2}{2}} = 12.2065$$

$$\text{Max Z} = \sum_{i=1}^{m} \frac{z_i}{\text{Q.M}\,(AA_i)} - \sum_{i=m+1}^{n} \frac{z_i}{\text{Q.M}\,(AL_i)}$$

$$= \frac{1}{7.6157} \sum z_i - \frac{1}{12.2065} \sum z_i$$

$$= \frac{1}{7.6157} (2x_1 + 2x_2) - \frac{1}{12.2065} (-2x_1 - 4x_2)$$

Thus, we get a single objective function as

$$\text{Max Z} = 0.4264x_1 + 0.5902x_2$$

Subject to the same given constraints

By simplex method, the optimal solution is

$$\text{Max Z} = 3.4762 \qquad x_1 = 4,\ x_2 = 3$$

New Quadratic averaging,

Here a=4, b=3

$$\text{Q.Av} = \sqrt{\frac{(4)^2+(3)^2}{2}} = 3.5355$$

$$\text{Max Z} = \left[\sum_{i=1}^{m} z_i - \sum_{i=m+1}^{n} z_i\right] / \text{Q.Av}$$

$$= \frac{1}{3.5355}\left[4x_1 + 6x_2\right]$$

Thus, we get a single objective function as

$$\text{Max Z} = 1.1313x_1 + 1.6970x_2$$

Subject to the same given constraints

By simplex method, the optimal solution is

$$\text{Max Z} = 9.6162 \qquad x_1 = 4,\ x_2 = 3$$

Example (2)

$$\text{Max } z_1 = x_1$$

$$\text{Max } z_2 = 2 + x_1 + 2x_2$$

$$\text{Max } z_3 = 3 + x_2$$

$$\text{Min } z_4 = -3x_2$$

$$\text{Min } z_5 = -x_1 - 3x_2$$

Subject to

$$2x_1 + 3x_2 \leq 6$$

$$x_1 \leq 4$$

$$x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Solution

Finding the value of each individual objective function by using simplex method,we obtain

	λ_i	X_i	AA_i=|λ_i|	AL_i=|λ_i|
1	2	(2,0)	2	
2	4	(0,1)	4	
3	4	(0,1)	4	
4	-3	(0,1)		3
5	-3	(0,1)		3

By Chandra Sen's technique,

$$\text{Max Z} = \sum_{i=1}^{m} \frac{z_i}{|\lambda_i|} - \sum_{i=m+1}^{n} \frac{z_i}{|\lambda_i|}$$

$$\text{Max Z} = \frac{x_1}{2} + \frac{2+x_1+2x_2}{4} + \frac{3+x_2}{4} - \left[\frac{-3x_2}{3} + \frac{-x_1-3x_2}{3}\right]$$

Thus, we get a single objective function as

$$\text{Max Z} = 1.25 + 1.0833x_1 - 2.75x_2$$

Subject to the same given constraints

By simplex method, the optimal solution is

$$\text{Max Z} = 4 \qquad x_1 = 0, x_2 = 1$$

Quadratic averaging,

$$\text{Q.M}\,(AA_i) = \sqrt{\frac{(2)^2+(4)^2+(4)^2}{3}} = 3.4641$$

$$\text{Q.M}\,(AL_i) = \sqrt{\frac{(3)^2+(3)^2}{2}} = 3$$

$$\text{Max Z} = \sum_{i=1}^{m} \frac{z_i}{\text{Q.M}\,(AA_i)} - \sum_{i=m+1}^{n} \frac{z_i}{\text{Q.M}\,(AL_i)}$$

$$= \frac{1}{3.4641} \sum z_i - \frac{1}{3} \sum z_i$$

$$= \frac{1}{3.4641}(2x_1 + 3x_2 + 5) - \frac{1}{3}(-x_1 - 6x_2)$$

Thus, we get a single objective function as

$$\text{Max Z} = 1.4433 + 0.9106x_1 + 2.8660x_2$$

Subject to the same given constraints

By simplex method, the optimal solution is

$$\text{Max Z} = 4.3093 \qquad x_1 = 0, x_2 = 1$$

New Quadratic averaging,

Here a=4, b=3

$$\text{Q.Av} = \sqrt{\frac{(2)^2+(3)^2}{2}} = 2.5495$$

$$\text{Max Z} = \left[\sum_{i=1}^{m} z_i - \sum_{i=m+1}^{n} z_i\right] / \text{Q.Av}$$

$$= \frac{1}{2.5495}[5 + 3x_1 + 9x_2]$$

Thus, we get a single objective function as

$$\text{Max Z} = 1.9611 + 1.1767x_1 + 3.5301x_2$$

Subject to the same given constraints

By simplex method, the optimal solution is

$$\text{Max Z} = 5.4912 \qquad x_1 = 0, x_2 = 1$$

Comparison between applied techniques

Examples	Chandra Sen's Technique	Quadratic Avg.	New Quadratic Avg.
(1)	Max Z = 0.9292 $x_1 = 4, x_2 = 0$	Max Z = 3.4762 $x_1 = 4, x_2 = 3$	Max Z = 9.6162 $x_1 = 4, x_2 = 3$
(2)	Max Z = 4 $x_1 = 0, x_2 = 1$	Max Z = 4.3093 $x_1 = 0, x_2 = 1$	Max Z = 5.4912 $x_1 = 0, x_2 = 1$

Conclusion

We have developed two techniques using Quadratic mean for solving MOLPP and the results have been compared in the above table. Thus, the new averaging technique gives better results than the averaging technique.

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