# THEORETICAL ANALYSIS OF MHD ON CHARACTERISTICS OF POROUS SINE CURVED SLIDER BEARINGS WITH NON-NEWTONIAN FLUID

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**Abstract:**

We examined the theoretical study on MHD effect of Porous sine curved circular plate with couple stress lubricant. Reynolds equation is derived from the Stokes couple stress theory. Derivation for distribution of pressure, load carrying capacity, frictional force and coefficient of friction is obtained. The results are discussed with various non-dimensional parameters like Hartmann number, couple stress parameter and permeability parameter. It is noted that the magnetic field and non-Newtonian lubricant which enhance the distribution of pressure, load support, frictional force and coefficient of friction. Further, the influence of permeability parameter reduces the characteristics of pressure, load, frictional force and coefficient friction.

**Keywords:** Magnetohydrodynamics, porous sine slider bearing, curved slider bearing, couple stress.

## 1. Introduction

Magneto hydrodynamics is the interface of conducting fluids with electromagnetic occurrences. From the last five decades of researches that concludes the MHD lubrication is more significant compared to hydrodynamic lubrication. The use of magnetic filed in bearing systems helps the increase of bearing attributes. Several investigators made a research on MHD effects on different bearing systems. Hughes [1] studied the effect of MHD on parallel rotating discs. Finite journal bearing by Kuzma [2] and Lin [3] examined the influence of MHD on slider bearing and also the applications of MHD lubrication has been done several researcher [ 4-6] they observed that the use of MHD lubrication provides the increase of pressure distribution, carrying of load support and reduce the coefficient of friction. MHD slider and diverse shaped bearing was analyzed by prakash [7] and Fathima et al [8] and concludes the carrying of load is more in parabolic slider bearing system also this improves the bearing functions. The application of magnetization leads the pressure distribution in fluid film region this provides increase carrying of load. All these studies are belongs to the Newtonian fluids. In practically this may not much satisfactory assumption. So the researcher found the new desirable lubricant by adding some polymers to the Newtonian fluid known as non-Newtonian fluid.

Several researcher used the concept of couple stress theory to study the influence on all kind of bearings first introduced the concept of couple stress by Stokes [9]. Ramanaiah and priti [10] analyzed the slider bearings with non-Newtonian lubricant, this theory indicates the use of couple stress which increases the carrying of load, frictional force also reduces the coefficient of friction equated than in case of Newtonian. Lin [11] and by Naduvinamani et al [12] presents the influence of couple stress lubricants on finite journal bearings and rotor bearings these study tells that the rise of load support and reduces the friction due to the effect of couple stress. Couple stress on rotor bearings. Nowadays numerous researchers used this theory to study the bearing performance [13-16].

In porous self lubricating bearings is not required continuous lubrication. The fluid is stored from the interconnecting pores of porous bearings. When we applying the load, through these pores bearing system the fluid is coming to the fluid film area to carrying of load support also carrying of load elimination fluid re-impregnation is accomplished. Nowadays the use of Porous bearings has more common because of their longer life work without re-lubrication. Thus these bearings have applications in vehicle industry, home appliances, machinery and so forth. Fourka and Bonis [17] have been studied the effect of porous on gas thrust bearings and different orifice, Short porous journal bearings and secant shaped slider bearings in presence of couple stress lubricants by Naduvinamani et al [18-19]. These two analysis says the porous parameter reduces the carrying of load and improve the life of the bearings. Fathima et al [ 20] by different finite plates with MHD Couple stresses and it seen that in different shapes bearing systems circular shaped bearing system has more load capacity and time. Porous materials has been used many investigators [21-22]. From these above studies we conclude the effect of porous covering permeability reduces the pressure, frictional force and coefficient frictions.

From the literature survey no investigation has been done on Porous sine curved slider bearing with MHD couple stress. So the current article study on porous sine slider bearings with MHD and couple stress lubrication.

## 2. Mathematical Analysis

The geometrical configuration of porous sine curved slider bearing of length $L_1$ with couple stress lubricant and the existence of external magnetic field is presented in Fig. 1. Lower plate is having bearing slides velocity $U$ along the direction of $x$-axis and external magnetic field $B_0$ is applied along y-axis. Under these assumptions the mathematical basic equations of MHD momentum and continuity equations are mentioned below

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\mu\frac{\partial^2 u}{\partial z^2}-\eta\frac{\partial^4 u}{\partial z^4}-\sigma B_0^2 u=\frac{\partial p}{\partial x}+\sigma E_y B_0 \tag{2}$$

$$\frac{\partial p}{\partial z}=0 \tag{3}$$

Let the expression for film thickness of porous sine bearings is

$$h=h_1+(h_0-h_1)\left\{1-Sin\left(\frac{\pi x}{2L}\right)\right\} \tag{4}$$

**Relevant boundary conditions :**

i) At the upper surface $z=H$

$$u=0 \qquad \frac{\partial^2 u}{\partial z^2}=0 \qquad w=0 \tag{5}$$

ii) At the lower surface $z=0$

$$u=U \qquad \frac{\partial^2 u}{\partial z^2}=0 \qquad w=0 \tag{6}$$

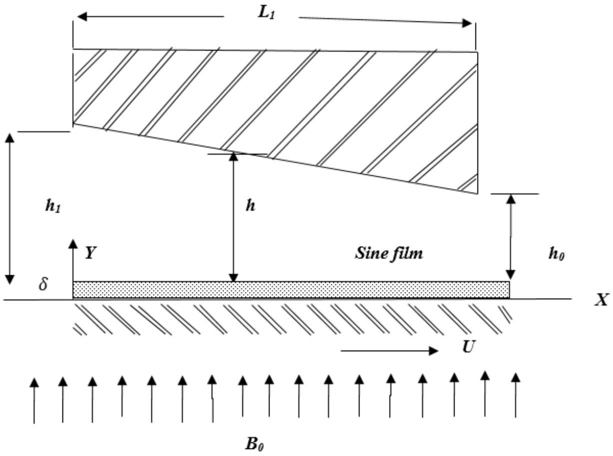


Figure 1: Physical Configuration of Porous Sine Curved Slider Bearing.

For the porous region:

$$\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}=0 \tag{7}$$

Where, the $u^*$ and $v^*$ represents the components of velocity in porous matrix and Biradar and Hanumangowda (2015) given the modified form of the Darcy's law

$$u^* = -\frac{k}{\mu\left(1-\beta+\frac{k\sigma B_0^2}{\mu m}\right)}\left(\frac{\partial p^*}{\partial x}+\sigma E_z B_0\right), \quad (8)$$

$$v^* = -\frac{k}{\mu(1-\beta)}\frac{\partial p^*}{\partial y}, \quad (9)$$

The conditions for $p^*$ in the porous is

$$\left.\frac{\partial p^*}{\partial y}\right|_{y=0} = 0, \text{ (Solid blocking)} \quad (10)$$

$$p^*(x,0) = p(x,0). \text{ (Continuity of pressure at the interface)} \quad (11)$$

Using $u^*$ and $v^*$ in Eq. (7) and integrating w.r.to $y$ across the porous layer thickness and using (10) and (11), we obtain

$$\left.v^*\right|_{y=0} = \frac{k\delta}{\mu c^2}\frac{\partial}{\partial x}\left(\frac{\partial p^*}{\partial x}+\sigma E_z B_0\right), \text{ Where, } c = \left(1-\beta+\frac{k\sigma B_0^2}{\mu m}\right)^{1/2}$$

Solving equation (2) using the Eqn (5), (6) then the velocity equation is obtained

$$u = -\frac{U}{2}\xi_1 - \frac{h_0^2 h}{2l\mu M_0^2}\frac{\partial p}{\partial x}\xi_2 \quad (12)$$

Where,

$$\xi_1 = \xi_{11}-\xi_{12},\ \xi_2 = \xi_{13}-\xi_{14} \quad \text{for } 4M_0^2 l^2/h_0^2 < 1 \quad (13a)$$

$$\xi_1 = \xi_{21}-\xi_{22},\ \xi_2 = \xi_{23}-\xi_{24} \quad \text{for } 4M_0^2 l^2/h_0^2 = 1 \quad (13b)$$

$$\xi_1 = \xi_{31}-\xi_{32},\ \xi_2 = \xi_{33}-\xi_{34} \quad \text{for } 4M_0^2 l^2/h_0^2 > 1 \quad (13c)$$

Where, $M_0$ be an Hartmann number and defined as $M_0 = B_0 h_0 (\sigma/\mu)^{1/2}$ and $\eta/\mu = l^2$

The associated expressions in equations (13a), (13b) and (13c) are mentioned in **Appendix A**.
Substituting (12) in Eqn(1) and integrating also using the conditions of Eqn.(5) and (6) this yields the modified Reynolds equations as

$$\frac{\partial}{\partial x}\left\{f(h,l,M_0)\frac{\partial p}{\partial x}\right\} = 6U\frac{dh}{dx} \quad (14)$$

Where,

$$f(h,l,M_0) = \begin{cases} \dfrac{6h_0^2h^2}{\mu l M_0^2}\left\{\dfrac{A^2-B^2}{\frac{A^2}{B}\tanh\frac{Bh}{2l}-\frac{B^2}{A}\tanh\frac{Ah}{2l}}\left(1+\dfrac{k\delta\sigma B_0^2}{hc^2\mu}\right)-\dfrac{2l}{h}\right\} & \text{for } 4M_0^2l^2/h_0^2 < 1 \\ \dfrac{6h_0^2h^2}{\mu l M_0^2}\left\{\dfrac{2\left(Cosh\left(h/\sqrt{2}l\right)+1\right)}{3\sqrt{2}Sinh\left(h/\sqrt{2}l\right)-h/l}\left(1+\dfrac{k\delta\sigma B_0^2}{hc^2\mu}\right)-\dfrac{2l}{h}\right\} & \text{for } 4M_0^2l^2/h_0^2 = 1 \\ \dfrac{6h_0^2h^2}{\mu l M_0^2}\left\{\dfrac{M_0\left(CosB_1h+CoshA_1h\right)}{h_2\left(A_2SinB_1h+B_2SinhA_1h\right)}\left(1+\dfrac{k\delta\sigma B_0^2}{hc^2\mu}\right)-\dfrac{2l}{h}\right\} & \text{for } 4M_0^2l^2/h_0^2 > 1 \end{cases}$$

$A_2 = (B_1 - A_1 Cot\theta)$, $B_2 = (A_1 + B_1 Cot\theta)$

Introducing non-dimensional quantities

$$x^* = \frac{x}{L},\ P^* = \frac{p^* h_0^2}{\mu UL},\ l^* = \frac{2l}{h_0},\ h^* = \frac{h}{h_0},\ \beta = \frac{H_c}{h_0},\ M_0 = B_0 h_0 \left(\frac{\sigma}{\mu}\right)^{1/2},\ \psi = \frac{k\delta}{h_0^3}$$

$$h^* = h_1^* + (1-h_1^*)\left\{1 - Sin\left(\frac{\pi x^*}{2}\right)\right\},\ C = \left(1 - \beta + \frac{\psi M_0^2}{\delta^* m^*}\right)^{1/2}$$

Substituting above values in (14) we get

$$\frac{\partial}{\partial x^*}\left\{ f(h^*, l^*, M_0, \psi)\frac{\partial P}{\partial x^*}\right\} = 6\frac{dh^*}{dx^*} \tag{15}$$

Where,

$$f^*(h^*, l^*, M_0, \psi) = \begin{cases} \frac{12h^{*2}}{l^* M_0^2}\left\{\frac{\left(A^{*2} - B^{*2}\right)}{\frac{A^{*2}}{B^*}\tanh\frac{B^* h^*}{l^*} - \frac{B^{*2}}{A^*}\tanh\frac{A^* h^*}{l^*}}\left(1 + \frac{\psi M_0^2}{h^* C^2}\right) - \frac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2} < 1 \\ \frac{12h^{*2}}{l^* M_0^2}\left\{\frac{1 + Cosh\left(\sqrt{2}h^*/l^*\right)}{\left(3/\sqrt{2}\right)Sinh\left(\sqrt{2}h^*/l^*\right) - \left(h^*/l^*\right)}\left(1 + \frac{\psi M_0^2}{h^* C^2}\right) - \frac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2} = 1 \\ \frac{12h^{*2}}{l^* M_0^2}\left\{\frac{M_0\left(CosB_1^* h^* + CoshA_1^* h^*\right)}{A_2^* SinB_1^* h^* + B_2^* SinhA_1^* h^*}\left(1 + \frac{\psi M_0^2}{h^* C^2}\right) - \frac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2} > 1 \end{cases}$$

$$A^* = \left\{\frac{1 + \left(1 - l^{*2} M_0^2\right)^{1/2}}{2}\right\}^{1/2},\quad B^* = \left\{\frac{1 - \left(1 - l^{*2} M_0^2\right)^{1/2}}{2}\right\}^{1/2}$$

$$A_1^* = \sqrt{2M_0/l^*}\,Cos\left(\theta^*/2\right),\ B_1^* = \sqrt{2M_0/l^*}\,Sin\left(\theta^*/2\right),\ \theta^* = \tan^{-1}\left(\sqrt{l^{*2} M_0^2 - 1}\right)$$

$$A_2^* = \left(B_1^* - A_1^* Cot\theta^*\right),\ B_2^* = \left(A_1^* + B_1^* Cot\theta^*\right)$$

Integrating both sides of equation (15) w. r. t. $x^*$we get

$$\left\{ f^*(h^*, l^*, M_0, \psi)\frac{\partial p^*}{\partial x^*}\right\} = 6h^* + C_1$$

Let the pressure conditions are

$$p^* = 0\ at\ x^* = 0,1$$

Using above conditions in equation (15) and integrating with respect to $x^*$ then we obtain the pressure distribution equation is mentioned below

We get

$$p^* = 6\int_{x^*=0}^{x^*}\frac{h^*}{f^*(h^*, l^*, M_0, \psi)}dx^* + C_1\int_{x^*=0}^{x^*}\frac{1}{f^*(h^*, l^*, M_0, \psi)}dx^*$$

Where,

$$C_1 = -\frac{6\int_{x^*=0}^{1}\frac{h^*}{f^*(h^*, l^*, M_0,, \psi)}dx^*}{\int_{x^*=0}^{1}\frac{1}{f^*(h^*, l^*, M_0, \psi)}dx^*}$$

$$p^* = 6\int_{x^*=0}^{x^*} \frac{h^*}{f^*(h^*,l^*,M_0,\psi)}dx^* - \frac{6\int_{x^*=0}^{1} \frac{h^*}{f^*(h^*,l^*,M_0,\psi)}dx^*}{\int_{x^*=0}^{1} \frac{1}{f^*(h^*,l^*,M_0,\psi)}dx^*} \int_{x^*=0}^{x^*} \frac{1}{f^*(h^*,l^*,M_0,\psi)}dx^* \qquad (16)$$

The load per unit width is mentioned below

$$w = \int_0^L p dx$$

Let the Equation for Dimensionless Load support

$$W^* = \int_{x^*=0}^{1} p^* dx^* \quad W^* = 6\int_0^1 \int_{x^*=0}^{x^*} \frac{h^*}{f^*(h^*,l^*,M_0,\psi)}dx^* dx^* - \frac{6\int_{x^*=0}^{1} \frac{h^*}{f^*(h^*,l^*,M_0,\psi)}dx^*}{\int_{x^*=0}^{1} \frac{1}{f^*(h^*,l^*,M_0,\psi)}dx^*} \int_0^1 \int_{x^*=0}^{x^*} \frac{1}{f^*(h^*,l^*,M_0,\psi)}dx^* dx^*$$

(17)

The components of stress tensor required for determining frictional force is

$$\tau_{yx} = \mu\frac{\partial u}{\partial y} - \eta\frac{\partial^3 u}{\partial y^3}$$

$$\tau_{yx}\,|_{y=0} = -\frac{\mu l M_0^2 U}{2h_2^2(A^2-B^2)}\left\{\frac{A^2}{B}\coth\frac{Bh}{2l} - \frac{B^2}{A}\coth\frac{Ah}{2l}\right\} - \frac{h}{2}\frac{\partial p}{\partial x}$$

The frictional force at $y = 0$ is given by

$$F = \int_0^1 \left(t_{21}\right)_{y=0} dx \qquad (18)$$

$$F = \int_0^L \left[-\frac{\mu l M_0^2 U}{2h_2^2(A^2-B^2)}\left\{\frac{A^2}{B}\coth\frac{Bh}{2l} - \frac{B^2}{A}\coth\frac{Ah}{2l}\right\} - \frac{h}{2}\frac{\partial p}{\partial x}\right]dx$$

The dimensionless frictional force is

$$F^* = -\frac{Fh_2}{\mu UL} = \int_0^1 \left\{G(h^*,l^*,M_0) + \frac{h^*}{2}\frac{\partial P^*}{\partial x^*}\right\}dx^*$$

Where,

$$G(h^*,l^*,M_0) = \frac{l^* M_0^2}{4(A^{*2}-B^{*2})}\left(\frac{A^{*2}}{B^*}\coth\frac{B^* h^*}{l^*} - \frac{B^{*2}}{A^*}\coth\frac{A^* h^*}{l^*}\right)$$

$$F^* = \int_0^1 G(h^*,l^*,M_0)dx^* + 3\int_0^1 \left\{\frac{h^*}{\xi(h^*,l^*,M_0,\psi)}\right\}dx^* - 3\left\{\frac{\int_{x^*=0}^{1} \frac{h^*}{f^*(h^*,l^*,M_0,\psi)}dx^*}{\int_{x^*=0}^{1} \frac{1}{f^*(h^*,l^*,M_0,\psi)}dx^*}\right\}\int_0^1 \left(\frac{1}{\xi(h^*,l^*,M_0,\psi)}\right)dx^*$$

(19)

$$\xi^*(h^*,l^*,M_0,\psi)=\begin{cases}\dfrac{12h^*}{l^*M_0^2}\left\{\dfrac{\left(A^{*2}-B^{*2}\right)}{\dfrac{A^{*2}}{B^*}\tanh\dfrac{B^*h^*}{l^*}-\dfrac{B^{*2}}{A^*}\tanh\dfrac{A^*h^*}{l^*}}\left(1+\dfrac{\psi M_0^2}{h^*C^2}\right)-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2}<1\\[2ex] \dfrac{12h^*}{l^*M_0^2}\left\{\dfrac{1+Cosh\left(\sqrt{2}h^*/l^*\right)}{\left(3/\sqrt{2}\right)Sinh\left(\sqrt{2}h^*/l^*\right)-\left(h/l^*\right)}\left(1+\dfrac{\psi M_0^2}{h^*C^2}\right)-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2}=1\\[2ex] \dfrac{12h^*}{l^*M_0^2}\left\{\dfrac{M_0\left(CosB_1^*h^*+CoshA_1^*h^*\right)}{A_2^*SinB_1^*h^*+B_2^*SinhA_1^*h^*}\left(1+\dfrac{\psi M_0^2}{h^*C^2}\right)-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2}>1\end{cases}$$

Let the Coefficient of Friction is given by

$$C=\frac{F^*}{W^*} \tag{20}$$

### 3.1 Dimensionless Pressure

Figure 2 depicts the deviation of dimensionless pressure $P^*$ along $x^*$ for distinct Hartmann Number $M_0$ and permeability parameter $\psi$ with fixed values $l^*=0.4$, $y=1.3$, $b=0.01$, $m=0.06$ and $\alpha=0.1$. From this we noticed that there is an increment of pressure for increasing of external magnetic field. Further, noted that the pressure is more significant in the absence of permeability parameter $(\psi=0)$. The deviation of $P^*$ versus $x^*$ for distinct $M_0$ and $l^*$ keeping fix $y=1.3$, $b=0.01$, $m=0.06$, $\alpha=0.1$ and $\psi=0.001$ is shown in figure 3, we found an increase of pressure due to increase of magnetic field. Furthermore, non-dimensional pressure is more Significant in non-Newtonian case compared to Newtonian case.

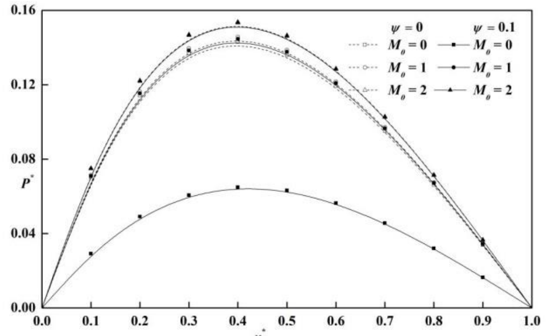

Figure 2. Variation of non-dimensional pressure $P^*$ with $x^*$ for different values of $\psi$ and $M_0$ with $l^*=0.4$, $y=1.3$, $b=0.01$, $m=0.06$ and $\alpha=0.1$

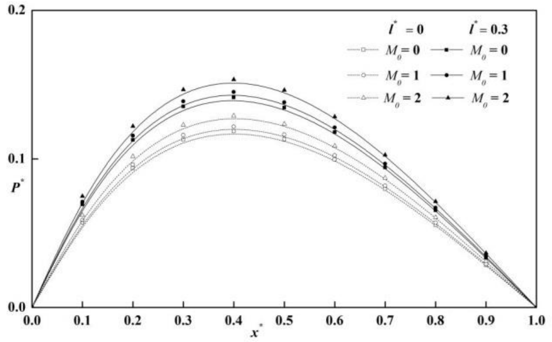

Figure 3. Variation of non-dimensional pressure $P^*$ with $x^*$ different values of $M_0$ and $l^*$ with $\psi=0.001$, $y=1.3$, $b=0.01$, $m=0.06$ and $\alpha=0.1$.

### 3.2 Non-dimensional Load carrying Capacity

The deviation of load support $W^*$ with $h_1^*$ for distinctive magnetic field $M_0$ and permeability parameter $\psi$ with fixed values $l^*=0.4$, $b=0.01$, $m=0.06$ and $\alpha=0.1$ is presented in figure 4 and found that the $W^*$ increases due to increase of $M_0$ than the non-magnetic case. The profile of load support $W^*$ against $h_1^*$ with distinct $M_0$ and $l^*$ keeping fixed $b=0.01$, $m=0.06$, $\alpha=0.1$ and $\psi=0.001$. From this graph, we observed the magnetization enhances the load carrying support $W^*$. Further it seems that the load support is more significant in non-Newtonian case. Further the use of magnetization which generates pressure distribution in the fluid film region this leads rise of carrying load support.

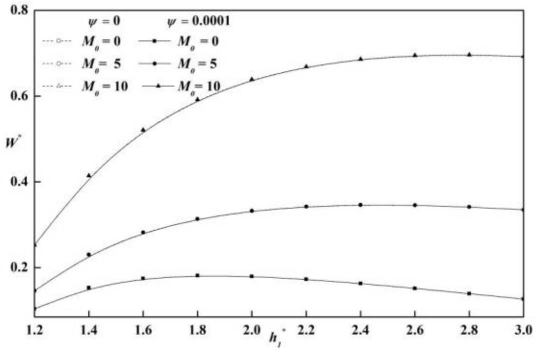


Figure 4. Variation of non-dimensional Load $W^*$ with $h_1^*$ for different values of $\psi$ and $M_0$ with $l^* = 0.4$, $b$=0.01, $m$ = 0.06 and $\alpha$ = 0.1

### 3.3 Non-dimensional Frictional Force

The profile of dimensionless frictional force $F^*$ versus thickness $h_1^*$ for different $M_0$ and permeability parameter $\psi$ with fixed values $l^* = 0.4$, $b = 0.01$, $m = 0.06$ and $\alpha = 0.1$ is shown in figure 6 and seen that the frictional force increases due to increase of magnetization and also observed the frictional force is more in the absence of permeability parameter $(\psi = 0)$. The variation of $F^*$ along thickness $h_1^*$ for distinct $M_0$ and $l^*$ keeping fix the $b = 0.01$, $m = 0.06$, $\alpha = 0.1$ and $\psi = 0.001$ is in figure 7. We noted here the increments of frictional force occurred due to increase of magnetization and also seen the frictional force exceeds in non-Newtonian case.

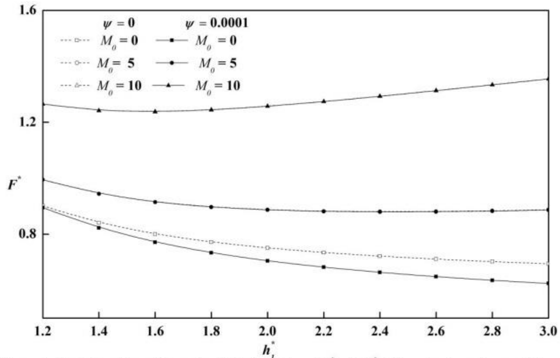


Figure 6. Variation of non-dimensional Frictional force F * with $h_1^*$ different values of $\psi$ and $M_0$ with $l^* = 0.4$, $m$ = 0.06, b = 0.01 and $\alpha$ = 0.1.

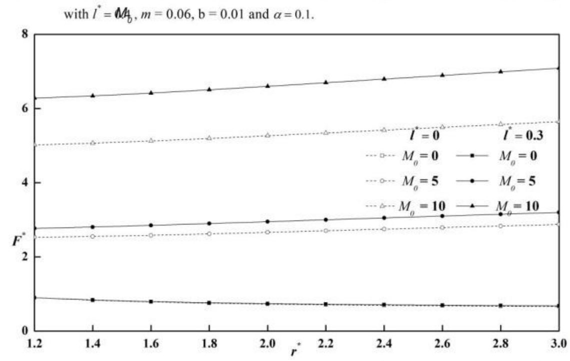


Figure 7. variation of non-dimensional Frictional force F * with $h_1^*$ different values of $M_0$ and $l^*$ with $\psi$ = 0.001, α = 0.1, $m$ = 0.06 and $b$ = 0.01 .

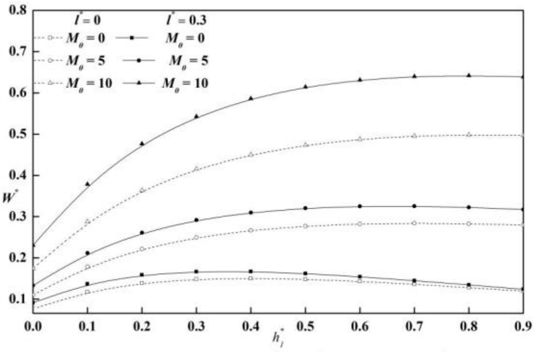


Figure 5. Variation of non-dimensional load $W^*$ with $h_1^*$ different values of $l^*$ and $M_0$ with $\psi$ = 0.001, $b$=0.01, $m$ = 0.06 and $\alpha$ = 0.01

### 3.4 Coefficient of Friction

Deviation of coefficient of friction $C$ versus film thickness $h_1^*$ for distinct Magnetization $M_0$ and permeability parameter $\psi$ with fixed $l^* = 0.4$, $b = 0.01$, $m = 0.06$ and $\alpha = 0.1$ is shown in figure 8, here we found the influence magnetization is reduces $C$ than the presence of $\psi$. The profile of $C$ versus film thickness $h_1^*$ for several Magnetization $M_0$ and couple stress $l^*$ from this figure 9, we have seen that the decrements of $C$ due to increment of magnetization $M_0$ than the Newtonian case.

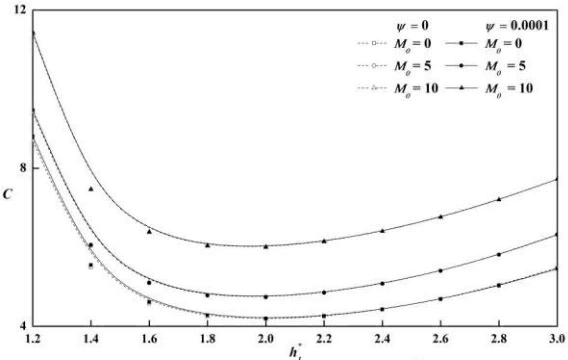


Figure 8. variation of non-dimensional Co-efficient of friction $C$ with $h_1^*$ different values of $\psi$ and $M_o$ with $l^* = 0.4$, $m = 0.06$, $b = 0.01$ and $\alpha = 0.2$.

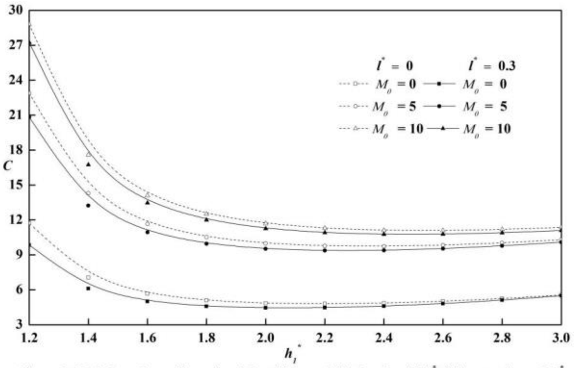


Figure 9. Variation of non-dimensional Co-efficient of friction C with $h_1^*$ different values of $l^*$ and $M_o$ with $m = 0.06$, $b = 0.01$, $\alpha = 0.1$ and $\psi = 0.001$.

## 4. Conclusions

Influence of MHD couple stress on Porous sine curved slider bearings is carried in this article. The following observation is mentioned from the results and discussions.

- Influence of magnetization provides the increase of pressure, load support, frictional force and decreases the coefficient of friction than non-magnetic case.
- We found an increase of pressure, carrying of load, frictional force and reduce of coefficient of friction due to couple stress lubricant.
- We found decrease of pressure, load support, frictional force and coefficient of friction in the absence of permeability parameters.

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**Appendix A:**

$$\xi_{11}=\frac{B^2}{\left(A^2-B^2\right)}\left\{\frac{\sinh\left(Ah/l\right)-\sinh(Ay/l)-\sinh\left(A(h-y)/l\right)}{\sinh\left(Ah/l\right)}\right\} \quad \text{(A1a)}$$

$$\xi_{12}=\frac{A^2}{\left(A^2-B^2\right)}\left\{\frac{\sinh\left(Bh/l\right)-\sinh(By/l)-\sinh\left(B(h-y)/l\right)}{\sinh\left(Bh/l\right)}\right\} \quad \text{(A1b)}$$

$$\xi_{13}=B^2\left\{\frac{\sinh\left(Ah/l\right)-\sinh(Ay/l)+\sinh\left(A(h-y)/l\right)}{\sinh\left(Ah/l\right)\left\{\left(B^2/A\right)\tanh\left(Ah/2l\right)-\left(A^2/B\right)\tanh\left(Bh/2l\right)\right\}}\right\} \quad \text{(A1c)}$$

$$\xi_{14}=A^2\left\{\frac{\sinh\left(Bh/l\right)-\sinh(By/l)+\sinh\left(B(h-y)/l\right)}{\sinh\left(Bh/l\right)\left\{\left(B^2/A\right)\tanh\left(Ah/2l\right)-\left(A^2/B\right)\tanh\left(Bh/2l\right)\right\}}\right\} \quad \text{(A1d)}$$

$$,\ A=\left\{\frac{1+\left(1-4l^2M_0^2/h_1^2\right)^{1/2}}{2}\right\}^{1/2},\quad B=\left\{\frac{1-\left(1-4l^2M_0^2/h_1^2\right)^{1/2}}{2}\right\}^{1/2} \quad \text{(A1e)}$$

$$\xi_{21}=\frac{\sinh\left\{(y-h)/\sqrt{2l}\right\}+\sinh(y/\sqrt{2l})-\sinh\left(h/\sqrt{2l}\right)}{\sinh\left(h/\sqrt{2l}\right)} \quad \text{(A2a)}$$

$$\xi_{22}=\frac{y\cosh\left\{(y-h)/\sqrt{2l}\right\}+y\cosh(y/\sqrt{2l})-h\cosh\left(h/\sqrt{2l}\right)-h}{2\sqrt{2l}\sinh\left(h/\sqrt{2l}\right)} \quad \text{(A2b)}$$

$$\xi_{23}=\frac{y\sinh\left\{(y-h)/\sqrt{2l}\right\}+y\sinh(y/\sqrt{2l})-h\sinh\left(h/\sqrt{2l}\right)}{6l\sinh\left(h/\sqrt{2l}\right)-\sqrt{2}h} \quad \text{(A2c)}$$

$$\xi_{24}=\frac{2\cosh\left\{(y-h)/\sqrt{2l}\right\}+2\cosh(y/\sqrt{2l})-2\cosh\left(h/\sqrt{2l}\right)-2}{3\sqrt{2}\sinh\left(h/\sqrt{2l}\right)-h/l} \quad \text{(A2d)}$$

$$\xi_{31} = \frac{\cosh A_1 y \cos B_1 (y-h) - \cos B_1 y \cosh A_1 (y-h)}{(\cosh A_1 h - \cosh B_1 h)} \quad \text{(A3a)}$$

$$\xi_{32} = \frac{\cot\theta \{\sinh A_1 y \sin B_1 (y-h) - \sin B_1 y \cosh A_1 (y-h)\} - (\cosh A_1 h - \cosh B_1 h)}{(\cosh A_1 h - \cosh B_1 h)} \quad \text{(A3b)}$$

$$\xi_{33} = \frac{\cot\theta \{\sin B_1 y \sinh A_1 (y-h) + \sinh A_1 y \sin B_1 (y-h)\} - (\cosh B_1 h + \cosh A_1 h)}{(B_1 - A_1 \cot\theta)\sin B_1 h + (A_1 + B_1 \cot\theta)\sinh A_1 h} \quad \text{(A3c)}$$

$$\xi_{34} = \frac{\cos B_1 y \cosh A_1 (y-h) + \cosh A_1 y \cos B_1 A_1 (y-h)}{(B_1 - A_1 \cot\theta)\sin B_1 h + (A_1 + B_1 \cot\theta)\sinh A_1 h} \quad \text{(A3d)}$$

$$A_1 = \sqrt{M_0 / lh_0}\cos(\theta/2),\ B_1 = \sqrt{M_0 / lh_0}\sin(\theta/2),\ \theta = \tan^{-1}\left(\sqrt{4l^2 M_0^2 / h_0^2} - 1\right) \quad \text{(A3e)}$$

**Nomenclature**

$B_0$ Magnetic field
$C$ Friction coefficient
$F$ Frictional force
$F^*$ Dimensionless frictional force $(= -Fh_0/\mu UL)$
$h$ Thickness of film
$h_1$ Inlet thickness of film
$h_1^*$ Dimensionless inlet thickness of film
$h^*$ Dimensionless thickness of film $(= h/h_0)$
$l$ Couple stress parameter $(\eta/\mu)^{1/2}$
$l^*$ Dimensionless couple stress parameter $(2l/h_0)$
$L$ Length of the Bearing
$M_0$ Hartmann number $\left(= B_0 h_1 (\sigma/\mu)^{1/2}\right)$
$p$ Pressure in the region of film
$P^*$ Dimensionless pressure ($= ph_0^2/\mu UL$)
$x, y$ Rectangular coordinates
$x^*$ Dimensionless rectangular coordinates $(x^* = x/L)$
$u, v$ Film region Velocity components
$W$ Load carrying capacity
$W^*$ Dimensionless load carrying capacity ($= -Wh_0^2/\mu UL^2$)
$\eta$ Material constant characterizing couple stress
$\mu$ Viscosity coefficient
$\sigma$ Electrical conductivity
$\delta$ Thickness of Porous layer
$\psi$ Permeability parameter $\left(= \frac{k\delta}{h_0^3}\right)$