SOLVING GAUSS ELIMINATION METHOD USINGTRAPEZOIDAL FUZZYMATRICES AND SOME PROPERTIES OF TRIANGULAR TRAPEZOIDAL MATRICES

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ABSTRACT

In this paper, gives the definition of trapezoidal fuzzy matrices and triangular matrices. sum and scalar multiplication of a triangular trapezoidal fuzzy matrices are proved and solving gauss elimination method using trapezoidalfuzzy matrices are satisfying property of triangular matrices.

Key words and phrases: Trapezoidal fuzzy number, trapezoidal fuzzy matrices, triangular trapezoidal matrices, Gauss elimination method.

1.INTRODUCTION

A fuzzy technology can be used in artificial intelligence computer science, control engineering, expert system and management science and so on. Developed the trapezoidal fuzzy matrices from Triangular fuzzy matrices.A lot of work on fuzzy matrices is to promote the development of fuzzy system for successful real-world applications.

2.PRELIMINARY

Definition 1. *Trapezoidal Fuzzy Number*

A fuzzy number A=(a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is defined as follow:

$$\mu_A(x)=\begin{cases}\frac{x-a}{b-a}, if\ a\le x\le b\\ 1, if\ b\le x\le c\\ \frac{x-d}{c-d}, if\ c\le x\le d\\ 0, if\ elsewhere,\end{cases}$$

Definition 2. *Trapezoidal Fuzzy Matrix*

A trapezoidal fuzzy matrix of order $(m\times n)$ *is defined as A=*$(a_{ij}^{TzL})_{m\times n}$ *where*

(a_{ij})=(*aij*1, *aij*2, *aij*3, *aij*4) is the ij^{th} element of A

Example: $\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$

3. ***Triangular Trapezoidal Fuzzy Matrix***

Definition 3. *Triangular Trapezoidal Fuzzy Matrix*

A square trapezoidal fuzzy matrix A=$\left(a_{ij}^{TzL}\right)$*is called a triangular trapezoidal fuzzy matrix if it either an upper triangular trapezoidal fuzzy matrix or a lower triangular trapezoidal fuzzy matrix.*

Definition 4. *Upper Triangular Trapezoidal Fuzzy Matrix*

A square trapezoidal fuzzy matrix A=$\left(a_{ij}^{TzL}\right)$*is called an upper triangular trapezoidal fuzzy matrix if all the entries below the principal diagonal are 0.*

Definition 5. *Lower Triangular Trapezoidal Matrix*

A square trapezoidal fuzzy matrix A=$\left(a_{ij}^{TzL}\right)$ *is called a lower triangular trapezoidal fuzzy matrix if all the entries above the principal diagonal are 0.*

4. Some properties of Triangular Trapezoidal Fuzzy Matrix

In this section present and proved the properties of Triangular Trapezoidal Fuzzy Matrix

Theorem 1.

Let A and B are twoupper triangulartrapezoidal fuzzy matrix of order n.Then the sum is also an upper triangular trapezoidal fuzzy matrix of order n.

Proof.

Let *A*=$\left(a_{ij}^{TzL}\right)$ and *B*=$\left(b_{ij}^{TzL}\right)$be two upper triangular *trapezoidal fuzzy matrices*

where $\left(a_{ij}^{TzL}\right)$=(*aij*1, *aij*2, *aij*3, *aij*4) and $\left(b_{ij}^{TzL}\right)$=(*bij*1, *bij*2, *bij*3, *bij*4)

since A and B are upper triangular *trapezoidal fuzzy matrices* $\left(a_{ij}^{TzL}\right)$= 0 and $\left(b_{ij}^{TzL}\right)$=0 for all$(i > j)$and *i+2≤j*;j= 1, 2,3, …, *m.*

Let A + B =C. Then $\left(a_{ij}^{TzL}\right)$+ $\left(b_{ij}^{TzL}\right)$=$\left(c_{ij}^{TzL}\right)$ for all*i>j*and *i+2≤j*; *j*=1, 2, 3, ...*m.*

$\left(c_{ij}^{TzL}\right)$ = $\left(a_{ij}^{TzL}\right)$+ $\left(b_{ij}^{TzL}\right)$=0+ 0=0.

Hence C is also an upper triangular *trapezoidal fuzzy matrix* of order n.

Theorem 2

Let A and B are two lower triangulartrapezoidal fuzzy matrix of order n. Then the sum is also a lower triangular trapezoidal fuzzy matrix of order n.

Proof.

Let $A=\left(a_{ij}^{TzL}\right)$ and $B=\left(b_{ij}^{TzL}\right)$be two lower triangular *trapezoidal fuzzy matrices*

where $\left(a_{ij}^{TzL}\right)$= (*aij*1, *aij*2, *aij*3, *aij*4) and $\left(b_{ij}^{TzL}\right)$= (*bij*1, *bij*2, *bij*3, *bij*4)

since A and B are lower triangular *trapezoidal fuzzy matrices* $\left(a_{ij}^{TzL}\right)$= 0 and $\left(b_{ij}^{TzL}\right)$=0 for all $(i < j)$ and $i \geq j + 2; i, j$=1, 2, 3, ...*m.*

Let A + B =C. Then $\left(a_{ij}^{TzL}\right)$+ $\left(b_{ij}^{TzL}\right)$=$\left(c_{ij}^{TzL}\right)$ for all *i*>*j* and$i \geq j + 2; i, j$=1, 2, 3, ...*m.*

$\left(c_{ij}^{TzL}\right)$ = $\left(a_{ij}^{TzL}\right)$+ $\left(b_{ij}^{TzL}\right)$=0+ 0=0

Hence C is also a lower triangular *trapezoidal fuzzy matrix* of order n.

Theorem 3

Let A be an upper triangulartrapezoidal fuzzy matrix of order n. Then the product KA is also an upper triangular trapezoidal fuzzy matrix of order n.

Proof.

Let $A=\left(a_{ij}^{TzL}\right)$ be an upper triangular *trapezoidal fuzzy matrix*

where $\left(a_{ij}^{TzL}\right)$= (*aij*1, *aij*2, *aij*3, *aij*4)

since A is an upper triangular *trapezoidal fuzzy matrix* $\left(a_{ij}^{TzL}\right)$ = 0 and for all $(i > j)$ and$i + 2 \geq j; i, j$=1, 2, 3, ...*m.*

Let K is the scalar and KA=B. Then K $\left(a_{ij}^{TzL}\right)$= $\left(b_{ij}^{TzL}\right)$ for all $i > j$and
$i + 2 \geq j; i, j$=1, 2, 3, ...*m.*

K$\left(a_{ij}^{TzL}\right)$= $\left(b_{ij}^{TzL}\right)$=0

Hence KA is also an upper triangular *trapezoidal fuzzy matrix* of order n.

Theorem 4

Let A be a lower triangulartrapezoidal fuzzy matrix of order n. Then the product KA is also a lower triangular trapezoidal fuzzy matrix of order n.

Proof.

Let *A*=(a_{ij}^{TzL}) be a lower triangular *trapezoidal fuzzy matrix*

where (a_{ij}^{TzL})= (*aij*1, *aij*2, *aij*3, *aij*4)

since A is a lower triangular *trapezoidal fuzzy matrix*(a_{ij}^{TzL}) = 0 and for all $(i < j)$and$i \geq j + 2; i, j$=1, 2, 3, ...*m.*

Let K is the scalar and KA=B. Then K(a_{ij}^{TzL})=$(b_{ij}^{TzL})$for all $i < j$and

$i \geq j + 2; i, j$=1, 2, 3, ...*m.*

K(a_{ij}^{TzL})=(b_{ij}^{TzL})=0

Hence KA is also a lower triangular*trapezoidal fuzzy matrix* of order n.

Theorem 5.

Let A be an upper triangular *trapezoidal fuzzy matrix then the transpose is a lower triangulartrapezoidal fuzzy matrix and vice versa*

Proof:

Let *A*=(a_{ij}^{TzL}) be an upper triangular *trapezoidal fuzzy matrix*

where (a_{ij}^{TzL})= (*aij*1, *aij*2, *aij*3, *aij*4)

since A is an upper triangular *trapezoidal fuzzy matrix* (a_{ij}^{TzL}) = 0 and for all $(i > j)$ and$i + 2 \geq j; i, j$=1, 2, 3, ..., *m.*

Let *B*be the transpose of A. Then Ā=B

(a_{ij}^{TzL})= (b_{ij}^{TzL}) for all $i > j$ and $i + 2 \geq j; i, j$=1, 2, 3, ..., *m.*

(a_{ij}^{TzL})= $0 = (b_{ij}^{TzL})$

That is for all $i < j \ and \ i \geq j + 2; i, j$=1, 2, 3, ...*m.*

(b_{ij}^{TzL})=0

Hence B is also a lower triangular trapezoidal fuzzy matrix.

5. Gauss Elimination

Gauss elimination is also known as row reduction is an algorithm for solving system of linear equations. This method is used to compute the rank of matrix, the determinant of a square matrix, and the inverse of an invertible matrix. Its convert the linear system of equation to upper triangular form from which solution of the equation is determined.

i.Augmented matrix must be written for the system of linear equation.

ii. Transform A to upper triangular form using row operation on *{A/b}* diagonal element may not be zero.

iii. Use back substitution for finding the solution of problem.

6. Numerical Example

consider the following linear system in the form of 4×4 trapezoidal fuzzy matrices.

$x_1 + 2x_2 + x_3 - x_4 = 5$

$3x_1 + 2x_2 + 4x_3 + 4x_4 = 16$

$4x_1 + 4x_2 + 3x_3 + + 4x_4 = 22$

$2x_1 + x_3 + 5x4 = 15.$

This system can be represented by the coefficient matrix A and right-hand side Vector b, as follows

$$A == \begin{bmatrix} 1 & 2 & 1 & -1 \\ 3 & 2 & 4 & 4 \\ 4 & 4 & 3 & 4 \\ 2 & 0 & 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 16 \\ 22 \\ 15 \end{bmatrix}$$

To perform row operation to reduce this system to upper triangular form, we define the augmented matrix.

$$\bar{A} = [Ab] = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 3 & 2 & 4 & 4 & 16 \\ 4 & 4 & 3 & 4 & 22 \\ 2 & 0 & 1 & 5 & 15 \end{bmatrix}$$

We first define $\bar{A}^{(1)}$=Ato be the original augmented matrix. Then, we denote by$\bar{A}^{(2)}$ the result of the first elementary row operation, which entails subtracting 3 times the first row from the second in order to eliminate $x_1$from the second equation:

$$\bar{A}^{(2)} = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 0 & -4 & 1 & 7 & 1 \\ 4 & 4 & 3 & 4 & 22 \\ 2 & 0 & 1 & 5 & 15 \end{bmatrix}$$

Next, we eliminate $x1$ from the third equation by subtracting 4 times the first row from the third

$$\bar{A}^{(3)} = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 0 & -4 & 1 & 7 & 1 \\ 0 & -4 & -1 & 8 & 2 \\ 2 & 0 & 1 & 5 & 15 \end{bmatrix}$$

Then we complete the eliminate of x_1 by the subtracting 2 times the first row from fourth

$$\bar{A}^{(4)} = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 0 & -4 & 1 & 7 & 1 \\ 0 & -4 & -1 & 8 & 2 \\ 0 & -4 & -1 & 7 & 5 \end{bmatrix}$$

We now need to eliminate $x_2$from the third and fourth equations. This is accomplished by subtracting the second row from the third, which yields

$$\bar{A}^{(5)} = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 0 & -4 & 1 & 7 & 1 \\ 0 & 0 & -2 & 1 & 1 \\ 0 & -4 & -1 & 7 & 5 \end{bmatrix}$$

And the fourth, which yields

$$\bar{A}^{(6)} = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 0 & -4 & 1 & 7 & 1 \\ 0 & 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 0 & 4 \end{bmatrix}$$

Finally, we subtract the third row from the fourth to obtain the augmented matrix of an upper triangular system,

$$\bar{A}^{(6)} = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 \\ 0 & -4 & 1 & 7 & 1 \\ 0 & 0 & -2 & 1 & 1 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

Finally, we obtain the upper triangular trapezoidal fuzzy matrix and also satisfied the property of triangular trapezoidal fuzzy matrix using Gauss elimination method also.

Note that in a matrix for such a system. all entries below the main diagonal the entries where the row index is equal to the column index are equal to zero.

That is,

(a_{ij}) for $i > j$.

Now, we can perform back substitution on the corresponding system,

$x_1 + 2x_2 + x_3 - x_4 = 5$

$-4x_2 + x_3 + 7x_4 = 1$

$-2x_3 + x_4 = 1$

$-x_4 = 3.$

to obtain the solution, which yields $x_4 = -3$, $x_3 = -2$, $x_2 = -6$ and $x_1 = 16$

7. Conclusion

In this paper a new method is applied to compute the fully linear system. Here Gauss elimination procedure is used as solver and the validity of the proposed algorithm is examined with numerical example. Many scientific and engineering domains of computation may take the form of linear equation also some properties of triangular Trapezoidal Fuzzy Matrix are proved and the theories of the discussed triangular Trapezoidal Fuzzy Matrix may be utilized in future works.

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